Representation of a Continuous Settling Tank by Hybrid Partial Differential Non Linear Equations for Control Design

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Abstract: The representation of the main physical phenomena of continuous sedimentation within a settling tank (hydrodynamics of two-phase suspensions) is essential for the further design of a control of the quality of the solid-liquid separation. The model is still made of highly non-linear partial differential equations after simplifying assumptions on the behaviour of the solid and liquid flows and considering a one dimensional settler. Moreover effective solid stress appears when the solid particles concentration is above a given threshold which depends on the quality of the sludge. Then a mobile interface appears between two different behaviours. The settler is divided into two zones to represent this discrete phenomenon. Our goal is to develop a model that can be used for a further design of a control of the water quality at the top outlet of the settling tank. A hybrid state space representation is explained with the different dynamics in each of the two zones, the constitutive equations and the boundary conditions. The steady state profile is calculated. A simplified version of a settling tank model is simulated.

Keywords: Model; Partial Differential Equations (PDE); Constitutive equations; Non linear systems; Mobile interface; Boundary conditions; Simulation; Model for the control design; Continuous settling tank

1. INTRODUCTION

Continuous sedimentation/consolidation is an important solid/liquid separation process widely used in the pulp and paper, mining, chemical, food, and many other process industries as well as in wastewater treatment plants or in estuarine or coastal zones. These processes have many features in common, particularly the relative flow of solid particles and fluid as the underlying basic principle, Burger (2000), Concha et al. (2003).

Inside the mixture, the solid particles settle and form a loose bed, like a porous network, from the depth at which their concentration reaches a given threshold named percolation value. The two phases solid/liquid mixture flows through the porous network that leads to an increase of the drag force on the solid particles and reduces their settling velocity, Toorman (1996). The excess pore pressure slowly dissipates as the sludge bed is compacting, Chauchat et al. (2013).

1-D dynamic models of settling tanks represent yet the best compromise between complexity and the representativeness of the phenomena. Two types of models are described in the literature; first, the models that are based on solid particles mass balance coupled to a constitutive equation for the solid particles velocity, Diehl (2000), Queinnec (2001), David et al. (2009), Burger et al. (2013), ..., secondly the models that are based on both solid particles mass and momentum balances, Burger (2000), Chauchat et al. (2013), Garrido et al. (2003), ... In the second case constitutive equations have to be used for the pressure, the effective solid stress and the drag force. In the first case Takacs et al. (1991) extended the flux approach to low concentrations and proposed a double exponential settling velocity model based on Vesilind expression, Vesilind (1968). It has been used to model settling tanks in normal operating conditions. But Queinnec (2001) showed the limitation of this first approach where the constitutive equation for the solid particles velocity depends only on the solids concentration, Kynch (1952). Cadet et al. (2015) concluded that the Takacs method often works satisfactorily in normal operating conditions but, during extreme events such as storms, the solid particles concentration may be decreasing with depth and the Takacs method fails. Moreover the simulations produced by the Takacs method are qualitatively different for different number of layers, and increasing the number of the layers deteriorates the model performance.

All these observations highlight the need to deepen methods that effectively represent all operating conditions in continuous sedimentation/consolidation to be able to de-

⋆ This work has been realized on behalf of the French Persyval program and MODSIM-DENSE project.

Manuscript 1178 submitted to 21st IFAC World Congress, 2020.
Received November 8, 2019.

sign a control effective at any time and in any operating condition.
This paper presents a nonlinear PDE model of the settling tank with a moving interface. It considers neither the semi-empirical Vesilind hindered settling velocity, Vesilind (1968) nor the Takacs semi-empirical expression, Takacs et al. (1991), and is based on mass and momentum balance equations. Our goal is to develop a model that can be used for a further design of a control of the water quality at the top outlet of the settling tank. To reach this goal, the model must take into account the various behaviours of the mixture inside the settling tank and not only the nominal operation. Therefore the solid particles concentration at the top of the tank has to remain equal to zero and this is the reference to follow and not a boundary condition of the model.
The variables (continuous and boolean) and equations (partial differential and algebraic) to describe the main physical phenomena taking place in the settling tank (hydrodynamics of two-phase suspensions) have been identified and are presented in a structured way. A discontinuous phenomenon has been detected and integrated into the dynamic model which makes it hybrid, Valentin et al. (2007). Appropriate initial and boundary conditions are discussed.
A first attempt was presented in Cadet et al. (2015), with a modeling approach that used the Vesilind equation.

