Inheritance Systems and the Dynamics of State Capacity in Medieval Europe

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Abstract. We analyse how two different inheritance systems might affect the creation of centralised and administratively capable states. We use a multi-country overlapping generations model in which country rulers want to expand the lands they control by military conquests. An administration is required to raise taxes upon which partially fund an army. The characteristics of inheritance rules allow to identify two mechanisms that affect the creation and development of the administrative apparatus: the probability that lands stay under the control of the ruling family, which stimulates investments in state capacity; and marriages between heirs. If rulers prefer theirs lands to belong to their family, inheritance systems that privilege men over women encourage a direct development of the administration. On the contrary, gender egalitarianism generates more marriages between heirs because both genders will be equally represented in the marriage market. Countries originating from a marriage between heirs can benefit from higher taxation income due to an scale effect. To assess the importance of each mechanism, we simulate an economy under each (exogenously given) system. According to our results, inheritance rules that privilege men favour investments in state capacity in the long run compared to rules that give the same probability of inheriting to men and women.

JEL classification codes: D10, H20, K19, J12
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1. Introduction

In Europe, the creation of unified, centralised and administratively capable countries began during the Middle Ages. One of the main purposes that lead to the creation of administrative apparatuses was tax-income generation to fund large and expensive armies deemed necessary to fight against other rulers. The goal was simple: increase landholdings which also brought more prestige and income. A peaceful alternative consists in marrying the heirs of two countries. In this paper, we explore how inheritance rules, which determine the gender of the heir, affect the creation and development of centralised administrations. Despite the focus we place on Europe, the results we obtain are extensible to any society in which power is transmitted by inheritance. For instance, one can think about Imperial China or Japanese shogunates.

European countries mostly applied one inheritance rule. That choice over other possible rules might have obeyed to different criteria and these need not include the development of a centralised country. Therefore, there is scope for some other inheritance system to outperform the chosen one in terms of facilitating and encouraging the creation of a bureaucratic apparatus. In this article we develop a theoretical model that will allow us to assess whether it is the case or not comparing two inheritance rules.

In the model we will build, multiple land-based societies are populated by two classes: a ruling aristocracy and commoners. Each society represents a different political entity controlled by different governing elites. We call these entities counties.

A ruler cares about the future ownership of the lands he controls, preferring them to belong to his own family. Social norms about the role of women made marriages a potential cause of lineage disappearance. Indeed, historically, men owned lands regardless of whom brought them into a family. From a noble's perspective, entitling a daughter as heiress implied exactly this: a transmission of his lands to his son-in-law's family. One can easily forestall such a risk avoiding to entail to daughters. Rulers also desire to expand their landholdings. To succeed, they need an army which has to be funded. The creation and development of an administration allows them to raise higher taxes which can be used to pay for it. The more certain rulers are about whether their family will hold their lands, the more willing to invest they will be. Thus, inheritance rules that privilege men encourage investments in state capacity.

On the other hand, a large country can generate more income. Rulers can use these resources to invest in developing the administration. War, as well as marriages between heirs increase the size of landholdings. In that sense, an egalitarian rule that attributes men and women the same probability of becoming heir(ess) maximises the expected number of marriages and this increases the probability of investing in state development.[1]

We propose to use two inheritance systems that differ in their degree of gender egalitarianism to model how it affects the development of the administration. The results we obtain can be extended to encompass any arbitrary level of gender bias in inheritances. Our first inheritance rule favours men over women. The real-world counterpart we use for it is an slightly refined version of male-cognatic primogeniture. Under the second inheritance rule no gender is favoured, this is, both enjoy exactly the same probabilities of becoming an heir. The real-world equivalent of such a rule is absolute primogeniture. Differences between both systems are clear: male-cognatic primogeniture only allows women to succeed if they have no living brothers and deceased brothers had no surviving legitimate descendants. In contrast, absolute primogeniture simply states that the oldest sibling (of whichever gender) has preference over the remaining younger siblings.

It is important to highlight that throughout this paper inheritance systems are exogenous and we do not allow them to be changed or decided by rulers. Our objective is to assess which inheritance rule allows a higher development of the administration. Thus, rules governing inheritance must remain fixed to allow any comparison. The endogenous choice of one system over the other is left for future research but its determinants might include fear for lineage extinction, pressure from the Church to avoid the creation of large and powerful countries through marriages and different bargaining power across genders, among others.

We should introduce the concept of state capacity, an intrinsic idea of political systems, which encompasses supersedes the mere creation and development of an administration. We borrow its definition from Walder [1995, p. 90-91] and describe state capacity as "the ability of a government to administer its territory effectively". Since its measurement is difficult, the concept is proxied by the capacity of the state to extract resources. Underlying both it is implicitly assumed that an "effective political system should be able to extract resources, aggregate them, and use them for national purposes". Since taxation is an effective way of resource extraction, regions with a developed central administration should also score high in terms of state capacity.

As was previously pointed out, in our model rulers decide to build on state capacity because it allows them to fund larger armies. In that vein, war favours the creation of efficient, bureaucratic states. Some literature already emphasized such relationship. For instance, Tilly

[1]The fact that a entailing with equal probability to men and women maximises the expected number of marriages is easily proven. If one has to marry $N \geq 2$ people out which $pN = M$ are males and $(1-p)N = F$ are females where $M + F = N$, $p \in [0,1]$, the maximum number of marriages is given by $\min\{M, F\} = N \min\{p, (1-p)\}$. Then $\max_{p\in[0,1]} N \min\{p,(1-p)\} \Rightarrow p = 0.5$. It is, a rule that gives each gender equal probabilities to inherit maximises the expected number of marriages.

et al. [1985] argue that the development of centralised administrations is associated with war-making capacity. Bonney [1999, p. 9] states that war was the main reason fiscal systems were established while Besley and Persson [2010] explain that, according to historians, states that could raise taxes had a successful military. A quotation from Don Bernardino de Mendoza also illustrates this same idea: "victory will go to whoever possess the last *escudo*".

The main result of our model sheds light on the relationship between the inheritance system at use and country's state capacity. The latter variable evolves influenced by two forces that are, in part, a result of the model. From one side, one finds the relationship between the probability of winning a war (more precisely, the amount of land to conquer) and the state capacity. Even though in our model wars are largely simplified, the possibility of increasing landholdings motivates the ruling class to invest in the creation of an administration. A more formal and accurate treatment of the relationship between conflict (in general) and state capacity is found in Lagerlöf [2014]. He explores how competition for resources affects long run development. In his set-up, there can be either small, fragmented countries or large ones. He compares the effects of state fragmentation on long run development. When faced with increased competition for resources from other small countries, a fragmented state has an incentive to increase its technology and population. On the contrary, less fragmentation implies fewer but larger countries, which can invest more due to a scale effect. Voth and Gennaioli [2012] analyse how the threat of war and the degree of state capacity evolve. Under the assumption that taxation income is lost when a country faces a military defeat, they show that expensive wars increase centralisation to generate more income to finance an increasingly expensive army. Despite that with higher taxes more resources are at stake, a more capable army compensates that risk. Besley and Persson [2008] study optimal state capacity decisions to make in a country where two groups can hold power and where risk of civil and external war coexist. The group in power optimally allocates taxation income either to defence or transfers it to his group. Civil war dissolves any revenue generated. State capacity investments decrease with the likelihood of a civil war: the existence of such risk makes the return of the investment more volatile. The contrary is true for external wars: higher investment means higher possibilities for defence.

Another consequence of our model is that wars and marriages generate endogenous variations in the number and size of the countries. The relationship between both is studied in Alesina and Spolaore [1997, 2006] who use state capacity and war to model how the number of existing countries, as well as their sizes, arise endogenously.

Finally, the choice of one system of inheritance over the other is out the scope of our model, although historically one observes a change from partitioned inheritances without gender exclusion to primogeniture favouring males and a later shift to the initial situation. Focusing on this last transition, Bertocchi [2006] analyses the joint evolution of economic development and political systems. The latter transitions from feudalism to democracy. The shift occurs when peasants generate enough income to participate in politics and, at the same time, they represent the majority in elections. Capital accumulation also replaces land as the major source of wealth which favours a transition from primogeniture towards partible inheritances.

The next section provides some historical context concerning how marriages were arranged between families and the importance of primogeniture upon which our model is founded. In Section 3 we introduce the optimisation problem faced by rulers which is solved period by period. This static part of the model is intertwined with the result of wars and the marriage market, presented in Section 4. The ensuing dynamics are governed, among other, by the gender attributed to each inheritor. Because it is random, one cannot find a closed expression for the exact dynamics of the mode. Therefore, in Section 5 we simulate the economy for both inheritance systems. One of the conclusions is a tendency towards the unification of different regions into larger ones. Although this phenomenon have been historically observed, one can also find examples of countries that remained fragmented for long periods of time. Consequently, in Section 6 we explore two cases of late unification, Italy and Germany, which run Counter to the main prediction of the model: country unification. Finally, Section 7 concludes.

2. Historical Background

Marriages between inheritors of different countries were common during the Middle Ages and were used to increase the size of the estates, as Habakkuk [1950], Clay [1968] and Girouard [1978] point out. The same idea is also stressed by Holt [1985] who clearly states that heiresses "brought [their] lands to [their] husband[s] and ultimately to [their] children" up to the point that a "maiden unbetrothed [. . .] was a wasted resource". Typically, the estate was bestowed to one heir and his siblings received sums of money and, eventually, some small landholdings. In this paper we will abstract from monetary transfers and small landholdings and we will solely focus on the effects of egalitarianism across genders in inheritances. Two opposite cases can be historically identified: the Salic Law, which forbade women from inheriting; and absolute primogeniture, which treats men and women alike. Between these extreme cases there are intermediate regimes with varying degrees of egalitarianism. Another classification takes into account whether the inheritance is transmitted intact or divided among children. The systems we referred above have in common that land is given to only one child. We group them under the term impartible inheritances meaning that inheritance is transmitted to one and only one heir.[2] Partitioned inheritances are the opposite case. A partitioned inheritance consists in the division of the inheritance between more than one heir. The exact

[2] Primogeniture and ultimogeniture share this characteristic. Under primogeniture the oldest child is the heir while under ultimogeniture it is the youngest.

number of heirs is irrelevant, as long as they are more than one. Moreover, the system can discriminate women.

From the definitions above we can broadly divide inheritance systems in two dimensions: a) whether the inheritance is transmitted intact or divided and b) whether women can inherit or not.

The evolution of inheritance systems in Europe is outlined by Cecil [1895] for some countries. According to his review, the majority of them transitioned to primogeniture from partitioned inheritances. In ancient Britain, siblings of both genders usually shared any inheritance. It is also remarkable that, if any of them was to be favoured, it was the youngest one; who would receive the homestead and some tools, including the plough. Patourel [1971] reaffirms the same idea stating that before the Norman conquest of England (completed in 1072) British landlords usually partitioned inheritances. He also points out one exception to this common rule: the West Saxons, who practised primogeniture since the 9th century. After the Conquest, William separated tenures into two categories: military and civil. Military tenures were subject to indivisibility and thus, entitled to only one inheritor under the premise that divisions weakened the kingdom against enemies. On the contrary, civil tenures followed the tradition of partition. Henry I decreed male primogeniture for military tenures. The distinction between both kinds of tenures was no longer drawn during Edward I's reign and primogeniture was commonly practised.

The French case mimics Britannia. Initially, partitioning a king's inheritance was a common practice. This was the case after the dead of Clovis (511), Clotaire (561), Charles Martel (741) and Pepin the Short (768). Patourel [1971] also notes that early Carolingians usually divided inheritances by designating more than one successor. The division of the Carolingian empire after the Treaty of Verdun (843) is probably the best exponent of such tendency. Despite this partitioning precedent, the Capetians were able to entail to only one successor, usually but not necessary, the oldest one. Hugh Capet, after his election as king, made the title hereditary by securing the election of his son, Robert II. Robert II, in turn, designated his eldest son Hugh as heir, as explained by Patourel [1971]. In Frankish territories, according to Cecil [1895], the North practised primogeniture while the South was actually under the Roman Law so it was possible to divide the inheritance and bestow it on whomever the landowner preferred. Even though, formally, partitioned inheritances were allowed in South Frankish lands, primogeniture was more commonly practised.

Cecil [1895] explains that in Normandy there was some kind of primogeniture and Patourel [1971] narrows this vision stating that primogeniture was not set by law but, rather, tradition. He also states that the countries around Normandy passed "the inheritance and acquisitions together and intact to one heir".

Germany makes the case for a completely different evolution of primogeniture. Cecil [1895] states that, in Teutonic Germany, property could be divided among different heirs but it was usually not. The Goth custom of impartible inheritance, named "geschlossene Güter", recalls primogeniture, although it required only not to divide lands and established nothing regarding who should receive them. Nonetheless, similarly to Britain, "lands held by military service should descent to the eldest son". The Emperor Barbarossa issued a decree to homogenise landholdings which declared honorary fiefs indivisible. Non honorary fiefs soon followed. Indivisibility was accorded by Emperor Frederick II in 1232. Although not stipulated, usually the eldest son was named heir. Indivisibility did not last long. As soon as 1242 German princes were willing to divide their states to exercise their authority within the lands they controlled. The Emperor was not in a position to avoid the reappearance of partitioned inheritances, nor was he inclined to do so since this practice reduced the power of princes. The partition scheme had its culminating moment during the 13th and 14th centuries with the split of many principalities.[3] This "reckless division" created more chances for a war to occur and also supposed a barrier to prosperity. During the Thirty Years' War,s primogeniture rose again, coinciding with Louis XV magnificence. German princes promptly emulated the identification between the state and its ruler and primogeniture aided in that sense.

The difficulty of women to inherit, as stated in Herlihy [1962], was a novelty of the Salic Law which forbid women from receiving any land; before it, inheritances followed Roman Law, which established equality between men and women. The requirements of the Salic Law were softened after the edict of King Chilperic (561-84) which allowed daughters to inherit if they had no male siblings. That is, the edict established a system similar to male-cognatic primogeniture. Drew [1991] provides a translation of the edict and, under Capitulary IV, section CVII, we can read, "if a man had neighbours but after his death sons and daughters remained, as long as there were sons they should have the land just as the Salic Law provides. And if the sons are already dead then a daughter may receive the land just as the sons would have done had they lived". Spring [1997] and Mate [1999] report that under the Norman Law a daughter would be an heiress if she had no brother but, in the case they were more than one daughter, they equally shared the inheritance. With the imposition of male preference over female these countries were able keep the integrity (except for the only-daughters case) of their lands.

Benefits of primogeniture overtook the risk of internal riots between heirs and siblings. Any internal instability between siblings was partially Counteracted sending young brothers to pilgrimage, in search of an heiress or entering the monastic life (Duby [1983]). The designation of a unique inheritor increased the security of fiefs against external attacks, as Brenner [1985]

[3]For instance, Saxony was divided between Saxe-Weimar, Saxe-Eisenach, Saxe-Gotha, Saxe-Meiningen, Saxe-Coburg, Saxe-Römhild, Saxe-Eisenberg, Saxe-Saalfeld, Saxe-Hildburhausen and others. Similarly, Bavaria was split into Munich, Ingolstadt, Landshut and Palatinate.

explains. Todd [2011] argues that impartible inheritances suited better empires based on military conquests. In a chapter about the economic effects of primogeniture Kenny and Laurence [1878] claim that the division of demesnes harmed economic development. They illustrate that idea comparing Italy and England. For Luchaire [1892, p. 239, 240], Heads of State had an incentive to diminish partitioning practices to ensure the continuity of dynasties. He noticed that all legislations, explicitly or implicitly, establish or extend the majorat. By the same token, males are preferred over females to keep counties or earls intact.

Despite the arguments in favour of primogeniture, history provides cases of partitioned inheritances. As mentioned before, the division of the Carolingian empire is one of them, as well as the successive divisions that took place in Germany during the 13th and 14th centuries. Evergates [2011] gives some examples that occurred in the province of Champagne between 1100 and 1300. Barlow [1983] explains the process that culminated in the division of William the Conqueror's (1087) possessions between his two eldest sons: Robert received Normandy and William obtained England.