2. A 1D DYNAMIC PHYSICAL MODEL

2.1 General 1D Mass and Momentum balances

The dynamic model describing the behaviour of the sludge in the settling tank comes from the mass and momentum balances under the following simplifying assumptions:

1. The flow is assumed to be plug flow
2. There is no biological activity in the settling tank Burger et al. (2011)
3. The solid particles have the same size and shape, Garrido et al. (2003), Burger et al. (2011), David et al. (2009), Diehl (2000)
4. There is a uniform particle concentration at a given depth, then 1D modeling is relevant, David et al. (2009), Diehl (2000)
5. The vessel wall friction is negligible
6. Suspensions are floculated completely before sedimentation
7. The solid particles are small with respect to the containing vessel and have the same density, Garrido et al. (2003)
8. Solid particles and fluid are incompressible (No creation of flocks or filaments), Garrido et al. (2003), David et al. (2009). Then solid density, ρ_s, and liquid density, ρ_l are constant, Diehl (2000), Chauchat et al. (2013)
9. There is no mass transfer between the solid particles and the fluid, Garrido et al. (2003)
10. The settler has a constant cross-sectional area
11. The liquid and solid phases completely fill the settler, then its volume is constant
12. The inlet (sludge feeding) is modelled by a point and the mixture leaving the inlet is distributed instantaneously and evenly over the entire cross-section, Diehl (2000).

The settling tank is modelled considering two zones, clarification and compression zones, associated with two different types of physical behaviours (with at least two different equations). It has one inlet (sludge feeding inside the tank) and two outlets (clarified water at the top and compressed sludge at the bottom). The sludge feeding takes place at $z = z_f$, which is at the end of the feeding pipe. There is a moving interface at depth $z = z_c(t)$ where a change of behaviour takes place due to contact forces between the solid particles appearing when they touch each other. Burger (2000) assumes that the water is clear at the top of the tank (zero solid particles concentration), then a three zone schematic view is presented. But, as our goal is to develop a model for control design, a zero solid particles concentration at the top of the tank is the objective to follow and a third zone in the upper part of the tank is not taken into consideration. Let $\varepsilon_s(z,t)$ denote the solid particles volume fraction with t the time and z the depth from the settling tank surface. $\varepsilon_l(z,t)$ denotes the liquid volume fraction. The solide particles concentration is then $C_s(z,t) = \rho_s \varepsilon_s(z,t)$. Let $V_s(z,t)$ (m/s) denote the solid phase Eulerian average velocity and $V_l(z,t)$ (m/s) the liquid phase Eulerian average velocity. A schematic view is given figure in 1.

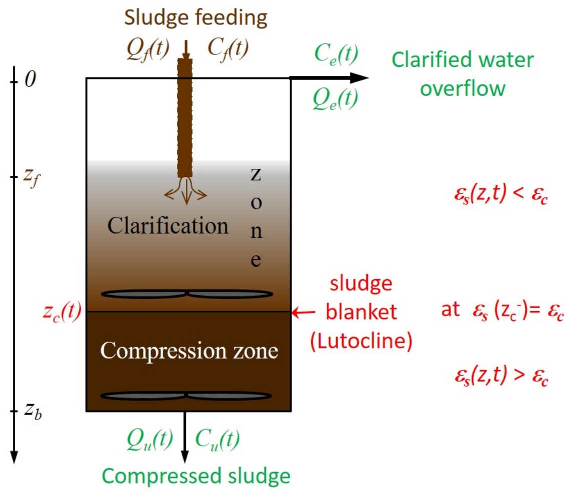

Figure 1. One-dimensional schematic view of a Settling Tank.

Remark 1. To improve the readability, a abuse of notations is done by omitting (z,t) in the following equations.