3. The Model

The model we present in this section is based on the overlapping generations framework. Let $t = 0, 1, 2, \ldots,$ represent time, which evolves discretely. Consider a (large) region of land which, at period $t = 0$, is divided among a certain number of smaller subregions. We call each of these subregions a *county.* For each period of time, let $i = a, b, \ldots, I_t$ where $a < b < \ldots < I_t \in \mathbb{N}$ be the label attached to each county.[4]. Then, $\mathcal{L}_t = \{a, b, \ldots, I_t\}$represents the entire region and counties are a partition of the set $\mathcal{L}_t$. We denote by $x_t^i > 0$ the surface of county $i \in \mathcal{L}_t$. We can normalise the total surface of the region $\mathcal{L}_t$ to be equal to one which implies that $\sum_{i \in \mathcal{L}_t} x_t^i = 1, \forall t \geq 0$. counties are populated by agents who live for two periods and who are divided between two classes: *Counts* and *commoners.* Mobility from one class to the other is in general not possible with one exception we discuss in Sub-section 4.2.3 and we assume classes to be an hereditary attribute. According to Bloch [1931, p. 113] and Blum [1961, p. 7], we also assume that commoners are attached to their Count and when the lord of a region changes, they swear an oath to the new Count. This implies that commoners are actually attached to their region of birth, representing another hereditary trait.

During their first period of life we suppose that both classes (Counts and commoners) are economically idle and they do not make any economic decision. We denote the total number of adult commoners in county i at time t by $n_t^i > 0$ and we assume that half of them are

[4]We should remark two aspects about this notation. Firstly, the number of counties, I_t, can change between t and $t + 1$. Secondly, we do *not* use $i = 1, 2, \ldots, I_t$ because some of these counties can disappear between two periods of time, leaving blanks in the list.

males and the remaining half are females. All commoners supply inelastically one unit of labour and a male-female couple forms a household.

At each period of time, a homogeneous final good is produced using labour and land in each county. Production Y_t^i takes place under the following Cobb-Douglas production function: $Y_t^i = {n_t^i}^{\alpha} \, {x_t^i}^{1-\alpha}$.

We neglect any positive feedback of state capacity on total production. Besley and Persson [2010] propose that state capacity represents a positive externality on total production through market-supporting policies or transportation facilities. The introduction of any mechanism such as these would give an advantage to the countries that are able to invest in state capacity by rewarding them with an increased income.

Total production is divided according to some rule between Counts and commoners. Different arrangements between landlords (Counts) and peasants on how to share the production are identified in Slicher van Bath et al. [1966]. Keeping aside considerations regarding the length of the tenancy, production could be divided using a lease or sharing the crops. Duby [1962] and Volokh [2009] find that share-cropping was more common in continental Europe and that leasing was the preferred choice in England. We assume that Counts and commoners divide production using the share-cropping system. Moreover, we impose shares to be constant and equal over time and across counties.[5] According to this observation, let $\psi \in (0,1)$ be the share of crops the Count takes for himself while the remaining $1-\psi$ goes to peasants. Alternatively, ψ can represent the lands the Count reserves for himself which are freely worked by peasants.[6] Furthermore, Counts can raise some taxes on the part that corresponds to commoners. We assume that the level of taxation is directly related to state capacity and it increases with investments made during period t. Specifically, we assume that taxes are given by $\tau_t^i = \frac{A_t^i + g_t^i}{1+\left(A_t^i+g_t^i\right)\kappa}$ and we write the Count's income as:

$$Z_t^i \;=\; \psi Y_t^i + (1-\psi)\, Y_t^i \frac{A_t^i + g_t^i}{1 + (A_t^i + g_t^i)\,\kappa},$$

where $g_t^i \geq 0$ represents the investment made during period t and $A_t^i \geq 0$ measures the accumulated investments in state capacity $\left(A_T^i = A_0^i + \sum_{t=0}^{T-1} g_t^i\right)$. We suppose that state capacity does not depreciate over time. Notice that if A_t^i does not depreciate, $g_t^i \geq 0$ imply that taxes can only increase. Finally, $\kappa > 1$ represents the reciprocal of the maximum tax burden that can be achieved under an almighty state, that is, when a county's state capacity tends to infinity. For instance, $\kappa = 3$ means that Counts can, at most, appropriate $1/3$ of the commoners' income. Without this maximum level of taxation, as Counts invest in state

[5] Slicher van Bath et al. [1966] explain that, in fact, shares varied according to land fertility. In more fertile lands Counts imposed a larger burden to peasants which could be as high as $1/2$, while in less fertile lands the share to be given to Counts was $1/3$.

[6] Working the landlord lands was viewed as a payment for the use of the lands, known as *corvée*.

capacity, commoners' income would decrease as the tax burden increases and, eventually, commoners could have no income.

We can write commoners' income as

$$S_t^i = (1-\psi)\, Y_t^i \left(1 - \frac{A_t^i + g_t^i}{1 + (A_t^i + g_t^i)\,\kappa}\right),$$

which is the remaining part out of total production and after taxes. Notice that this quantity represents the total amount going to commoners which should be divided between them. We assume that S_t^i is equally shared between all households.

3.1. **Counts.** As described in the previous section, all Counts are assumed to be adults (in their second period of life)[7] when they have to make economically relevant decisions. Furthermore, we assume that all Counts exhibit exactly the same preferences which can be represented by the following utility function:

$$\mathcal{U}_t^i = \ln\left(c_t^i\right) + \pi\eta \ln\left(x_{t_c}^i\right), \tag{3.1}$$

where $c_t^i > 0$ represents consumption during a Count's adulthood and $x_{t_c}^i \in (0,1)$ represents the surface of the county after the war process (but before marriages). These lands will be inherited by Count's heir(s). As is clear from the utility formulation, Counts care about their consumption possibilities and about the future size of their county[8] which is discounted using two different discount factors: $\pi \in (0,1)$ represents the psychological discount rate and $\eta \in (0,1)$ represents the probability that, at period $t+1$, the county still belongs to the same family. Consequently, with probability $1-\eta$ the county becomes the property of a different family from which the current Count derives no utility.

Recalling that total area is constant (and normalised to 1) if any of the counties increase their size $\left(x_{t_c}^i > x_t^i, i \in \mathcal{L}_t\right)$ it must be at the expense of some other counties whose territory shrink. All Counts have exactly the same preferences which, aside from other considerations, include a desire to increase the amount of land each possesses. We model county size changes through a war process[9] such that, at every period of time, Counts battle all-against-all.[10] The outcome of the battle process depends only on two factors: how capable states are and the number of weapons in use. Following Skaperdas [1996], for any county i, post-war county size is described by:

[7]This assumption is in contradiction with Russell and Cipolla [1971] who find that half of the sons inherited estates before reaching the adulthood. The assumption we have chosen can be justified if, instead of considering periods of life delimited by age, periods are given by the activities carried. Under this variation, all Counts will effectively be in their second period of life.

[8]A description of the possibilities a large county entitled is presented in Girouard [1978]. These included political influence and prestige, among others.

[9]See for instance Kaminsky [2002].

[10]To accommodate this battle process which does not take into account the distance to the battle field we can think of $\mathcal{L}_t$ as a doughnut with counties on its surface and battles taking place in the central hole.

$$x_{t_c}^i = \frac{\left(1 + A_t^i + g_t^i\right) {b_t^i}^\phi}{\sum_{i \in \mathcal{L}_t} \left(1 + A_t^i + g_t^i\right) {b_t^i}^\phi}, \forall i \in \mathcal{L}_t, \tag{3.2}$$

where $b_t^i > 0$ represents the amount of weapons bought by the Count. The parameter $\phi > 0$ measures the relative importance of human resources in the war process. We assume that war during European Middle Ages was a human intensive process and, consequently, we impose $\phi > 1$. Following Bean [1973], we do not include commoners in the war outcome function since they had an insignificant role. To simplify the problem at hand, we assume that Counts ignore the externality caused by a unilateral increase in the number of weapons or in state capacity, that is, Counts take the denominator of (3.2) as given.[11] We can then substitute (3.2) in the objective function (3.1) and obtain

$$\mathcal{U}^i = \ln\left(c_t^i\right) + \gamma \ln\left(\left(1 + A_t^i + g_t^i\right) {b_t^i}^\phi\right), \forall i \in \mathcal{L}_t,$$

where $\gamma \in (0, 1) \equiv \pi\eta$ and we use the fact that $\sum_{i \in \mathcal{L}_t} \left(1 + A_t^i + g_t^i\right) {b_t^i}^\phi$ is taken as a constant by all Counts.

A Count's revenue generation has two origins: one from crops he reserves for himself and a second one from taxation of his subjects. The budget constraint a Count faces is given by

$$Z_t^i = \psi Y_t^i + (1 - \psi) Y_t^i \frac{A_t^i + g_t^i}{1 + \left(A_t^i + g_t^i\right)\kappa} = c_t^i + p_b b_t^i + p_g g_t^i,$$

where $p_b > 0$ represents the costs associated to buy a weapon. The parameter $p_g > 0$ stands for the cost of increasing county's state capacity. Notice that there are no payments to maintain a certain level of state capacity once it is reached, as would be the case if, for instance, officials were paid each period. Nonetheless, if we suppose they received as a payment a constant fraction of the total taxes they collected our results would not qualitatively change[12].

[11] This assumption greatly simplifies the model and allows us to derive analytical results at the expense of an inconsistent behaviour when there is only one county. Clearly, if only one county exists it faces no risk of losing any territory due to an attack and its optimal number of soldiers is 0. Notwithstanding, with the assumption that the externality is ignored all counties will always have a positive number of soldiers and, if they can afford it, they will also invest in state capacity.

[12] In that case Count's income would read $Z_t^i = \psi Y_t^i + (1 - \psi)(1 - \zeta) Y_t^i \frac{A_t^i + g_t^i}{1 + \left(A_t^i + g_t^i\right)\kappa}$ where $\zeta \in (0, 1)$ represents the part of taxes officials keep as payment.

Therefore, the optimisation problem from the point of view of county i is:

$$\max_{c_t^i, b_t^i, g_t^i} \quad \ln\left(c_t^i\right) + \gamma \ln\left(\left(1 + A_t^i + g_t^i\right) {b_t^i}^{\phi}\right), \tag{3.3}$$

s.t.

$$\begin{aligned} Z_t^i &= \psi Y_t^i + (1-\psi) Y_t^i \frac{A_t^i + g_t^i}{1 + \left(A_t^i + g_t^i\right)\kappa}, \qquad (3.4)\\ Z_t^i &= c_t^i + p_b b_t^i + p_g g_t^i, \\ c_t^i &> 0,\ b_t^i > 0,\ g_t^i \geq 0. \end{aligned}$$

The following Proposition characterises the solution to the problem above. It takes the form of a system of implicit equations when it is optimal to invest in state capacity while, when optimal investment is equal to 0, the solution for b_t^i is a simple equation.

Proposition 1. *The optimal decision for all Counts is given by the unique triplet* (c_t^i, b_t^i, g_t^i) *such that:*

$$b_t^i = B\left(g_t^i\right) = \begin{cases} \frac{\left(p_g\left(1+\left(A_t^i+g_t^i\right)\kappa\right)^2 - Y_t^i(1-\psi)\right)\phi\left(1+A_t^i+g_t^i\right)}{\left(1+\left(A_t^i+g_t^i\right)\kappa\right)^2 p_b} & if\ g_t^i > 0 \\ \frac{Y_t^i\gamma\phi\left(A_t^i+\psi+A_t^i(\kappa-1)\psi\right)}{p_b\left(1+A_t^i\kappa\right)(1+\gamma\phi)} & if\ g_t^i \leq 0 \end{cases} \tag{3.5}$$

$$g_t^i = G\left(g_t^i\right) = \max\left\{0, g_t^i \middle| G_1\left(g_t^i\right) = 0\right\} \tag{3.6}$$

$$c_t^i = C\left(b_t^i, g_t^i\right) = Z_t^i - p_b b_t^i - p_g g_t^i, \tag{3.7}$$

where $G_1\left(g_t^i\right) = \frac{\gamma}{1+A_t^i+g_t^i} + \frac{\left[p_g\left(1+\left(A_t^i+g_t^i\right)\kappa\right) - Y_t^i(1-\psi)\right][1+\gamma\phi]}{\left(1+\left(A_t^i+g_t^i\right)\kappa\right)\left[g_t^i\left(p_g\left(1+\left(A_t^i+g_t^i\right)\kappa\right) - Y_t^i\right) - A_t^i Y_t^i + Y_t^i\psi\left(A_t^i+g_t^i-1-\left(A_t^i+g_t^i\right)\kappa\right)\right]}$.

Proof. See Appendix A

□

Proposition 2. *Optimal number of weapons,* b_t^i, *is an increasing function of* Y_t^i, *total production, and* A_t^i, *current level of state capacity. Optimal investment in state capacity is an increasing function of* Y_t^i *and decreasing function of* A_t^i.

Proof. See Appendix B

□

Proposition 2 has an important consequence. Army size is increasing with Y_t^i and A_t^i. Moreover, g_t^i also increases with Y_t^i. However, despite the fact that optimal investment in state capacity decreases with its accumulated level, for $A_t^j > A_t^i$ we have $A_t^j + g_t^j > A_t^i + g_t^i$, see Appendix B. This implies that, everything else being equal, a county with both higher resources and a more developed state will be able to capture more lands from its opponents since war outcome is positively associated with b_t^i and $1 + A_t^i + g_t^i$. Therefore, such a county will tend to increase its size creating an empire. Recalling the production function $Y_t^i = {n_t^i}^{\alpha} {x_t^i}^{1-\alpha}$ it is clear that the effects of an increase in county size or population are the equivalent to increases in Count's wealth.

Finally, the following Proposition presents some comparative statistics with respect to some parameters.

Proposition 3. *Optimal values for b_t^i, the optimal number of weapons, are non-decreasing with the discount rate γ, with the relative importance of men in the war process ϕ, with the fraction of income Counts take for themselves ψ and decreasing with weapon cost p_b. Optimal values for g_t^i, the amount invested in state capacity, are non-decreasing with γ, ψ, independent of p_b and decreasing with p_g, ϕ and κ.*

Proof. Implicit derivation of b_t^i and g_t^i with respect to the variables of interest gives our result. □

The economic intuition behind Proposition 3 is clear. Optimal number of weapons decreases with its own price, as we would expect. The positive relationship with γ and ϕ is also clear. If a Count cares more about the size of the state to be given to his heir, he will optimally allocate more resources to increase it. On the other hand, if weapons become more important in war outcome, the optimal response is to purchase more to take advantage of the additional value they provide in the battle process. Finally, a higher ψ means that Counts own more resources and depend less on taxation income. Since weapons are a normal good, an increase in income should raise their demand. We can reason similarly for the case of g_t^i. The negative relationship between it and ϕ follows from the substitutability between weapons and state capacity. We should also comment on the negative relationship between by g_t^i and κ. An increase in κ means that, everything else being equal, taxation revenue decreases. Therefore, investing in officials to collect taxes is less attractive and optimal investment decreases.