If ρ_s is the solid phase (particles) density (kg/m^3) and ρ_l the liquid phase density (kg/m^3), the mass balances can be written as the next two partial differential equations (PDE's) for the liquid phase and the solid phase, in the most general way:

Liquid phase mass Balance:

$$\partial_t(\rho_l \varepsilon_l) = -\partial_z(\rho_l \varepsilon_l V_l) + f_1(C_f, Q_f)\delta(z - z_f) \quad (1)$$

Solid phase mass Balance:

$$\partial_t(\rho_s \varepsilon_s) = -\partial_z(\rho_s \varepsilon_s V_s) + f_2(C_f, Q_f)\delta(z - z_f) \quad (2)$$

Manuscript 1178 submitted to 21st IFAC World Congress, 2020.
Received November 8, 2019.

f_1 and f_2 are the contribution of the sludge feeding to the variation of volume fraction. They depend on the solid phase mass concentration of the sludge at the input, C_f and on the sludge flow Q_f at the input.
As well, the momentum balances can be written as two PDE's for the liquid phase and the solid phase, Chauchat (2007):

Liquid phase momentum balance:

$$\partial_t(\rho_l \varepsilon_l V_l) = -\partial_z(\rho_l \varepsilon_l V_l^2) + \varepsilon_l \rho_l g - \partial_z P_l - r(V_l - V_s) + f_3(C_f, Q_f)\delta(z - z_f) \quad (3)$$

with:

$\varepsilon_l \rho_l g$	volumic gravitational force (body force)
$-\partial_z(P_l)$	gradient of the hydrodynamic pressure within the liquid phase
$-r(V_l - V_s)$	Stokes like drag force i.e. liquid-solid dynamic interaction force per unit volume standing for viscous friction between the two phases (generalization of the Stokes equation (as well Darcy equation)) Garrido et al. (2003), Chauchat et al. (2013). r is the resistance coefficient.

Solid phase momentum Balance:

$$\partial_t(\rho_s \varepsilon_s V_s) = -\partial_z(\rho_s \varepsilon_s V_s^2) + \varepsilon_s \rho_s g - \partial_z P_s + r(V_l - V_s) + f_4(C_f, Q_f)\delta(z - z_f) \quad (4)$$

where f_3 and f_4 are the contribution of sludge feeding to the momentum variation. They depend on the solid phase mass concentration of the sludge at the input, C_f and on the sludge flow Q_f at the input.

As the sludge volume is constant, the following algebraic equation is valid at any time and any depth:

$$\varepsilon_l(z,t) + \varepsilon_s(z,t) = 1 \quad (5)$$

As solid particles and fluid are incompressible, the average velocity of the mixture, denoted $V_m(z,t)$, can be calculated by the following algebraic equation:

$$V_m(z,t) = \varepsilon_l(z,t)V_l(z,t) + \varepsilon_s(z,t)V_s(z,t) \quad (6)$$

Moreover by using assumption 8 and equation (5), the sum of both equations (1) and (2) gives:

$$\partial_z V_m(z,t) = \left[\frac{f_1(C_f,Q_f)}{\rho_l} + \frac{f_2(C_f,Q_f)}{\rho_s}\right]\delta(z - z_f) \quad (7)$$

Then V_m is piecewise constant according to z and can be written:

$$V_m(t) = \varepsilon_l(.,t)V_l(.,t) + \varepsilon_s(.,t)V_s(.,t) \quad (8)$$

Therefore, $\varepsilon_l(z,t)$ can be calculated from (2) and (5), and $V_l(z,t)$ can be deduced from (4) and (8).

2.2 Constitutive equations

P_s et P_l pressures may be written by the two following algebraic expressions in terms of the pore pressure (hydrodynamic pressure) P and the effective solid stress σ_e Burger (2000):

$$P_l = \varepsilon_l P \quad (9)$$
$$P_s = \varepsilon_s P + \sigma_e \quad (10)$$

Constitutive (closure) expressions come from experimental data. Different equations have been proposed by various authors in several contexts (non-exhaustively):

For $\partial_z P$:	Burger (2000), Garrido et al. (2003)
For σ_e:	Burger (2000), Garrido et al. (2003), Burger et al. (2013), Chauchat et al. (2013)
For r:	Garrido et al. (2003), Chauchat et al. (2013),Toorman (1996)

We selected the constitutive equations presented by Garrido et al. (2003) related to the treatment of tailings from a Chilean copper mine for P and σ_e. The constitutive equation for r is presented by Chauchat et al. (2013) and based on Toorman (1996):

$$\partial_z P = Q_0^{-1}\Delta\rho g \varepsilon_s (1-\varepsilon_s)^{-n_p}(V_s - V_m) \quad (11)$$
$$\sigma_e = \alpha\sigma_0 \frac{\varepsilon_s^{n_s} - \varepsilon_c^{n_s}}{\varepsilon_c^{n_s}} \quad (12)$$
$$r = \rho_l g/K \text{ with } K = A_k \varepsilon_s^{-2/(3-n_r)} \quad (13)$$

with Q_0, n_p, σ_0, n_s, A_k and n_r different constant parameters characterizing the sludge or the settling tank and α, a boolean parameter which depends on the settler's zone and defined as follow:

$$\alpha(\varepsilon_s) = \begin{cases} 0 & \text{for } \varepsilon_s \le \varepsilon_c \\ 1 & \text{for } \varepsilon_s > \varepsilon_c \end{cases} \quad (14)$$