3.2. **Commoner households.** In order to simplify the model, we assume that commoners are passive agents who reproduce at a constant rate which is equal in all counties:

$$\pi_n^i \equiv \frac{n_{t+1}^i}{n_t^i} - 1 \equiv \pi_n \, \forall i \in \mathcal{L}_t. \tag{3.8}$$

4. Timing, Marriages and Dynamics

4.1. **Timing.** The evolution of the economic system depends on the timing of events. The decision to invest in state capacity depends crucially on the uncertainty about the gender of the inheritor. Similarly, war should take place before marriages are decided to avoid the formation of alliances through marriage arrangements. Although this is assumption might not seem completely realistic, one can think that wars had a random, non foreseeable component. Moreover, high mortality rates implied that even if such marriages were celebrated, their length was uncertain. Therefore, no one could be sure whether such an alliance would last until the time a war began. Finally, family bonds *per se* do not assure a peaceful relationship between parts.[13]

We propose the following phases that take into account these considerations:

(1) Counts' decision: Counts decide c_t^i, b_t^i and g_t^i.
(2) Production and fertility: production takes place and commoners reproduce.
(3) War: war takes place between all counties. The outcome of the war process is a new distribution of land and population which depends on the effort put into war by each county as described in section 3.1
(4) Post-war:
    (a) Counts have children, the number of children each Count has is equal for all Counts and exogenously given.
    (b) Counts receive the amount of land they conquered and commoners are proportionally divided between counties, according to their size. Counties that are too small disappear.
    (c) Counties can be randomly divided into smaller, new counties to be received by Count's inheritors.[14]
    (d) Marriage decisions are made based on the gender of the Counts' children and the current law.

This sequence of events does not allow Counts to subscribe treaties based on future marriages. To put it in other terms, since no Count can be sure that his county will be inherited by a son or a daughter, non-aggression treaties or alliances cannot take place when a future marriage is promised. This avoids the creation of large dynastic unions prior to the war phase that would otherwise influence the outcome of war.

[13]For instance, Hicks [1998] explores some conflicts that arose within the Neville family in England; McLaughlin [1990, p. 199] cites the case of Richilde of Hainaut who fought her brother-in-law and Arnould III of Guines who battled against her own son; and Miller [1983, footnote 16] refers to the case of Styr Þorgrímsson who fought against his son-in-law (and afterwards changed sides).

[14]See section 4.2.2

Notice that the 4th phase introduces changes in county size and population. We should keep track of them in order to compute the dynamics of the model. For this reason we introduce the following notation to deal with these changes: $t \geq 0$, as before, indicates the current time period but, for sub-periods within phase 4, we denote each of them as t_a, t_b, t_c and t_d, respectively. Even though we introduce the notation for sub-period (a), the unique relevant sub-periods in terms of changes in variables are (b), (c) and (d). Finally, t_a, t_b, t_c and t_d indicate the state of the world at the *beginning* of each sub-period.

4.2. **Dynamics.** Until now, we have focused on the static part of the model. In this section we describe its dynamic aspects which are greatly influenced by the outcome from the marriage market. The following subsections will introduce different mechanisms that impact the exact evolution of the economy.

4.2.1. *County disappearance.* It is natural to think that counties smaller than a certain size are either completely conquered or they are too small to function so they get integrated in some bigger county. As said before, we suppose that these counties disappear. Let $\mathcal{D}_t = \left\{ i | x^i_{t_b} < \delta \right\}$ be the set that represents these counties, where $\delta > 0$ denotes the minimum size a county must have before disappearing. When this occurs, we equally distribute its land and population among the remaining counties, it is:

$$x^i_{t_b} = x^i_{t_b} + \frac{\sum_{i \in \mathcal{D}_t} x^i_t}{|\mathcal{L}_t| - |\mathcal{D}_t|} \forall i \mathcal{L}_t \setminus \mathcal{D}_t$$

and

$$n^i_{t_b} = n^i_{t_b} + \frac{\sum_{i \in \mathcal{D}_t} n^i_t}{|\mathcal{L}_t| - |\mathcal{D}_t|} \forall i \mathcal{L}_t \setminus \mathcal{D}_t.$$

[15] We should also update the list of counties removing counties in $\mathcal{D}_t$, it is, $\mathcal{L}_{t_b} = \mathcal{L}_t \setminus D_t$.

4.2.2. *Inheritance.* We base the inheritance system on the historic characteristics presented in 5. Let $\Phi > 0$ represent the number of children each Count has.[16] We assume it to be exogenous and independent of any variable.[17] Given Φ, it is possible to compute the probability of observing an heiress under each regime. Under male-cognatic primogeniture, this event takes place only when all Φ children are daughters. The probability of such event is equal to $(1 - 0.5)^{\Phi}$. Consequently, with probability $(1 - 0.5)^{\Phi} = 0.5^{\Phi}$ the county

[15] We use $|\cdot|$ to indicate set cardinality.

[16] Actually not all children survived their father since infant mortality was high, see Russell [1965]. Therefore, Φ actually represents the number of children who survive their father but to ease the exposition we will refer to Φ simply as the number of children.

[17] Notice that, under absolute primogeniture, the number of children has no effect on the effective disCount rate γ. In fact, the only variable that matters is the gender of the first born, which is independent of the number of children. For male-cognatic primogeniture the number of children is an important variable because it decreases the likelihood of having an heiress. As long as the number of children is independent of income, our assumption is innocuous.

will be inherited by a female and her family will lose its control. With the complementary probability, $1-0.5^{\Phi}$, a male will inherit and the county will remain under the control of his family. Under absolute primogeniture, if the first child is a female the county will have an heiress. Thus, the county is lost with probability 0.5 (from the point of view of the current Count). Recalling that $\gamma \equiv \pi\eta$ where π is a constant psychological discount rate and η represents the probability of the county being kept, let γ^M represent the effective discount rate under male-cognatic primogeniture and, similarly, let γ^A be the effective discount rate under absolute primogeniture. It is trivial to show that $\Phi > 1$ implies $\gamma^M > \gamma^A$.[18] Finally, to take into account the fact that some counties could be divided between Φ heirs, let $\omega \in [0,1]$ represent the exogenous probability that the county is divided among its inheritors. We abstract from historic records and we simply assume that when the county is bequeathed to more than one heir, lands are divided proportionately between them which implies the creation of $\Phi - 1$ new counties of size $\frac{1}{\Phi}x_t^i$.

We should comment about the random divisions of counties. If we allowed Counts to optimally decide whether to partition their inheritance or not, under male-cognatic primogeniture the optimal choice is never to do so. To see this, rewrite the optimisation problem allowing $\omega = \{0,1\}$ to be chosen. Then, the function to optimise is given by

$$\log\left(c_t^i\right) + \pi \log\left(x_{t_b}^i\right)\left(\eta^M\left(1-\omega\right) + \omega\left(1-0.5^{\Phi}\right)\right) + \pi\omega\left(1-0.5^{\Phi}\right)\log\left(\frac{1}{\Phi}\right).$$

[19] Noticing that $\eta^M = 1-0.5^{\Phi}$ and simplifying we arrive at the following equivalent expression:

$$\log\left(c_t^i\right) + \pi\eta^M \log\left(x_{t_b}^i\left(\frac{1}{\Phi}\right)^{\omega}\right).$$

[18] This result is in line with Chu [1991], where primogeniture arises as the optimal inheritance rule to minimise the risk of lineage extinction. This *fear* of lineage extinction as a consequence of giving birth to females only is also supported by historical evidence: Clay [1968] details the case of the Marquess of Halifax, who instead of leaving the estate to his daughter and heiress Elizabeth left the properties to his great-nephew William Luckyn, with the condition that he adopt the surname of the family. Some landowners decided to name as inheritor someone different from who should have been under common-law to avoid the loss of their name. An example of this can be found in Payling [2001] in which the inheritor of Lord Basset of Drayton was his kinsman in preference to his common-law heir, with the condition that the former bear his arms and surname. Another mechanism devised to avoid, at least partially, the disappearance of a family consisted in marrying an heiress with a younger, non-heir, son and name as inheritor some male relative. This was the case of Constance, heiress of Henry Green of Drayton who was married to John, a younger son of Humphrey Stafford, Duke of Buckingham. Constance received a small landholding from her parent but John brought lands worth 400 marks per year. With this arrangement, Green's lands were not integrated into Buckingham's. Hicks [1998, p. 32] summarises the idea by simply stating that "preservation of the line and the family name really mattered". In the model we do not consider such subterfuges to keep the family name so Counts can do nothing to avoid their lands going to their heiresses.

[19] If one allows to chose whether to partition or not, $\omega = \{0,1\}$. Then, using a partitioned inheritance a Count has a utility (ignoring consumption) level of $\pi\left(1-0.5^{\Phi}\right)\ln\left(x_{t_b}^i\frac{1}{\Phi}\right)$ since only one child will continue the lineage and the probability of giving birth to only daughters is still present. If, instead, a Count chooses not to partition his utility reads $\pi\eta^M \ln\left(x_{t_b}^i\right)$. The combination of both gives us the previous expression.

Since $\Phi \geq 1$, a Count who decides to divide his inheritance ($\omega = 1$) offers to the son who will continue his lineage a small portion of size $\frac{1}{\Phi}x^i_{t_b}$. Instead, if he decides to transmit the inheritance intact, his heir will receive $x^i_{t_b} \geq \frac{1}{\Phi}x^i_{t_b}$ if $\Phi \geq 1$. Therefore, the optimal choice is to never divide inheritances except for the case $\Phi = 1$, when Counts are indifferent because there is only one heir and partitioning is impossible.[20] This contrasts with some historically observed partitioned inheritances under male-cognatic primogeniture and this is the reason why we decided that partitioned inheritances will take place randomly.

It remains to be explained why under absolute primogeniture Counts do not consider the (exogenous) probability of dividing the inheritance. Aside from the convenient fact that it allows us to write one unique utility function that encompasses both cases, such a utility formulation would imply that Counts had no control at all regarding their wills (the decision to divide the inheritance would have been exogenously given), especially since this model does not belong to the representative agent framework. This seems far from being realistic. Rather, they probably took into account other considerations outside the scope of this model, for instance, social pressure or preferences for an egalitarian treatment among their children. [21] Instead, we can think that these variables influencing the decision to divide the inheritance only manifest after all the relevant decisions are made and, moreover, that these variables cannot be inferred. Then, the exogenous probability of dividing a county will not be taken into account by Counts. One possible example is how capable a son turns out to be; the decision to divide the inheritance being forcefully postponed until he is old enough to reveal his capacities and not based on general statistics. Thus, Counts behave as if they would always abide to the prevalent inheritance rule from which they only deviate in some exceptional cases.

4.2.3. *Marriages.* The outcome of a marriage market depends on its characteristics. We will suppose that the marriage market functions frictionless and all participants have perfect information. We will also introduce some rules governing the marriage process in terms of simple preferences about whom to marry. We posit that a Count prefers marrying his inheritor with (1) any other inheritor from another county. The second best option a Count

[20]If we allow Counts to decide whether or not to partition their inheritance under absolute primogeniture, partitioning increases with income and decreases with the number of children. If, instead, we allow to postpone the decision until children's gender is revealed, then it is optimal to partition the inheritance when the first child is a woman and at least one of the remaining siblings is a man in order to assure the continuity of the lineage.

[21]For instance, McNamara and Wemple [1973, p. 131] comment the case of father who was willing to divide his inheritance among all their children because he equally loved all of them. Similarly, Alfred the Great divided his lands in his testament among their children against the Chilperic edict, specifying that if males wanted to keep lands intact, they could purchase the parts inherited by their sisters. In any case, the decision has to be based on some observables that are out of the control of the Count and that cannot be inferred from statistics. Finally, van Steensel [2012] explains the case of Jan Ruijchrok who asked permission to divide his inheritance among all their children to provide them a substantial inheritance.

has is to marry his inheritor with (2) a non-inheritor who is, nonetheless, the child of some Count. Finally, if this is not possible due to a lack of candidates, we assume that the heir is married to (3) some commoner (lawyer, trader, etc.).[22] (4) All Counts sort candidates who inherit in the same manner. Options (2) and (3) can be treated as equivalents because these marriages do not merge any lands. Although throughout the paper we called all decision makers Counts, the dynamics of the model will generate economic dispersion among them. During the period of time we simulate, elite members actually followed some strategies when marrying their children: usually marriages were only arranged between spouses of similar rank. [23] We impose a milder requirement in the marriage market: the distance (in wealth or any other relevant trait) between the two spouses must be below a certain threshold if they marry.[24]

With the conditions above spouse selection follows a refined version of Gale and Shapley [1962]. In our case, after applying the procedure described, the outcome is intuitive. Suppose a Count has to choose a spouse for his heiress. He will prefer marrying his daughter with the best candidate possible among those who belong to the same rank, that is, his first candidate will be the male heir with the largest amount of land (or with the highest income if income determines preferences) within his rank. Similarly, male heirs will prefer to marry wealthy heiresses (again, either measured in terms of land or income). Therefore, market equilibrium displays quasi-perfect sorting: the richest male heir will marry the richest female heiress, the second richest male heir the second richest female heiress, etc. Perfect assortativeness is not guaranteed since it is possible that some of the richest heirs remain single because their potentials spouses belong to a lower rank. Finally, if there are more heiresses than heirs, unmatched heiresses will marry non-heirs sons or commoner males. Alternatively, if there are more heirs than heiresses, unmatched male heirs will marry non-inheriting daughters or commoner females. Heirs and heiresses who are too wealthy or too poor to marry are assumed to find an spouse from some other place in the world. Formally, let $\mathcal{M}_t \subset \mathcal{L}_{t_d}$ be the set of counties that have a male heir and $\mathcal{F}_t \subset \mathcal{L}_{t_d}$ be the set of counties that have a female heiress, both at the beginning of sub-phase *(d)*, that is, before marriage decisions are taken. Let $\Psi(\cdot)$ measure the relevant variable that determines marriages and let $D(i,k)$ measure the distance between two inheritors and $d_t > 0$ the value of the threshold during period t.

[22]Marriages between a noble and a parishian were infrequent but are historically recorded, see Stone et al. [1995]. Notice that when an heiress marries a non-heir or parish male it creates a cadet branch or a new dynasty.

[23]For instance, Bouchard [1981] explains the difficulties of early Capetians in finding suitable queens for their sons. Hurwich [1998] analyses the marriage pattern of German lower nobility and ascertains that men tended to marry upwards in the social ladder (and women downwards) due to how children from unequal marriages were treated. In Zeeland, marriages between spouses of different strata are recorded but they were mostly arranged between spouses of similar level, see van Steensel [2012].

[24]Alternatively, inheritors can be classified into $\mathcal{N} > 1$ groups according to their wealth. Only marriages between spouses who belong to the same group would be valid.

Finally, let $\mu_t^{i,k} = 1$ if there is a marriage between the male heir of county $i \in \mathcal{M}_{t_d}$ and the female heiress of county $k \in \mathcal{F}_t$ and $\mu_t^{i,k} = 0$ otherwise. The outcome of the marriage market is summarised in the following Proposition:

Proposition 4. *The outcome of the marriage market fulfilling properties 1-4 above is given by:*

$\mu_t^{i,k} = \arg\max_{\mu_t^{i,k}} \sum_i \sum_k \left(\Psi\left(i\right) + \Psi\left(k\right)\right)^2 \mu_t^{i,k}$,

such that

$\mu_t^{i,k} = \{0,1\}$, $\sum_k \mu_t^{i,k} \leq 1$, $\sum_i \mu_t^{i,k} \leq 1$, $\mu_t^{i,k} D\left(i,k\right) \leq d_t$, $\sum_i \sum_k \mu_t^{i,k} = \min\left\{\left|\mathcal{M}_t\right|, \left|\mathcal{F}_t\right|\right\}$, $\forall i \in \mathcal{M}_t$, $\forall k \in \mathcal{F}_t$.

Proposition 4 deserves some comments. Firstly, we should emphasize that the valuation for any given heir (heiress) is common to all heiresses (heirs), that is, they all prefer in the first instance the richest heir (heiress), then the second richest, etc.[25] This, together with super-modularity of the function generates positive assortativeness. Optimisation is subject to some constraints which say that a potential couple is either married or not, each heir (heiress) can only marry once, spouses must be close enough in terms of wealth in order to marry and the total number of marriages cannot exceed the number of heirs or heiresses, whichever is smaller. Due to the maximisation nature of the problem we will marry only couples for which $\Psi\left(i\right) + \Psi\left(k\right)$ is high, that is, couples in which the heir is preferred by all heiresses and the heiress is also preferred by all heirs. In other words, $\Psi\left(\cdot\right)$ measures how good a candidate is: the greater $\Psi\left(\cdot\right)$, the better the candidate. All heirs and all heiresses attach the same value to each potential candidate, as would be the case if, for instance, the only relevant trait was the size of their lands. This implies that all heiresses would like to marry the richest heir in the first place; the second in second place, etc. This also applies to heirs who evaluate heiresses. Therefore, marriages will display positive assortativeness: rich males will tend to marry rich females. Appendix C provides an alternative and more direct method to find the outcome of the marriage market.