Let us note that the constitutive equation of σ_e depends on the zones of the settling tank as there is no effective solid stress in the clarification zone for small solid particles concentration. The formulation ensures that σ_e is a continuous function at $\varepsilon_s = \varepsilon_c$. Then the mixture inside the settling tank has two different behaviours. The need for continuous and boolean variables in the settler model makes it hybrid, Valentin et al. (2007). A full discussion of the significance of expressions (11) and (12) can be found in Garrido et al. (2003).

2.3 1D Hybrid Non Linear PDE Model

Then the settling tank hybrid nonlinear PDE's model can be written from:

- both solid particles mass and momentum general conservative laws (2) and (4),
- the two algebraic equations specific to this system (5) and (8),
- the four constitutive equations (11)-(14)
- the simplifying assumptions 1 to 12.

Both equations (2) and (4) can be rewritten in the simplified form below:

$$\partial_t \varepsilon_s = -\partial_z(\varepsilon_s V_s) + \frac{f_2(C_f,Q_f)}{\rho_s}\delta(z - z_f) \quad (15)$$

$$\partial_t(\varepsilon_s V_s) = -\partial_z(\varepsilon_s V_s^2) + \varepsilon_s g - \frac{\partial_z(\varepsilon_s P)}{\rho_s} - \frac{\partial_z \sigma_e}{\rho_s} + \frac{r(V_m - V_s)}{\rho_s(1-\varepsilon_s)} + \frac{f_4(C_f,Q_f)}{\rho_s}\delta(z - z_f) \quad (16)$$

Boundary conditions:
As solid particles and fluid are incompressible and as the total sludge volume is constant, mixture mass conservation of the solid phase and of the liquid phase lead to the algebraic relation below:

$$Q_f = Q_e + Q_u \tag{17}$$

From there, we consider that the inlet is located at $z = 0$ and that $P(0) = P_{atm}$, $\varepsilon_s(0,t) = C_f/\rho_s$, $V_s(0^+,t) = V_0$ and $V_m(z_b^-,t) = Q_u/A_T$.

3. THE HYBRID STATE SPACE MODEL

Equations (15) and (16) lead to the following system:

$$\partial_t\varepsilon_s = -\partial_z(\varepsilon_s V_s) + \frac{f_2(C_f,Q_f)}{\rho_s}\delta(z-z_f) \tag{18}$$

$$\begin{aligned}\partial_t V_s = &-V_s\partial_z V_s + g - \frac{\partial_z(\varepsilon_s P)}{\rho_s\varepsilon_s} - \frac{\partial_z\sigma_e}{\rho_s\varepsilon_s}\\ &+\frac{r(V_m-V_s)}{\rho_s\varepsilon_s(1-\varepsilon_s)} + f_{41}(C_f,Q_f)\delta(z-z_f)\end{aligned} \tag{19}$$

with $f_{41}(C_f,Q_f) = \frac{f_4(C_f,Q_f)-V_s f_2(C_f,Q_f)}{\varepsilon_s\rho_s}$, and $\sigma_e(\varepsilon_s)$, $\partial_z P(\varepsilon_s,V_s,V_m)$, $r(\varepsilon_s)$ and $\alpha(\varepsilon_s)$ given by the constitutive equations (11)-(12)-(13)-(14).
Let $\mathcal{X}$ be a Hilbert space with scalar product $\langle,\rangle_{\mathcal{X}}$ and $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ be linear operators, $\mathcal{A}$ from $D(\mathcal{A}) \subset \mathcal{X} \to \mathcal{X}$, $\mathcal{B}$ from $\mathbb{R}^n$ to $\mathcal{X}$ and $\mathcal{C}$ from $D(\mathcal{C}) \subseteq \mathcal{X}$ to $\mathbb{R}^m$, where n and m are the dimensions of the input and output states resp.