Let the outcome of the marriage market be summarised by the matrix μ_t. Formally, we define μ_t as a $\left|\mathcal{L}_{t_d}\right| \times \left|\mathcal{L}_{t_d}\right|$ square matrix in which $\mu_t^{i,j} = 1$ if the heir (heiress) of county i marries the heiress (heir) of county j and $\mu_t^{i,j} = 0$ otherwise. We impose that $\mu_t^{i,i} = 1$, that is, the elements on the diagonal are equal to one. This matrix will allow us to compute the changes in county size, population and technology due to marriages.

[25] In Gale and Shapley [1962] each individual can have a valuation for people from the other gender that could be different among them, it is, subjective to each agent. For instance, we can think that each heir evaluates heiresses according to how beautiful he thinks they are; clearly, perceived beauty of a given heiress can differ from one heir to another.

Recall that changes in land (and in population) occur due to three reasons: war, partitioned inheritances and marriages. The effects of war can be computed using (3.2).

Due to the assumption that commoners are immobile, we should also characterise how land ownership changes during the course of war because it has an impact on population dynamics. We assume that counties that increase in size do so uniquely at the expense of counties that reduce their size and that population is uniformly distributed within each county. In simpler terms, this assumption means that Counts prefer to focus their efforts against weak enemies. For instance, if counties 1 and 2, due to their military supremacy, are able to increase the size of their lands, the assumption implies that county 1 does *not* steal any land from county 2, nor does county 2 from county 1.[26] Let $\mathcal{X}_t$ denote the set of counties that increase their size due to war in period t, that is, $\mathcal{X}_t = \left\{ i \in \mathcal{L}_t | \, x^i_{t_b} > x^i_t \right\}$, and let $\mathcal{Y}_t = \left\{ i \in \mathcal{L}_t | \, x^i_{t_b} \leq x^i_t \right\}$ be the set of counties that reduce their size or remain equal. According to the assumption above, the number of young commoners at a post-war phase in period t in each county is given by

$$n^i_{t_b} = n^i_t \pi_n \frac{x^i_{t_b}}{x^i_t} \forall i \in \mathcal{Y}_t$$

and

$$n^i_{t_b} = n^i_t \pi_n + \left[\frac{x^i_{t_b} - x^i_t}{\sum_{j \in \mathcal{X}_t} x^j_{t_b} - x^j_t} \right] \left[\sum_{k \in \mathcal{Y}_t} \frac{x^k_t - x^k_{t_b}}{x^k_t} n^k_t \pi_n \right] \forall i \in \mathcal{X}_t.$$

The first equality traces the changes in land size for counties that lost the war. These counties, by assumption, do not conquer any land. Therefore, their population density remains constant pre- and post-war if we do not consider population growth. As a consequence, it is possible to compute their post-war population, $n^i_{t_b}$, as its population density applied to their post-war size multiplied by population growth rate. For counties that increase in size, post-war population computation is more involved: it includes the population increase due to fertility in each county $(n^i_t \pi_n)$ to which we add the population they gain through the conquest of new lands. The first term between square brackets represents the amount of land county i relative to the total land wpn by winners. This is, if counties in $\mathcal{X}_t$ increase in total their size by θ units of land, the first term in brackets measures how much of θ is gained by county i. The second term between brackets represents the population lost by each county in $\mathcal{Y}_t$, which is its population density multiplied by population growth rate $\frac{1}{x^k_t} n^k_t \pi_n$ and by how much land they lose $\left(x^k_t - x^k_{t_b} \right)$. It can be shown that $\sum_{i \in \mathcal{L}_t} n^i_t \pi_n = \sum_{i \in \mathcal{L}_t} n^i_{t_b}$, so the

[26]To gain more insight on what this assumption means suppose that it does not hold. Assume, for instance, that among three counties two increase their size and the third one decreases it. It is possible to think that county 1 gains a bit of land from county 2 and some land to county 3 and, at the same time, county 2 gains territory from county 3 in such a way that compensates the lost suffered against county 1. If this were the case, it would still be possible to compute the total size of each county but not their population because battles are settled in an all-against-all system and not sequentially.

equations above only redistribute total population. For the sake of completeness, note that $A^i_{t_b} = A^i_t$.

Finally, the evolution of the number of counties depends on marriages and on the random divisions of counties. Let $\sigma^i_t \in \{0,1\}$ be a binary variable that takes value 1 if county i is divided among *more than one heir* and value 0 if not. Then, it follows that $|\mathcal{L}_{t_d}| = |\mathcal{L}_t| + (\Phi - 1)\sum_i \sigma^i_t$, that is, the number of counties after divisions is equal to the number of counties before the division plus the number of *new counties created.* We define, for each county, the set $\mathcal{S}^i_t$ as the successor(s) of county i as:

(1) If county i is *not* divided, then $\mathcal{S}^i_t = \{i\}$,
(2) If county i is divided, then $\mathcal{S}^i_t = \{i, I_t + 1, I_t + 2, \ldots, I_t + \Phi - 1\}$.[27]

The updated list of counties at the beginning of the 4th sub-period is the union of the counties which were not divided and the list of counties divided plus the newly created counties. Notice that for each period the number of sets $\mathcal{S}^i_t$ is equal to (the initial value at period t of) $|\mathcal{L}_t|$.

At the end of sub-period (d) or, equivalently, at the beginning of period $t+1$, the number of counties might also change because of marriages. We can write the number of counties at the beginning of period $t+1$ as $|\mathcal{L}_{t+1}| = |\mathcal{L}_{t_d}| - \frac{1}{2}\sum_i \sum_{j\neq i} \mu^{i,j}_t$, where the double sum computes the number of people who marry.[28]

Whenever a marriages between an heir and an heiress occurs two counties are unified into one. We will take the convention that counties that do *not* merge keep the name they had during period t_d and, for those that are united, the newly created county takes the lowest value of its components as its name. That is, if the heirs of counties $i = 2$ and $j = 5$ marry, the merged county will be named 2. We can write the list of counties at the beginning of period $t+1$ as $\mathcal{L}_{t+1} = \mathcal{L}_{t_d} \setminus \bigcup \left\{ \max_{j\neq i} (i,j) \,|\mu^{i,j}_t = 1 \forall i \in \mathcal{L}_{t_d} \right\}$. The set operation simply removes from the list of county labels one of mergers: that with the largest label.

We can also compute the size of all counties (newly created and already existing) before marriage decisions as well as their population and technological level. We assume that counties that are divided conserve the same state capacity level as their predecessors. Recalling divisions create successors of equal size we can write:

[27] In this definition I_t must be modified after each iteration on i.

[28] Half of the number of people who marry is actually the number of marriages since each marriage involves two people.

$$x^i_{t_d} = \left(\frac{1}{\Phi}\right)^{\sigma^j_t} x^j_{t_b} \forall i \in \mathcal{S}^j_t, \forall j \in \mathcal{L}_t, \tag{4.1}$$

$$n^i_{t_d} = \left(\frac{1}{\Phi}\right)^{\sigma^j_t} n^j_{t_b} \forall i \in \mathcal{S}^j_t, \forall j \in \mathcal{L}_t, \tag{4.2}$$

$$A^i_{t_d} = A^j_{t_b} \forall i \in \mathcal{S}^j_t, \forall j \in \mathcal{L}_t. \tag{4.3}$$

We provide an example in order to better illustrate some of the concepts we introduced before.

Example 1. Suppose that there are only two counties, Saxony and Bavaria, during period t so we can write $I_t = 2$, $\mathcal{L}_t = \{1, 2\} = \{\text{Saxony, Bavaria}\}$. Assume their size, population and state capacity are $x^1_t = 0.7$, $x^2_t = 0.3$, $n^1_t = 10$, $n^2_t = 12$, $A^1_t = 1$, $A^2_t = 1.3$. If one ignores the effects of war on county size and population, Saxony is divided between $\Phi = 3$ inheritors at period t, while Bavaria applies strict absolute primogeniture. This implies $\sigma^1_t = \sigma^{\text{Saxony}}_t = 1$ and $\sigma^2_t = \sigma^{\text{Bavaria}}_t = 0$. The succession set corresponding to Saxony will be $\mathcal{S}^1_t = \mathcal{S}^{\text{Saxony}}_t = \{1, 3, 4\} = \{\text{Saxony, Saxe} - \text{W, Saxe} - \text{G}\}$ and the succession set corresponding to Bavaria will be $\mathcal{S}^2_t = \mathcal{S}^{\text{Bavaria}}_t = \{2\} = \{\text{Bavaria}\}$. Finally, we can write the size of all counties using Equations (4.1)-(4.3) as $x^1_{t_d} = x^{\text{Saxony}}_{t_d} = \left(\frac{1}{3}\right)^1 0.7$, $x^3_{t_d} = x^{\text{Saxe-W}}_{t_d} = \left(\frac{1}{3}\right)^1 0.7$, $x^4_{t_d} = x^{\text{Saxe-G}}_{t_d} = \left(\frac{1}{3}\right)^1 0.7$, $x^2_{t_d} = x^{\text{Bavaria}}_{t_d} = \left(\frac{1}{3}\right)^0 0.3$. We can proceed similarly for population and state capacity.

Finally, we assume that when two counties merge, the state capacity of the new entity is a weighted average of the previous levels of state capacity. Given initial conditions on county size $x^i_0 \forall i \in \mathcal{L}_0$, population $n^i_0 \forall i \in \mathcal{L}_0$ and initial state capacity $A_0 = 0 \forall i \in \mathcal{L}_0$ the economy can be described as a sequence of temporary equilibria $\{b^i_t, g^i_t, c^i_t, n^i_t\}^\infty_{t=0}$, a sequence of matrices $\{\mu_t\}^\infty_{t=0}$ and a sequence of sets denoting counties $\{\mathcal{L}_t\}^\infty_{t=0}$ such that, in all periods all agents behave optimally and markets clear, that is, in which equations 3.5-3.7 and 3.8 are fulfilled, marriages are given according to Proposition 4 and land, population and technology evolve according to

$$\forall i \in \mathcal{L}_{t+1},\ x^i_{t+1} = x^i_{t_d} + \sum_{j \neq i} x^j_{t_d} \mu^{i,j}_t, j \in \mathcal{L}_{t_d}, \tag{4.4}$$

$$\forall i \in \mathcal{L}_{t+1},\ n^i_{t+1} = n^i_{t_d} + \sum_{j \neq i} n^j_{t_d} \mu^{i,j}_t, j \in \mathcal{L}_{t_d}, \tag{4.5}$$

$$\forall i \in \mathcal{L}_{t+1},\ A^i_{t+1} = \sum_{j \neq i} \frac{A^i_t n^i_t + A^j_t n^j_t \mu^{i,j}}{n^i_t + \sum_j n^j_t \mu^{i,j}}, j \in \mathcal{L}_{t_d}. \tag{4.6}$$

We will continue with our example above to show the effects of marriage.

Example 2. Suppose that the heir of county 2, Bavaria, marries the heiress of county 4, Saxe-Gotha, and that there are no other marriages between heirs. We can write the marriage matrix as

$$\mu_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$

and the number of counties at period $t+1$ will be given by $|\mathcal{L}_{t+1}| = |\mathcal{L}_{t_d}| - \frac{1}{2}\sum_i \sum_{j\neq i} \mu_t^{i,j} = 4 - \frac{1}{2}2 = 3$. Moreover, $\mathcal{L}_{t+1} = \{1,2,3,4\} \setminus \{\max(2,4)\} = \{1,2,3\}$. Suppose we rename the county created through the marriage as Bavaria $-$ S $-$ G and it takes position 2, that is, the position at which Bavaria was at period t :

$\mathcal{L}_{t+1} = \{1,2,3\} = \{\text{Saxony, Bavaria} - \text{S} - \text{G, Saxe} - \text{W}\}$. The size of counties at the beginning of period $t+1$ is given by Equation 4.4. In our case we have $x^1_{t+1} = x^1_{t_d} + \left(x^2_{t_d}\mu_t^{1,2} + x^3_{t_d}\mu_t^{1,3} + x^4_{t_d}\mu_t^{1,4}\right) = \frac{1}{3}0.7 + 0 = \frac{1}{3}0.7$, $x^2_{t+1} = x^2_{t_d} + \left(x^1_{t_d}\mu_t^{2,1} + x^3_{t_d}\mu_t^{2,3} + x^4_{t_d}\mu_t^{2,4}\right) = \frac{1}{3}0.7 + 1{\cdot}0.3 = \frac{1}{3}0.7 + 0.3$ and $x^3_{t+1} = x^3_{t_d} + \left(x^1_{t_d}\mu_t^{3,1} + x^2_{t_d}\mu_t^{3,2} + x^4_{t_d}\mu_t^{3,4}\right) = \frac{1}{3}0.7 + 0 = \frac{1}{3}0.7$.

5. Simulation

The history of events in this model is randomly given by inheritors' genders and by randomly partitioned inheritances. Since the aim of this paper is to analyse the possible impact of two different inheritance systems on economic performance, we need to simulate the economy: as shown before, under male-cognatic primogeniture the disCount rate γ^M is lower than under absolute primogeniture γ^A, $\gamma^M > \gamma^A$. By Proposition 3, this encourages investments in state capacity. On the other hand, absolute primogeniture maximises the number of marriages, hence counties become larger. Since investments in state capacity are positively related with county size (captured in Y^i_t), that indirect scale effect may compensate the lower disCount factor.

5.1. **Parametrisation.**

The parametrisation of the model is based on historical facts for most of the parameters. Russell [1958] shows that, assuming an infant death rate of 25% in France, 4.15 children survived their father on average. Data for England indicates that the number of surviving children averages 2.35. Accordingly, we choose the number of surviving children per Count to be equal to 3, which is the nearest integer to the average between these two cases. Crops are shared between Counts and commoners taking into account at the rate$\psi = 5/12$, which is the average of the extreme cases presented in Slicher van Bath et al. [1966]2.. For simplicity, we will assume that the psychological discount factor, π, is equal to 1. The effective discount factor under male-cognatic primogeniture, γ^M, and under absolute primogeniture, γ^A, become $1 - 0.5^{\Phi} = 1 - 0.5^3$ and 0.5, respectively.

According to Sánchez Martínez et al. [2003], the wage of a soldier was 2 *sous* daily.[29] We compare this cost with the prices and typical noble food consumption from Banegas López [2010], who shows that nobles spent around 1.6 *sous* daily in food. Therefore, the real cost of raising an army is set to 1.2.

We proceed similarly to compute the cost of taxation. Following Verdés Pijuan [2004, p. 153], in the town of Cervera during the year 1424, the *racional*, that is, an auditor, received no less than 330 *sous*[30] in yearly wages. It is also said that a *racional* had to work three days a week or around 150 days yearly, which gives us an equivalent wage of 2.2 *sous* per day. With this information, we set $p_g = 1.375$. The value for α in the production function is taken from Tintner and Brownlee [1944], Mundlak et al. [1997, 1999] who show that the elasticity of output (crops) with respect to land, $(1-\alpha)$, is around 0.5, so $1 - \alpha = \alpha = 0.5$. Population growth is computed as the yearly average increase in the European and minor Asian population between 600 and 1500 using data from Russell [1958]. As a period in the model comprises 30 years and the average increase is 0.0842% per year, we will assume $g_n = 1.026$. The value for κ is taken from Bartlett [2000] who comments on the exceptionally high tax-burden Richard's ransom represented which was equal to 25%, accordingly we set $\kappa = 4$. Finally, for each simulation we will draw values for the initial period variables (x_t^i, n_t^i) from uniform distributions between 1 and 2 and we will set the initial level of state capacity equal to 0, $A_0^i = 0 \forall i \in \mathcal{L}_0$.[31]

county divisions and children gender are random variables. We pose the probability of a random division, ω, to be equal to 10%: a number relatively low but that might account for the changes that occurred in Germany. The disappearance threshold is set to $\delta = 0.01$, it is, one-hundredth of the minimal original size. We impose a marriage threshold which changes over time and we set it equal to one standard deviation in wealth.[32] Finally we should comment on ϕ, which measures the relative importance of men in war. The value of ϕ is larger than 1 by assumption. In the simulations we set it equal to $1 + 1/10^{11}$ which will generate a slow evolution of the economy. Table 1 summarizes our model parametrization.

[29] In Sánchez Martínez et al. [2001] are also recorded the costs for a battle-armed horseman (7 *sous*), battle-armed man with a horse (5 *sous*) or a battle-armed man (4 *sous*).

[30] The source says they received no less than 30 *florins*. One *florin* corresponded to 11 *sous*.

[31] We do not attempt to normalise total land size to be equal to 1.

[32] This imposes the minimum number of counties to be equal to two. Suppose that there are only 2 Counts whose wealth differs in $d > 0$ units. The threshold that determines whether two inheritors can marry is given by the standard deviation of wealth, which is equal to $\sqrt{\frac{d^2}{2}} < d$. Therefore, whenever there are two Counts, they belong to different strata and can never marry. Nonetheless, in some simulations we reach a number of counties equal to one. These cases are a consequence county elimination and not marriages.

Table 1. Parameters

Parameter	Value	Source
α	1/2	Tintner et al. (1944) and Mundlak et al. (1997, 1999).
ψ	5/12	Slicher van Bath et al. (1966)
ω	1/10	Arbitrarily set.
g_n	1.026	Russell (1958).
Φ	3	Russell (1958).
ϕ	1+1/10	Arbitrarily set to have slow transitions.
p_b	1.375	Comparison between noble food expenditures from Banegas López (2010) and soldier wages from Sanchez Martínez et al. (2003).
p_g	1.2	Comparison between noble food expenditures from Banegas López (2010) and auditor wages from Verdés Pijuan (2004).
κ	4	Tax burden imposed to pay Richard's ransom from Bartlett (2000).
γ^M	7/8	Computed using $\gamma^M = 1 - 0.5^\Phi$.
γ^A	1/2	Computed using $\gamma^A = 1 - 0.5$.
δ	0.01	Arbitrarility small number equal to 1/100 of the minimum initial size.

Finally, we should discuss why we opted to parametrize the model instead of calibrating it. Due to the stochastic nature of the model, it is not straightforward how interpret the results of a calibration. Suppose that one can set proper target values for the calibration exercise using a good dataset. Then, the model still needs to be simulated a certain number of times for each possible set of parameters to account for how the gender of inheritors as well as partitioned inheritances are determined. As a consequence, one can only aspire to correctly calibrate a generated average history. Even assuming that real-world data is available for the entire transition dynamics, nothing guarantees that it would correspond to the average case previously computed. In other words, even if all relevant data were at hand, a miss-match between the results from the calibration and these data can still be attributed to the fact that the data is one of the many possible histories. This concern invalidates any assessment regarding the relative performance of the calibration and thus, the exercise is relatively futile. Instead, a simulation exercise does not require a validation against real data to determine whether the retrieved values for the parameters are or not correct since it imposes real world values to them.

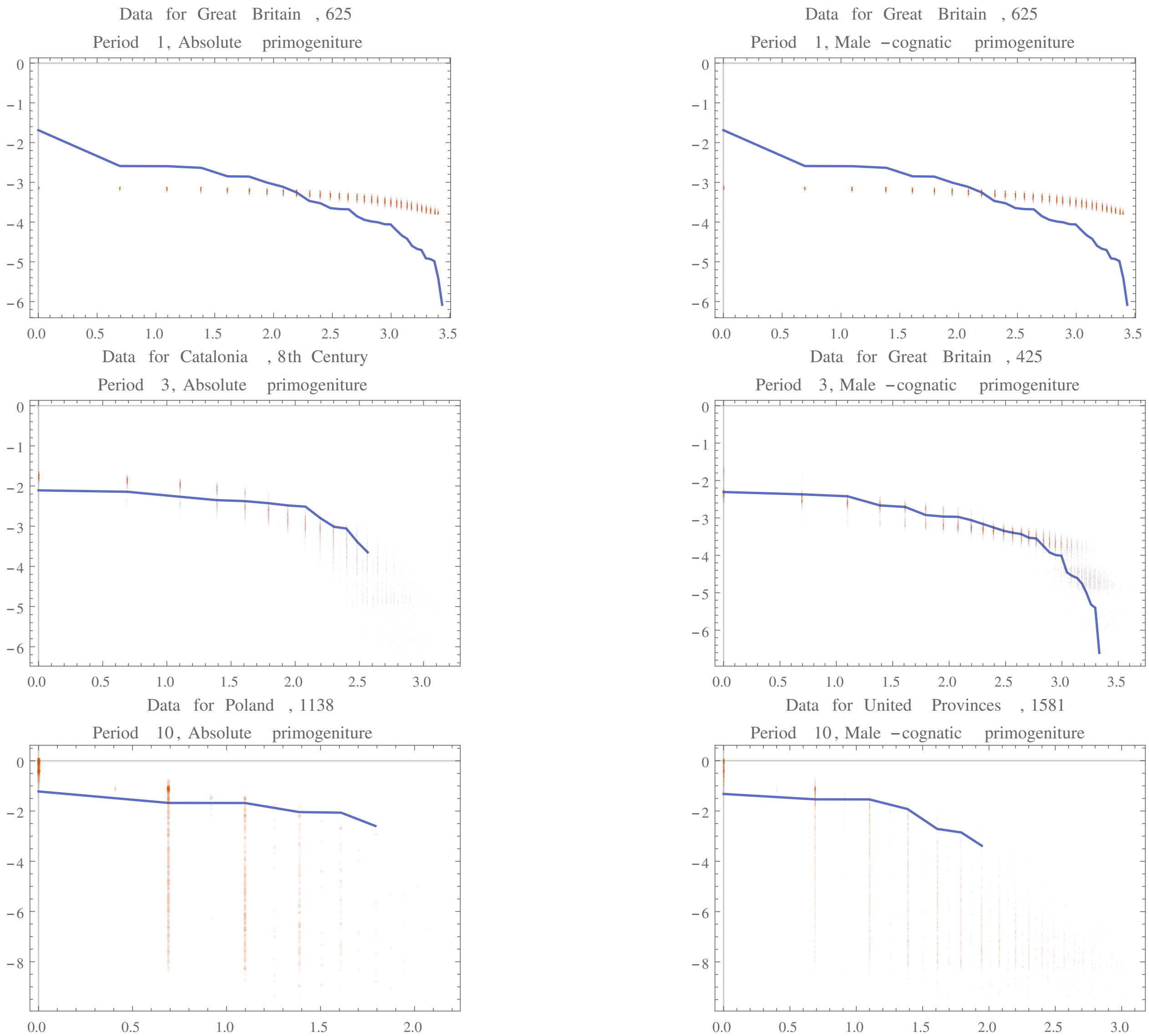
In all simulations, we will assume that marriages are arranged according to spouses' potential production (which is positively related to their future potential income).

For each regime we simulated the corresponding economy using a code written in Python programming language (source codes are available upon request). We ran 1000 simulations for each regime. In the remaining part of this Section we comment the results obtained. Unless otherwise stated, when we refer to marriages we mean marriages between an heir and an heiress.

5.2. **Sensitivity analysis.** Before turning to the main conclusions, we wonder how sensitive our model is to the choice of parameters. First, we will compare the results of the model regarding country sizes with real-world data. The objective is to assess whether the pattern that emerges from our simulations is similar to that observed in reality. Since, typically, the size of regions follows a Zipf distribution, Figure 5.1 compares the logarithm of county sizes with the logarithm of their rank, where surfaces were normalised to be between 0 and 1. Real data was selected in order to match the average number of counties at each period, that is, each panel compares situations with approximately the same number of counties. For instance, the model's first period comprises 30 of them and our Counter-factual data are the 28 kingdoms in which the Great Britain was divided around year 600 CE. The graphs depict real data using a solid line while simulated data are represented by faded dots. The fit, although not perfect, is satisfactory.

In order to increase our confidence on the fact that the results we obtain are not conditioned by the choice of one set of parameters instead of another slightly different, we also perform a sensitivity analysis. In particular, we change the values of almost all parameters one by one. Each time we modify them, we run 300 simulations under each inheritance system and we plot the evolution of average state capacity over time since this is our main variable of interest. These figures are relegated to the Appendix D. In general, we are confident that the model would deliver the same qualitative result we will present even if parameters were somewhat different from those we chose. The general evolution of state capacity follows always exactly the same trend, with only one notable exception. When we set $\phi = 0.5$, absolute primogeniture outperforms male-cognatic primogeniture. A tentative explanation would be that, for such a low value of ϕ, battles have little impact on changes on the distribution of land. Therefore, since the onset, marriages have a major role in shaping the evolution of county sizes. Following the fact that there are more marriages under absolute primogeniture, these counties should be larger which explains why they can invest more and, thus, reach higher levels of state capacity. Also, although not modifying the general picture, the price of increasing state capacity, p_g, has a relatively large impact on the values the former reaches. Nonetheless, as expected, the lower the price, the higher the investments to develop an administration which will have a positive impact on state capacity level. Otherwise, the prediction of our main set of simulations remains true: male-cognatic primogeniture outperforms absolute primogeniture in the long-run. It is noticeable to observe how, as we increase the number of children, the evolution of state capacity remains similar under absolute primogeniture but is boosted under male-cognatic primogeniture. This effect is partially due to changes in the discount rate, which directly induces higher expenditures in the creation of an administration.

Figure 5.1. county size comparison



Finally, one might wonder under which circumstances the results can be reversed. Variables that affect the number of marriages under one regime and not the other can produce such a shift. Consider, for instance, that Counts fathered a large number of children. Under male-cognatic primogeniture this would imply a highly imbalanced sex-ratio in the marriage market which would allow only few marriages. On the other hand, increasing the number of children per Count has no impact on the marriage market under absolute primogeniture. In that case, it might be possible for counties under absolute primogeniture to display higher levels of state capacity in the long run following from more mergers.

5.3. **Analysis of the results.** As predicted by the theory, under absolute primogeniture there are more marriages once we control for the number of counties.

Table 2 presents the result of an OLS and Poisson regressions that relate the number of marriages to the total number of marriageable people (it is, total number of counties plus counties created due to partitioned inheritances minus the number of counties that disappear), the per-period wealth standard deviation and a dummy variable to capture the prevailing inheritance system. The coefficients for marriageable people and absolute primogeniture are positive. The explanation of this result is simple: for each marriage a male heir and a female heiress are required. Under absolute primogeniture the probability of becoming a heir for both genders is the same but, under male-cognatic primogeniture, males are over-represented among inheritors. Female supply in absolute primogeniture matches (on average) its demand while under male-cognatic primogeniture females are in short-supply. As a result, under male-cognatic primogeniture some males cannot find a female heiress, so they must marry someone else. Finally, a higher standard deviation in wealth implies that finding a suitable spouse is more difficult because partners are more likely to belong to different strata. Therefore, the expected number of marriages should decrease.

Table 2. Number of marriages between heirs

Number of marriages	OLS		Poisson	
Number of inheritors	0.293***	(314.98)	0.061***	(163.16)
Absolute primogeniture (=1 if yes)	1.455***	(88.67)	0.586***	(89.96)
Std. Dev. in wealth	-0.009***	(-5.03)	-0.112***	(-75.95)
Constant	-1.227***	(-50.87)	0.041**	(3.02)
Observations	42149		42149	
R^2	0.802		0.553	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

A direct consequence of having more marriages under absolute primogeniture is that, compared to male-cognatic primogeniture, there are fewer counties at each period despite the fact that we eliminate more due to their size being below the threshold under the latter. The explanation is trivial if we ignore the county disappearance process: each period, a constant proportion of counties are divided. On the other hand, two counties can only be merged through a marriage. It follows that under male-cognatic primogeniture the number of counties should be larger because fewer of them gather and the division rate is the same. Once we restore counties' disappearance, we find that more of them are eliminated under male-cognatic primogeniture. Indeed, this system tends to create big counties which annihilate the remaining. This effect is also present under absolute primogeniture but at a lower scale. We will analyse this mechanism more carefully below.

The coefficient associated to wealth dispersion has the opposite sign than expected. A larger standard deviation implies that counties differ more but, at the same time, Counts are willing

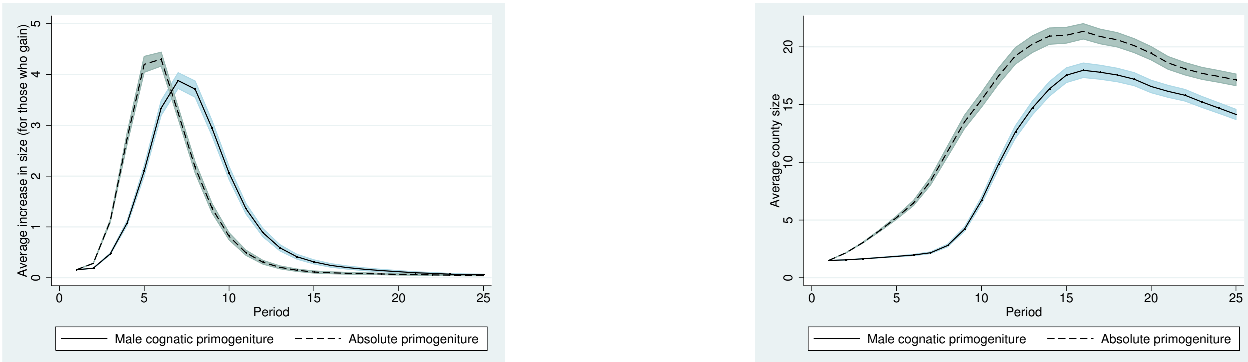
Table 3. Wealth standard deviation

	Absolute primogeniture (1 if yes)	
Number of inheritors	1	0
0-2	18.97	18.37
2-3	12.14	11.35
3-5	10.05	9.42
5-10	6.33	6.40
10-15	3.06	4.39
More than 15	0.57	1.12

to accept partners who are further away in the income distribution. A priori, this effect should increase the number of marriages since it is easier for Counts to find a suitable candidate. Notice that under the extreme case of $d = \infty$ any pair of heir and heiress can marry regardless of their relative positions. The negative value of the coefficient could be partly explained once two facts are considered. In the first place, wealth variance is maximal when there are only two heirs, see Table 3. Secondly, with two inheritors, marriages are impossible. These two effects together might drive the coefficient downwards. Nonetheless, if we consider the same regression but we exclude the cases with only two counties, the coefficient associated to wealth dispersion becomes non-significant using OLS and remains negative under a Poisson regression.

County size distribution is also a consequence of marriages. Under both regimes, during the initial periods, the average county size is drove by war patterns: there is a county which tends to capture a relatively large share of total land due to its military supremacy. As time advances, marriages become the main cause for changes in average county size. Each period, the amount of land that large counties can conquer decreases because total land is fixed and they have already conquered part of it. We should, thus, expect the relative importance of war on land distribution changes to decrease over time. This can be seen in Panel A) of Figure 5.2 which depicts the average change in county size over time due to war for counties that increase size (recall that total land is fixed and, therefore, whatever is gained by some counties is lost by the remaining). The figure is consistent with our explanation, showing a steady rise in land won through battles which then decreases. The other factor that modifies county size distribution are marriages. This effect is less important under male-cognatic primogeniture because it generates fewer marriages. Panel B) of Figure 5.2 plots the evolution of the average county size over time. Notice the decline of the average county size from period 15 onwards. This decreasing trend results from a slow increase in the average number of counties that takes places during these periods. This, in turn, is a consequence of two causes: random partitioned inheritances and few marriages resulting from having only

Figure 5.2. Change in size and median county size



two inheritors, as explained before. From our simulated data, marriages cannot overcome the generation of new counties through partitioned inheritances.