We consider the Cauchy problem with output $\Sigma(\mathcal{A},\mathcal{B},\mathcal{C})$ in Kalman form, as follows:

$$\varphi_t = \mathcal{A}\varphi + \mathcal{B}U + w,\ Y = \mathcal{C}\varphi, \tag{20}$$

where $w \in \mathcal{X}$ is considered as an unknown constant vector (disturbance) and $U : \mathbb{R}_+ \mapsto \mathbb{R}^n$ is the input that will be determined by the controller in the future, and Y the output to regulate. φ, the state space representation, is given by:

$$\varphi = \begin{pmatrix}\varepsilon_s(z,t)\\ V_s(z,t)\end{pmatrix}$$

After some manipulations of (18) and (19) and replacing $\partial_z\sigma_e$ with the derivative of equation (12), the hybrid state space representation of the settling tank is:

$$\begin{cases}\partial_t\varphi = \mathcal{A}(\varphi,\partial_z\varphi,P(\varphi,U),r,\alpha) + \mathcal{B}(\varphi,r)U + w\\ Y = \mathcal{C}(\varphi)\end{cases} \tag{21}$$

$$\text{with } \mathcal{A} = \mathcal{A}_1(\varphi,P(\varphi,U),\alpha)\partial_z\varphi + \mathcal{A}_2(\varphi,P(\varphi,U),r) \tag{22}$$

$$\text{and } \mathcal{A}_1 = -\begin{pmatrix} V_s & \varepsilon_s\\ \frac{P}{\rho_s\varepsilon_s} + \alpha\frac{\sigma_0 n_s\varepsilon_s^{n_s-2}}{\rho_s\varepsilon_c^{n_s}} & V_s\end{pmatrix}, \tag{23}$$

$$\text{and } \mathcal{A}_2 = \begin{pmatrix}0\\ g - \frac{\partial_z P}{\rho_s} - \frac{rV_s}{\rho_s\varepsilon_s(1-\varepsilon_s)}\end{pmatrix}, \tag{24}$$

$$\text{and } \mathcal{B} = \begin{pmatrix}0\\ \frac{r}{\rho_s\varepsilon_s(1-\varepsilon_s)}\end{pmatrix}. \tag{25}$$

The matrix $\mathcal{C}(\varphi)$ depends on the control objectives, $U(t) = V_m(t)$ ($\mathbb{R}^n = \mathbb{R}$) and:

$$w = \begin{pmatrix} f_2(C_f,Q_f)/\rho_s\\ f_{41}(C_f,Q_f)\end{pmatrix}\delta(z-z_f) \tag{26}$$

$\partial_z P(\varepsilon_s,V_s,U)$, $r(\varepsilon_s)$ and $\alpha(\varepsilon_s)$ are given by the constitutive equations (11), (13) and (14).

The determinant of $\mathcal{A}_1(\varphi,P(\varphi,U),\alpha)$ is:

$$\begin{aligned}det(\mathcal{A}_1) = &\frac{P(\varepsilon_s,V_s,U)}{\rho_s} - V_s(z,t)^2\\ &+\alpha(\varepsilon_s)\frac{\sigma_0 n_s\varepsilon_s(z,t)^{n_s-1}}{\rho_s\varepsilon_c^{n_s}}\end{aligned} \tag{27}$$

Remark 2. Let us note that the constitutive equation of σ_e and the control signal U (through the constitutive equation of P) can change the nature of the system.
Moreover, the determinant of $\mathcal{A}_1$ depends on α which means that the nature of the system may depend on the zone of the settling tank, above or below the moving interface located in $z = z_c(t)$, where $\varepsilon_s(z,t) = \varepsilon_c$. Such kind of remark was also made in Cadet et al. (2015) for such class of system with a modeling approach that uses the Vesilind equation.

4. STEADY-STATE PROFILE

After some manipulations, the steady-state profile is:

$$\partial_z\varepsilon_s = \frac{1}{\rho_s det(\mathcal{A}_1)}\left[\frac{r(V_m-V_s)}{(1-\varepsilon_s)} + \rho_s\varepsilon_s g - \varepsilon_s\partial_z P\right] \tag{28}$$

$$\partial_z V_s = -\frac{V_s}{\varepsilon_s}\partial_z\varepsilon_s \tag{29}$$

with $\partial_z P(\varepsilon_s,V_s,U)$, $r(\varepsilon_s)$ and $\alpha(\varepsilon_s)$ given by the constitutive equations (11), (13) and (14).