Understanding how marriages shape county size distribution was important because the latter, together with population, affects the willingness to invest in state capacity. Its evolution can be studied from two different perspectives: the average state capacity *achieved* under each regime or the amount of resources *invested* in state development. We will begin the analysis with the second point of view since it also plays a role in the first.

Firstly, we should notice that on an observation by observation basis, the amount invested in state capacity depends on a County's size and population as well as on the value of γ. Panel A) of Figure 5.3 plots the average investment in state capacity. On average, counties under absolute primogeniture invest more in state capacity during the first periods (except the first one) than do counties under male-cognatic primogeniture. After the 10th period the trend is reversed. We should emphasize that comparing average investment takes into account, indirectly, the number of counties, which differs across regimes. We tackle this issue in Panels B) and C), which present the share of counties that invest any positive amount of resources in state capacity development and the average investment made by counties wealthy enough to spend any amount of resources in the creation of an administration, respectively.

In this model, investments are a distribution censored below 0. By Propositions 1 and 3 we know that population, size and γ affect positively the probability of observing an investment.

The first period is interesting because it is the only one for which the distribution of size and population are the same across regimes. Since $\gamma^M > \gamma^A$, a larger share of counties should be able to invest under male-cognatic primogeniture which is confirmed in Panel B). During the remaining periods we observe the opposite: a higher percentage of counties is able to invest under absolute primogeniture. Changes in size and population distributions underlie the process. There are more marriages under absolute primogeniture and, because of them, more counties merge. This shifts more of them to the upper tail of the distributions. In other words, because of marriages, more counties are rich enough to invest. The indirect effect brought in by the marriage market overcomes the direct effect of a lower γ. Despite the process outlined before, it is remarkable the decreasing portion in the average number of counties that invest a positive amount that takes place from the 15th period onwards. As noticed before, during these periods the number of counties increases. The creation of new counties through random partitioned inheritances affects the distribution of population, size and income, shifting them to lower values. Thus, since marriages are unable to overcome the division of counties, we should expect those that remain to be less likely to invest, which is reflected in the graph. To further test this hypothesis, we run a regression on the probability of investing where we control whether a certain county originated from a partition or from a marriage. We should expect that partitioned counties should be able to invest less: these are smaller than the original county and they can pushed back in the distribution of wealth. The contrary should hold for countries formed through a marriage. Table 4 presents the result of a linear probability model that relates the likelihood of investing and whether there were partitions during the previous period, while controlling for inheritance rule, state capacity, wealth levels and period fixed effects. The results we obtained are in line with the theory previously outlined.

Table 4. Likelihood of investing

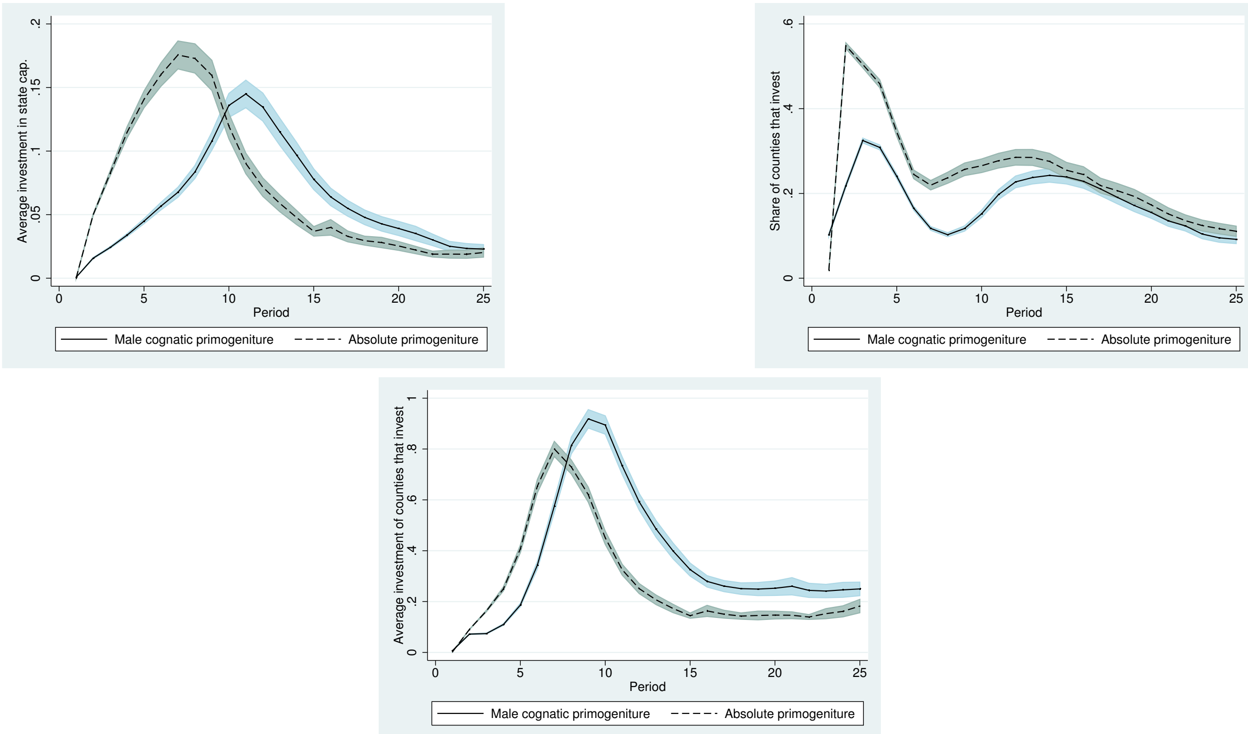
	(1)	
Invests	LPM	
County was divided last period (1=yes)	-0.225***	(-219.03)
County was merged last period (1=yes)	0.337***	(259.22)
Wealth	0.04***	(324.14)
State Capacity	-0.02***	(-35.40)
Absolute primogeniture (=1 if yes)	-0.02***	(-20.64)
Constant	0.05***	(-34.70)
Observations	401155	
Pseudo R^2	0.4598	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Finally, in Panel C) we present the investing behaviour of the counties that are able to spend any positive amount in building state capacity. We can distinguish three different phases: an initial one comprising periods 1 and 2; a transition phase from period 3 until period 7; and a final phase which lasts until the final period. During the first phase, all counties that are able to invest are similar, regardless of the regime under which they are. Therefore, as was the case before, $\gamma^M > \gamma^A$ entirely explains the difference in investment behaviour. During the second phase, absolute primogeniture counties do better than their counterpart. The

FIGURE 5.3. Average investment in state capacity



change is a consequence of changes in county's distribution. Figure E.1 in E provides the Lorenz curves for population and county size for the 2nd, 5th and 10th period. During the periods in the second phase, inequality between counties rises. Both marriages and, mainly, war contribute to it but it is more acute under male-cognatic primogeniture.[33] The general pattern is similar under both systems: following the first phase, only some counties are able to invest in state capacity. These counties increase their size due to their military supremacy. Nonetheless, absolute primogeniture also allows for more marriages so large counties would tend to be larger and, consequently, could invest more.

The final phase is similar to the initial one: size and population distributions are similar for counties that are able to invest across regimes. Then, from $\gamma^M > \gamma^A$ we should expect more counties to be able to invest in state development under male-cognatic primogeniture. Moreover, we can identify a second cause that explains the investment pattern. At the end of the previous phase, counties under absolute primogeniture tended to invest more, thus they achieved higher levels of state capacity. Proposition 3 states that optimal investments are decreasing in achieved state capacity. This provides another reason why more counties under male-cognatic primogeniture invest. Notice that in our previous regression we obtained a significant, negative coefficient associated to the level of state capacity.

[33]Indeed, the 6th period corresponds with the maximum surface change due to wars (see Figure 5.3).

Having analysed how counties invest in state capacity, we can now shed light on how marriages affect the evolution of the average state capacity. In order to do so, we will concentrate on the transition from sub-phase (c) at period t to the period $t+1$. In this lapse of time marriages are decided. We can compute the effect of marriages comparing the average level of state capacity that would have been reached at period $t+1$ if *no* marriages were arranged with its actual level. Differences between both are due to marriages.

Table 5 shows the result of an OLS regression that relates the actual average level of state capacity with the average level assuming no marriages, and including as controls the number of counties, a dummy variable to account for inheritance rule as well as time dummies.

Table 5. State capacity

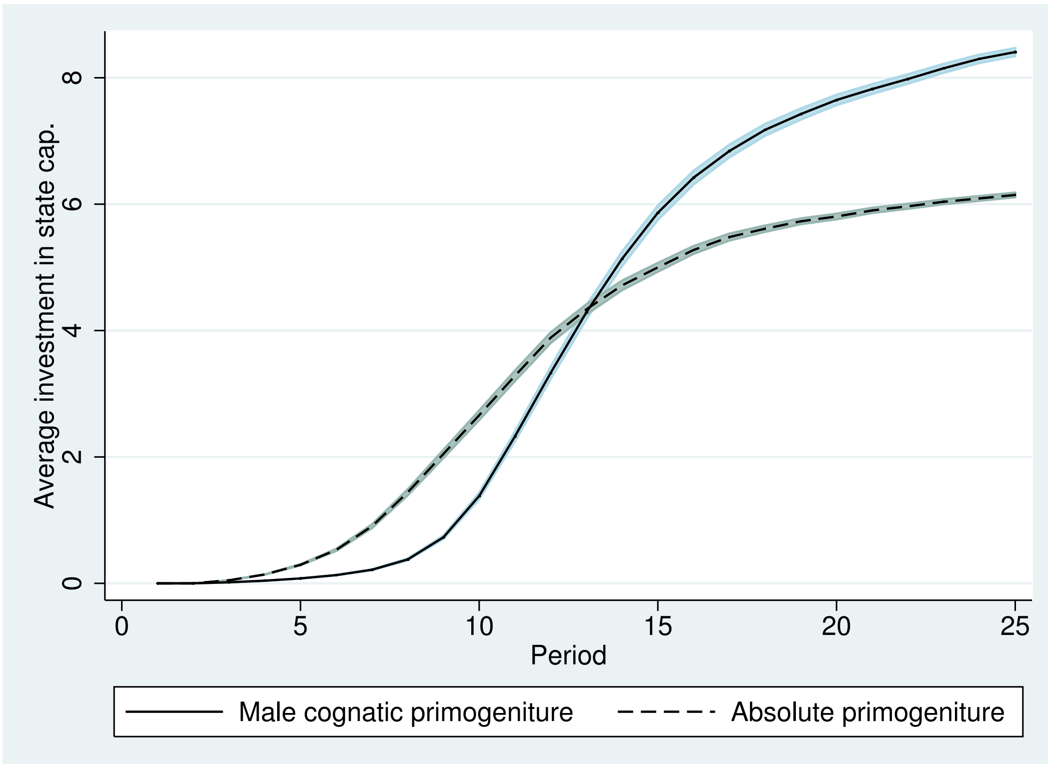
	(1) Avg. state capacity		(2) Avg. state capacity	
Average state cap. without marriages	0.274***	(138.37)	0.271***	(135.84)
Total number of counties	0.00369***	(7.32)	0.00478***	(9.38)
Absolute primogeniture (1 if yes)	0.120***	(20.16)	0.117***	(19.68)
Partitioned inheritance (1 if yes)			-0.0754sym***	(-13.07)
Constant	0.275***	(12.69)	0.764***	(31.35)
Observations	401155		401155	
R^2	0.360		0.361	

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

From the results presented in Column (1) absolute primogeniture has a positive impact on the evolution of average state capacity. The mechanism underlying this effect is related to how counties are divided and marriages decided. At each period, independently of the inheritance system, on average 100ω percent of the counties are partitioned. Population, size and state capacity level of counties should be the same for partitioned an non-partitioned counties because divisions are random. The partition system generates counties some of which are rich and some of which are poor. Gender is also assigned randomly to counties created from partitioned inheritances so half of them would be ruled by men and the other half by women. However, non-separated counties under male-cognatic primogeniture tend to be ruled by men, while under absolute primogeniture men and women rule them with equal probability. Therefore, in the marriage market under male-cognatic primogeniture, women are the short-side. We can expect all of them to find a suitable partner and marry, leaving poor heirs unmarried. Instead, under absolute primogeniture genders are balanced so all men and women should be able to marry. Poor heirs who could not marry under male-cognatic primogeniture push the average state capacity level downwards because county size (the relevant trait to decide marriages) is highly correlated with state capacity (0.7263 for the

Figure 5.4. Average state capacity



entire sample). Additionally, the sensitivity test on the probability of partitioned inheritances shows that the higher its value, the lower the levels of average state capacity that capacity that be achieved, see Figure D.2. The positive coefficient for absolute primogeniture indicates that the shift towards lower values is more important for male-cognatic counties.

It is possible to assess whether this hypothesis is correct or not using our simulations and running the regression present in Column (2) of Table 5 which includes a dummy variable to indicate whether in a given period there were partitioned inheritances or not. From the exposition before, we should expect divisions to affect negatively the evolution of state capacity since, although they create more counties, those less likely to marry are the poorest ones. The negative coefficient on the variable partitions confirms our hypothesis, it is, in periods without partitions the decrease in state capacity due to marriages is reduced.

Finally, in Figure 5.4 we present the evolution over time of the average state capacity under each regime. In line with the results presented above, initially absolute primogeniture generates counties which are characterised by a higher state capacity than these under under male-cognatic primogeniture. At period 10, male-cognatic primogeniture catches up and overcomes the trend. This pattern follows from investments made, although it presents some delay for the reversal of the trend.

6. country Unification

In accordance with the theory presented above, Europe should be composed by a small number of countries. We do not observe such a convergence at a global level. Regionally, some countries arose as unified entities, for instance France and the United Kingdom. On the other hand, the theory is unable to explain the late unification of Italy and Germany, areas that were fragmented into a large number of small regions. Nonetheless, it is possible to reconcile these two observations with our framework once we consider their specificities.

Italy, before its unification in 1861, was formed by independent city-states ruled by local elites. Conflicts among elites to control a city were common and included the use of violence. According to Hughes [1975], medieval Genoa consisted in "a system of long-lasting alliance within lineages and enmity between them [which] gave pattern to the seemingly pointless warfare played out on the streets of medieval Genoa and in its *contado*". Lansing [2014] explains the case of Florence and we find again competition among lineages for the control of neighbourhoods with none achieving a long-lasting supremacy. Similar conflicts arose in Venice, although according to Greif [1995], Venetians managed to reduce inter-clan tension with a mechanism to choose the Doge. It is remarkably that the city of Siena called Florence to help put to an end an internal riot between Tolomei and Salimbeni families (Bowsky [1981, p. 166]). Finally, Jones [1965] also considers clan structures important during Italian Middle Ages: a clan acted together in all activities, specially for its defence.

From this evidence, we can presume that elites focused on gaining influence in their city. From the point of view of the model, we can associate a city-state with the region $\mathcal{L}$. Similarly, former counties can be represented by clans.

We should also impose some restrictions on marriage possibilities because an heir from one family was not allowed to marry an opposing clans heiress. It was uncommon for the structure of power to change as a result of marriage restrictions. Suppose that all clans have similar initial conditions in terms of wealth. According to the model, each period we should expect minor changes in the distribution of wealth because war outcome across clans will not differ much and we can expect similar marriages to be decided within each opposing group. Then, unification will be a long lasting process. At the same time, we would observe investments in state capacity to increase the control each family exerts on its domains, be it by means of taxation, clientelism or even patronage. Finally, it is also possible to generate alternating power between groups if a marriage gives temporary advantage to one of them, to be overtaken in following periods by another marriage within the rival group. That is, a marriage in one alliance can give it a temporary advantage. In the model without frictions in the marriage market, this temporary advantage can be easily transformed into a long-term superiority because higher income allows better marriage opportunities, thus perpetuating it. On the contrary, if the marriage market has significant frictions, transitory supremacy need not perpetuate itself. The reason is simple: the preferred marriage without frictions (which might allow long-term superiority) may be with an inheritor of the rival group. The general model can be modified to accommodate clan rivalry in the marriage market by defining $\mathcal{N} > 1$ groups and assuming that marriages can only be concerted within each of these groups.

The case of Germany is substantially different. During the Middle Ages, its closest equivalent was the Holy Roman Empire. According to Eulau [1941], we can assimilate the Empire to a quasi-federation, although this view is controversial. The Holy Roman Empire was not a uniform country as was France or the United Kingdom. Instead, it was conformed by principalities, duchies, counties and Free Imperial Cities, each ruled by a prince (a generic title). Rulers enjoyed a large degree of autonomy. Another important difference lies in the way in which the Emperor was succeeded. Early unified states relied on inheritance rules to pass the title of king across generations while the Holy Roman Empire elected the Emperor from its princes. Upon the Emperor's death, seven electors had the right to choose, in an election, who would succeed him. To be elected as Emperor, a prince would have to convince their peers, usually through promises to grant them more rights. Therefore, the emperor himself wielded little power within the empire in terms of capacity to intervene in state affairs. He even lacked an imperial tax-system and his revenue depended on the income generated by the lands he controlled as a prince, a reason for which Johanek concludes that "the emperor was not really in a position to govern the Empire" (see Hansen [2000, p. 295]). As a final characteristic, we should remember that Holy Roman Empire princes practised partible inheritance with the exception being the lands of secular electors since the Golden Bull of 1356, which forced them to practice primogeniture to avoid land division (Whaley [2012, p. 27]). In the model, these characteristics can be replicated setting $\gamma = 0$, it is, certainty for a Count that his kin will not rule the next period.[34] Accordingly, we should expect no investment in state-building nor in army. This is partly consistent with reality: the Empire lacked an imperial tax-system and Ertman [1997, p. 237] indicates that only since 1450 the Empire began building an administration as a response to outside pressure. Notice that the process of state creation began under the wealthy and long lasting Habsburg dynasty. Regarding military conquests, we should also expect none following $\gamma = 0$. Indeed, inheritance (the Kingdom of Burgundy) and marriage (Italy) represented the largest territorial expansions for the Empire. Lack of military growth is consistent with the organisation of the Empire since neither the Emperor nor princes would have benefit from it: conquered lands would have been introduced as new states.

Despite the fact that the Empire as a whole only began to build an state relatively late, the states that formed it were more willing to do so. In that case, and despite the fact

[34] This is not completely accurate since some dynasties hold the title of Emperor for many years, notably de Habsburgs who occupied the throne since 1438 until 1806 with only one exception. In any case, the election process made unsure the continuity of a dynasty as a ruler.

that partible inheritance was common, the enlargement of state or administration would be reasonable since heirs would enjoy any improvement arising from them. Intra-state battles and feuding were not a major concern since the Landfrieden act of 1152 prevented them, although conflicts arose, for instance the Swabian War of 1499 or the War of the Jülich succession of 1609-1614. Nonetheless, princes sought to increase their state capacity as is explained in Evans and Wilson [2011, Ch. 4]. A more efficient and bureaucratic apparatus increased their taxation income which could be used to raise armies. This process was specially true for Savoy and Brandenburg who tried to expand their domains at the expenses of weak neighbours. Finally, lack of unification through marriages arranged between state heirs was a consequence of the inheritance policy. If the number heirs exceeds two, marriages cannot overcome the trend towards state multiplication caused by state division. Moreover, partitioned inheritances in the Holy Roman Empire avoided entailing lands to daughters as much as possible which exacerbates the effect by over-representing men in the marriage market.

7. Conclusion

The present paper analyses how two different inheritance systems can affect the state capacity of a society based in land production. All relevant decisions are taken by agents called Counts who are modelled assuming that they care about consumption and the influence of his (or her) dynasty, which can have measured by the size of their county. Since this measure is also correlated with the income the next generation can attain, the previous statement can be rephrased as that Counts also care about the well-being of their successors. In order to increase county size, Counts must invest in state capacity, which allows to rise taxes, and in the creation of an army. We ignored any externality state creation could have on the production process, for instance the construction of infrastructures. The model suggests that male-cognatic primogeniture could lead to higher state capacity levels than absolute primogeniture does in the long run. Despite that, in a short and mid-term perspective absolute primogeniture seems to perform best. Since during the Middle Ages most countries adopted male-cognatic primogeniture it should also be superior to the absolute primogeniture in the short run in some sense: male inheritors may be better prepared to defend a fief, the importance of maintaining a county within a family might be the first priority (as would be with lexicographical preferences), etc. We should also consider the static version of the model: at each period of time, given a distribution of counties, it is always optimal to opt for male-cognatic primogeniture rather than its alternative. This is so because, under such regime, the probability to have a male successor is maximal. Therefore, if Counts ignore the dynamical aspect of the model, male-cognatic primogeniture becomes the best option.

The mechanism that generates the effects described is twofold. From one point of view, a greater certainty that Count's kin will continue ruling (captured as different disCount rates) encourages expenditures in state and army creation. However, a Counterbalancing force is found in marriages. A marriage creates a new county which is wealthier than its constituents alone and this increases both kinds of expenditures. According to our simulations, the indirect effect of size is more important than the direct effect during the initial periods and changes afterwards. Initially, counties under absolute primogeniture tend to be larger than its male-cognatic Counterparts. As time advances, the difference is softened and both regimes generate a similar distribution in the long run for counties that are able to invest. Thus, counties being equal, the direct effect implies greater expenditures in state creation.

It can be argued that the model generates a marriage whenever a male and a female are available which is not realistic. Firstly, we do not take into account possible enmities between counties. This can be alleviated by introducing $\mathcal{N} > 1$ allied groups and allowing marriages only within each group, as in the Italian case. Secondly, in the model nothing prevents a marriage between two siblings when a county splits or between close relatives in a more general case. This is in clear contradiction with the concepts about endogamic marriages. Roman Civil Law forbid marriages within 3 degrees of consanguinity and Canon Law forbid them within 4 degrees, see De Colquhoun [1851, p. 513]. Moreover, the church increased the degrees of consanguinity up to seven, to be reduced to four in 1215 (Bouchard [1981]). According to Davidson and Ekelund Jr [1997] this made dynastic development more difficult which in turn benefited the Church by reducing external threats. Durey [2008] explains that prohibiting these marriages allowed to divert rents from nobles to the Church. In any case, a ban on consanguinity clearly represented an impediment to marriages we are not considering in the model.

To maintain tractability and simplicity, the model ignores many strategic aspects Counts could adopt: firstly, they could internalise the effects of their decisions on war expenditures; and secondly, marriages could be used to form alliances. Neither of these aspects are considered so extensions can investigate its effects. Also, in the present model commoners are passive agents who do not make any decisions. The model can be augmented allowing them to choose their fertility and consumption endogenously. Finally, we should comment that the results here presented represent the slowest possible evolution of state capacity. This follows from the value of $\phi \approx 1$, as well as from ignoring any positive externality of state capacity on production. The introduction of such externality would reward states which are able to invest in state capacity with additional income. As a consequence, this countries could raise larger armies or invest more, giving them a subsequent and long-lasting advantage. Similarly, a higher value of ϕ would have allowed winning countries to gain more land, which in turn raises their income.

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Appendix A. Proof of Proposition 1

Throughout the Appendices we do not use sub and super-indices in order to simplify notation.

We shall show that there exists one and only one solution to the optimisation problem. We first consider the case of an internal solution in which $b^\star > 0$ and $g^\star > 0$. We can write the optimisation problem in Equations 3.3 and 3.4 as max $F(b,g) = \ln b + \gamma \ln \left(b^\phi (1 + A + g)\right)$, s.t. $b > 0, g \geq 0, b > 0$.

(1) Suppose that optimal $b^\star > 0$, $g^\star > 0$ and $c^\star > 0$. These variables are determined by the first order conditions which can be written as:

$$\begin{cases} 0 = & \frac{\gamma}{1+A+g} - \frac{p_g(1+(A+g)\kappa)^2 - R(1-\psi)}{(1+(A+g)\kappa)^2\left(\psi R+(1-\psi)R\frac{A+g}{1+(A+g)\kappa} - p_b b - p_g g\right)}, \\ 0 = & \frac{\gamma\phi}{b} - \frac{p_b}{\left(\psi R+(1-\psi)R\frac{A+g}{1+(A+g)\kappa} - p_b b - p_g g\right)}. \end{cases}$$

We can isolate b in the second equation and obtain

$$b = B(g) = \frac{\phi(1+A+g)\left[p_g(1+(A+g)\kappa)^2 - R(1-\psi)\right]}{p_b(1+(A+g)\kappa)^2}.$$

Notice that $\text{C1}: b > 0 \implies p_g(1+(A+g)\kappa)^2 > R(1-\psi)$. Upon substitution in the first equation the existence of a solution amounts to show that the function defined by

$$G(g) = \frac{\gamma}{1+A+g} - \frac{p_g(1+(A+g)\kappa)^2 - R(1-\psi)}{(1+(A+g)\kappa)^2\left(R\psi+(1-\psi)R\frac{A+g}{1+(A+g)\kappa} - p_g g - \frac{(1+A+g)\phi(p_g 2 - R(1-\psi))}{(1+(A+g)\kappa)^2}\right)}$$

has a root. Since we assumed an interior solution, $g > 0$. Considering the condition for $b > 0$ we must also have $\text{C2}: -c(1+(A+g)\kappa)^2 = p_g(1+(A+g)\kappa)^2(g+(1+A+g)\phi) - R(1-\psi)[(1+A+g)\phi+(A+g)(1+(A+g)\kappa)] - \psi R(1+(A+g)\kappa)^2 < 0$. We next show that there exists one and only solution for this problem. For A large enough, condition C2 is violated. In this case the solution to $G(g) = 0$ implies $g^\star < 0$. Since by assumption $g^\star > 0$, the optimal solution is a corner solution with $g^\star = 0$. Assuming that both conditions hold, notice that $G(g)$ has a discontinuity. Let $\hat{g}$ be such that $\left(\psi R + (1-\psi)R\frac{A+\hat{g}}{1+(A+\hat{g})\kappa} - p_b B(\hat{g}) - p_b\hat{g}\right) = 0$. We can compute $\lim_{g\to\hat{g}^-} G(g) = -\infty$, $\lim_{g\to\hat{g}^+} G(g) = +\infty$, $\lim_{g\to 0} = +\infty$ and $\lim_{g\to+\infty} = 0$. Then, $g^\star \in [0,\hat{g}]$. Moreover, since $G(g)$ is decreasing in g in all its domain, by the intermediate value theorem there must be one and only one $g^\star | G(g^\star) = 0$. Existence and uniqueness of $b^\star$ follows from existence and uniqueness of $g^\star$and the functional form for $B(g)$.

(2) We consider the corner solution with $b^\star > 0$, $c^\star > 0$ and $g^\star = 0$. The optimal solution can be obtained setting $g = 0$ and re-optimising $F(b, 0)$. The first order condition in that case can be written as

$$b^\star = \frac{R\gamma\phi(A+\psi+A(\kappa-1)\psi)}{p_b(1+A\kappa)(1+\phi\gamma)} > 0.$$

(3) We finally show that the optimal point corresponds to a maximum.

(a) Consider, first, the interior case and write

$$gg = \frac{\partial^2 F(b,g)}{\partial g^2} = -\frac{\gamma}{(1+A+g)^2} - \frac{2R\kappa(1-\psi)}{(1+(A+g)\kappa)^3 c} - \frac{\left(p_g(1+(A+g)\kappa)^2 - R(1-\psi)\right)^2}{(1+(A+g)\kappa)^4 c^2} < 0,$$

$$bb = \frac{\partial^2 F(b,g)}{\partial b^2} = -\frac{\gamma\phi}{b^2} - \frac{p_b{}^2}{c^2} < 0,$$

$$bg = \frac{\partial^2 F(b,g)}{\partial b \partial g} = -\frac{p_b\left(p_g(1+(A+g)\kappa)^2 - R(1-\psi)\right)}{(1+(A+g)\kappa)^2 c^2} < 0.$$

Let $\mathcal{H} = \begin{bmatrix} bb & bg \\ bg & gg \end{bmatrix}$ be the Hessian matrix evaluated at the optimum. Clearly $\operatorname{tr}(\mathcal{H}) < 0$ and its determinant is given by

$$\left(\frac{p_b{}^2}{c^2} + \frac{\gamma\phi}{b^2}\right)\left(\frac{\gamma}{(1+A+g)^2} + \frac{2R\kappa(1-\psi)}{(1+(A+g)\kappa)^3 c}\right) + \frac{\gamma\phi}{b^2}\left(\frac{\left(p_g(1+(A+g)\kappa)^2 - R(1-\psi)\right)^2}{(1+(A+g)\kappa)^4 c^2}\right) > 0.$$

Therefore, its eigenvalues are negative and the point corresponds to a maximum.

(b) In the case of a corner solution, the second derivative with respect to b is equal to $-\frac{\gamma\phi}{b^2} - \frac{p_b{}^2}{c^2} < 0$ and the solution is a maximum, too.

Appendix B. Proof of Proposition 2

All the derivatives for the interior solution can be written as follows:

$$\frac{\partial b}{\partial \omega} = \frac{\frac{\partial^2 F(b,g)}{\partial b \partial g}\frac{\partial^2 F(b,g)}{\partial g \partial \omega} - \frac{\partial^2 F(b,g)}{\partial g^2}\frac{\partial^2 F(b,g)}{\partial b \partial \omega}}{\frac{\partial^2 F(b,g)}{\partial b^2}\frac{\partial^2 F(b,g)}{\partial g^2} - \frac{\partial^2 F(b,g)}{\partial b \partial g}},$$

and

$$\frac{\partial g}{\partial \omega} = \frac{\frac{\partial^2 F(b,g)}{\partial b \partial g}\frac{\partial^2 F(b,g)}{\partial b \partial \omega} - \frac{\partial^2 F(b,g)}{\partial b^2}\frac{\partial^2 F(b,g)}{\partial g \partial \omega}}{\frac{\partial^2 F(b,g)}{\partial b^2}\frac{\partial^2 F(b,g)}{\partial g^2} - \frac{\partial^2 F(b,g)}{\partial b \partial g}}.$$

Notice that the denominator is the value of the determinant of $\mathcal{H}$ defined above, which is positive. Therefore, we can focus solely on the numerators for which we present simplified forms.

(1) Derivatives for b, internal solution:

(a) With respect to A : the numerator is equal to

$$\frac{p_b p_g\left(c\gamma\left(1+(A+g)\kappa\right)^3 + 2R\kappa\left(1+A+g\right)^2\left(1-\psi\right)\right)}{c^3\left(1+A+g\right)^2\left(1+(A+g)\kappa\right)^3} > 0.$$

(b) With respect to R : we were unable to show analytically whether the expression is positive or negative when $g > 0$. A numerical analysis suggests that b increases in R when $g > 0$.

(2) Derivatives for g, internal solution:

(a) With respect to A : the numerator is equal to

$$\left(\frac{{p_b}^2}{c^2}+\frac{\gamma\phi}{b^2}\right)\times$$
$$\left(-\frac{\gamma}{(1+A+g)^2}+\frac{R\left(-2c\kappa(1+(A+g)\kappa)+p_g(1+(A+g)\kappa)^2-R(1-\psi)\right)(1-\psi)}{c^2(1+(A+g)\kappa)^4}\right)-$$
$$-\frac{{p_b}^2R(p_g2-R(1-\psi))(1-\psi)}{c^4(1+(A+g)\kappa)^4}.$$

Notice that $\left(-2c\kappa\left(1+\left(A+g\right)\kappa\right)+p_g\left(1+\left(A+g\right)\kappa\right)^2-R\left(1-\psi\right)\right)<0$ and, therefore, the numerator is negative.

(3) With respect to $R:$ the numerator is equal to

$$\frac{1}{c^3(1+(A+g)\kappa)^3}\left[{p_b}^2\left(1+\left(A+g\right)\kappa\right)\left(1-\psi\right)+\frac{1}{b^2}c\gamma\phi\times\right.$$
$$\times\left[c\left(1+\left(A+g\right)\kappa\right)\left(1-\psi\right)+\left(p_g\left(1+\left(A+g\right)\kappa\right)^2-R\left(1-\psi\right)\right)\right.$$
$$\left.\left(A+g+\psi+\left(A+g\right)\left(\kappa-1\right)\psi\right)\right]]>0.$$

(4) If $A_t^i>A_t^j$, then $A_t^i+g_t^i>A_t^j+g_t^j:$

We show that $A+g$ increases in A. Consider first the partial derivative of g with respect to A which is negative. If its absolute value is smaller than one, then $A+g$ will be increasing in A. The partial derivative can be written as

$$\frac{\partial g}{\partial A}=$$
$$=-\left(\frac{p_b^2}{c^2}+\frac{\gamma\phi}{b^2}\right)\left(\frac{\gamma}{(1+A+g)^2}+\frac{2R\kappa(1-\psi)}{c(1+(A+g)\kappa)^3}\right)+\frac{\gamma\phi}{b^2}\frac{R\left(p_g(1+(A+g)\kappa)^2-R(1-\psi)\right)(1-\psi)}{c^2(1+(A+g)\kappa)^4}=$$
$$=\frac{-d_1+d_2}{d_1+d_3},$$

were its numerator is negative and its denominator positive. Finally, $-d_1+d_2<0$ and $d_1+d_3>0$ implies that $0>\frac{-d_1+d_2}{d_1+d_3}>-1$.

(5) Derivatives for b, corner solution:

(a) With respect to $A:$

$$\frac{\partial b}{\partial A}=\frac{R\gamma\phi\left(1-\psi\right)}{p_b\left(1+\kappa A\right)^2\left(1+\gamma\phi\right)}>0.$$

(b) With respect to $R:$

$$\frac{\partial b}{\partial R}=\frac{\phi\gamma\left(A+\psi+A\left(\kappa-1\right)\psi\right)}{p_b\left(1+\kappa A\right)\left(1+\gamma\phi\right)}>0.$$

Appendix C. Alternative Characterisation of the Marriage Market Outcome

The linear nature of the problem presented in Propositon 4 allows it to be solved using linear optimisation techniques instead of applying the Gale-Shapley algorithm. We also provide an alternative solution method which does not rely on numerical optimisation:

Proposition 5. *The optimisation problem in Proposition 4 can be solved applying the following iterative pseudo-code:*

(1) *Sort the arrays* $\mathcal{M} = \{m_1, m_2, \ldots, m_M\}$ *and* $\mathcal{F} = \{f_1, f_2, \ldots, f_F\}$ *in decreasing order, it is,* $z_1 > z_2 > \ldots > z_Z$ *where* $z = \{m, f\}$, $Z = \{M, F\}$.

(2) *Set* $\mu_t^{i,k} = 0 \forall i \in \mathcal{M}, \forall k \in \mathcal{F}$.

(3) *While* $|\mathcal{M}|$ and $|\mathcal{F}| > 0$:
   - (a) *If* $\Psi(\mathcal{M}[1]) - \Psi(\mathcal{F}[1]) > d$ *remove the first male heir:* $\mathcal{M} = \mathcal{M} \setminus \mathcal{M}[1]$.
   - (b) *Else, if* $\Psi(\mathcal{F}[1]) - \Psi(\mathcal{M}[1]) > d$ *remove the first female heiress: update the array* $\mathcal{F} = \mathcal{F} \setminus \mathcal{F}[1]$.
   - (c) *Else,* $\mu_t^{i,k} = 1$ *where* $i = \mathcal{M}[1]$ *and* $k = \mathcal{F}[1]$.
     Update the arrays $\mathcal{M} = \mathcal{M} \setminus \mathcal{M}[1]$ *and* $\mathcal{F} = \mathcal{F} \setminus \mathcal{F}[1]$.

   We use $\mathcal{M}[1]$ *and* $\mathcal{F}[1]$ *to denote the first element of each respective array.*

The procedure takes advantage of the fact that, if the richest heir cannot marry the richest heiress because the distance between both is too large, then he cannot marry any heiress. The same argument is applied to heiresses. In the second step, we impose that every inheritor remains single, during the *while* statement we will update it for these who marry. Condition (a) checks that requirement for the richest male, if he fails that check we remove him from the list of candidates. In condition (b) we use the same procedure with heiresses. Finally, if both candidates belong to the same strata, we marry them. Since both lists are sorted, we are sure that they marriage maximises the objective function in Proposition 4. Finally, in the updating procedure we simply remove them from the list of candidates. We apply the algorithm as long as there are heirs and heiresses available. The third step states that if at the end of the procedure there are still male heirs available, they will not marry any heiress. The same is true for heiresses.

Appendix D. Sensitivity Analysis

Figure D.1. Sensitivity analysis

Absolute primogeniture

Male-cognatic primogeniture

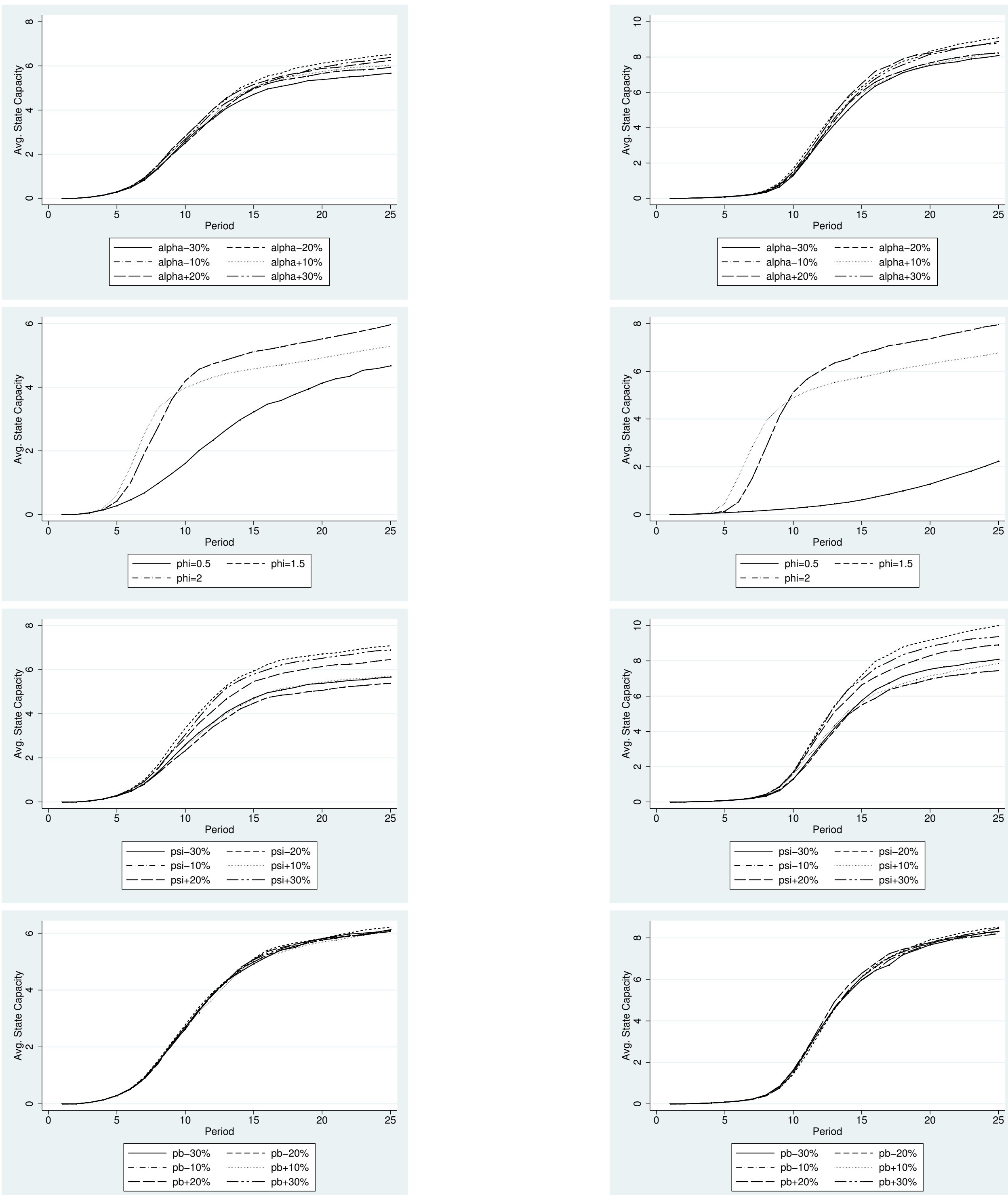
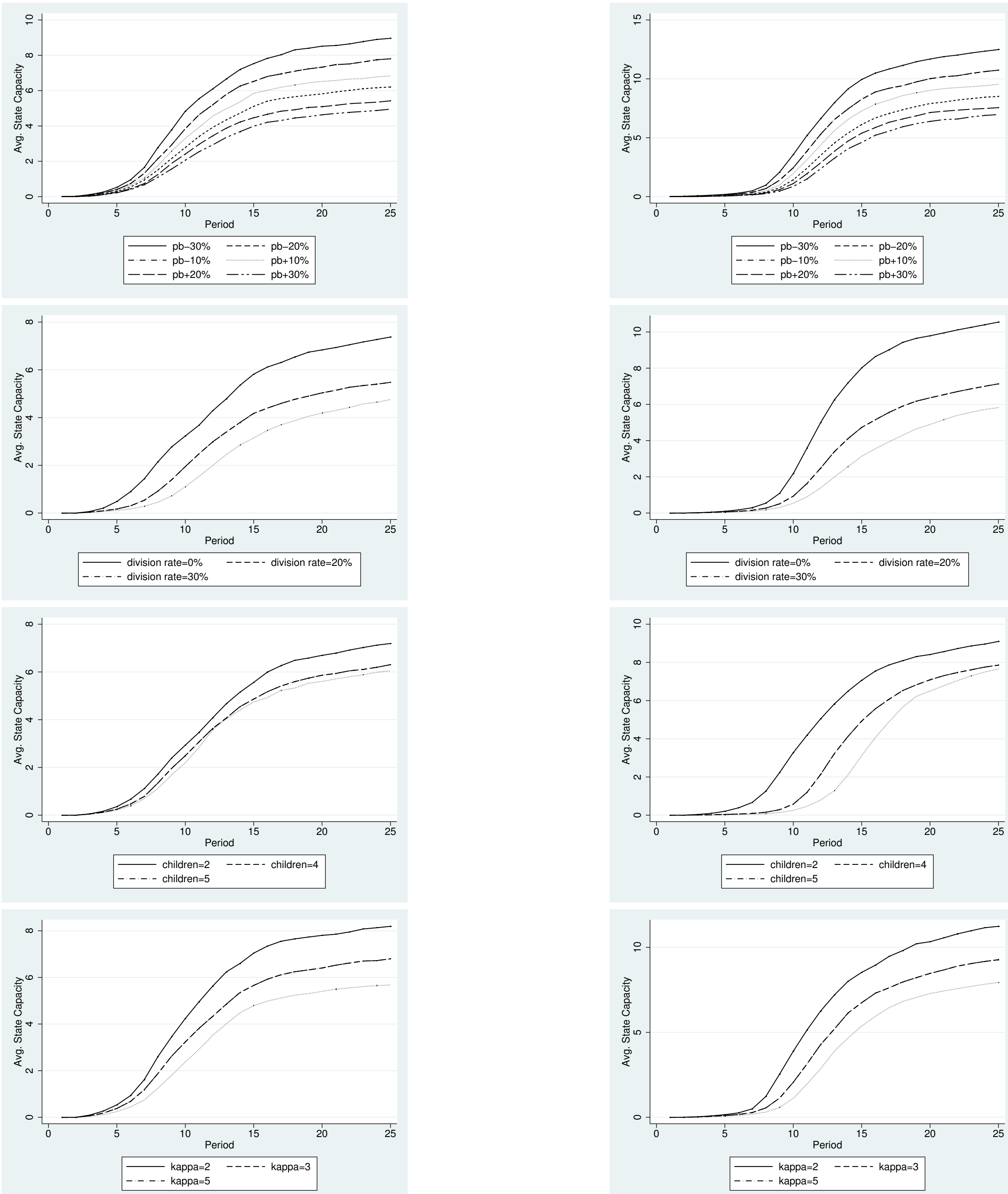


FIGURE D.2. Sensitivity analysis

Absolute primogeniture | Male-cognatic primogeniture



Appendix E. Lorenz Curves

Figure E.1. Lorenz curves for periods 2, 5 and 10

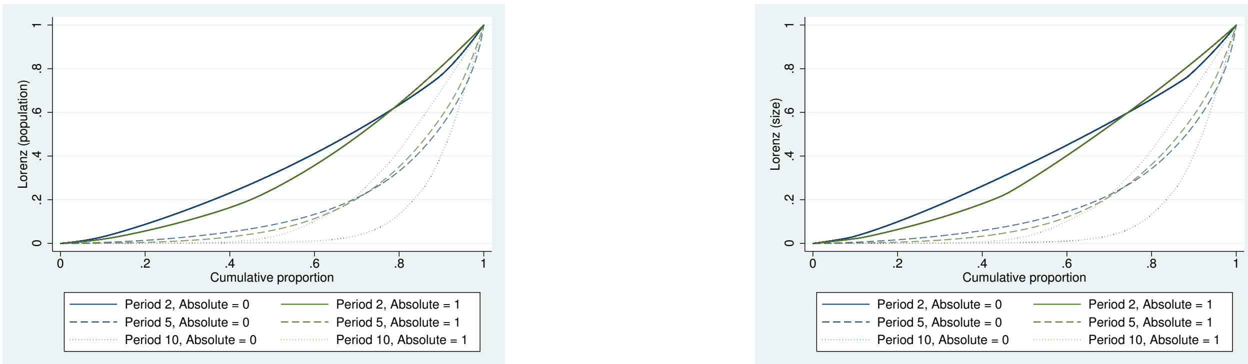
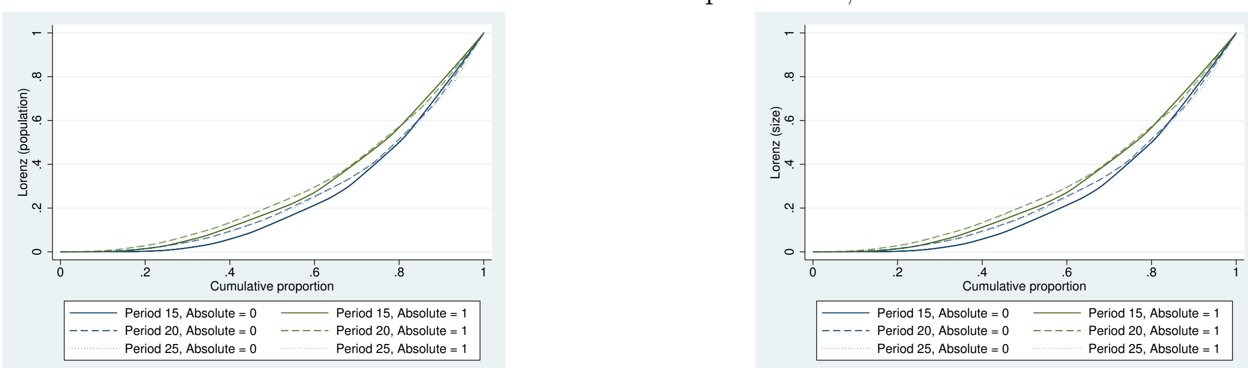


Figure E.2. Lorenz curves for periods 15, 20 and 25



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