In this paper, a simplified steady-state profile with a sludge feeding at the top of the settling tank is considered. The settling tank parameters are $A_T = 707m^2$ and $z_b = 3$. The sludge characteristic parameters are $\rho_s = 2590kg/m^2$, $\rho_l = 1000kg/m^2$, $\varepsilon_c = 0.23$, and the values of the others parameters (in the constitutive equations) are taken similar to Garrido et al. (2003) paper. Two profiles are considered and given in figures (2)-(3).
The difference between both is the smaller variation of V_m for the second profile (indexed by the subscript a e.g. $\varepsilon_{s,a}$, P_a.)
Figure 2 shows that the sludge thickens and compresses

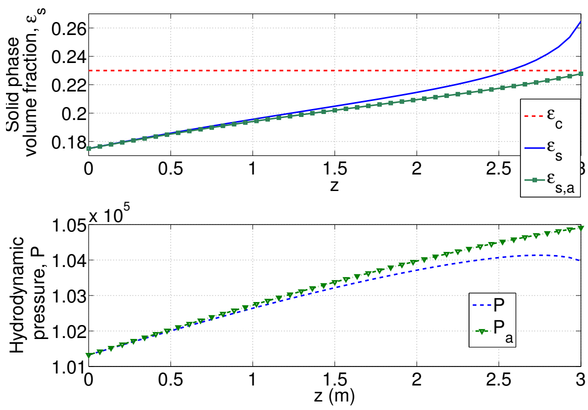

Figure 2. ε_s and P simplified steady-state profiles.

Manuscript 1178 submitted to 21st IFAC World Congress, 2020.
Received November 8, 2019.

regularly inside the tank. In the initial steady-state profile the interface takes place at depth $z_c(t) = 2,5m$ and, below the interface, the excess pore pressure slowly dissipates as the sludge bed is compacting.

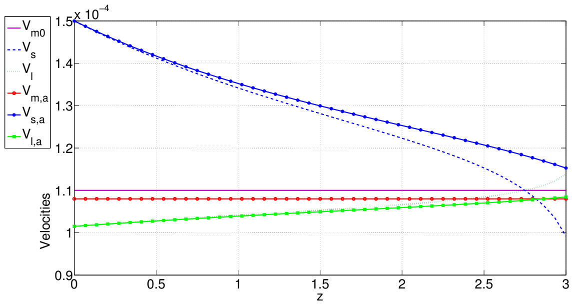

Figure 3. Velocities simplified steady-state profiles.

Figure 3 shows that the solid particles reduce more significantly their settling velocity below the moving interface. Figure 4 shows the presence of an effective solid stress and

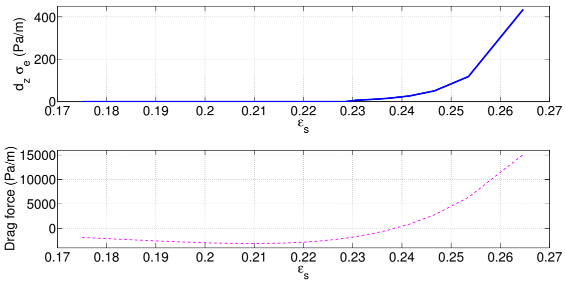

Figure 4. Effective solid stress and drag force simplified steady-state profile.

an increase of the drag force on the solid particles below the moving interface in the initial steady-state profile, in the bottom zone where the two phases solid/liquid mixture flows through the compressed porous network.
The following section is dedicated to the PDE simulation.

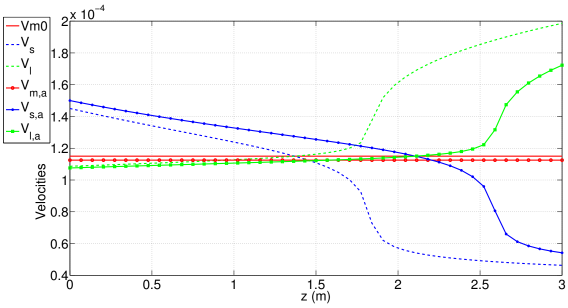

Figure 5. Velocities simplified steady-state profiles.

5. SIMULATIONS

The PDE are simulated with a theta-scheme, also called Chang & Cooper scheme, that has shown good properties for granular media and in particular for this class of systems, Buet et al. (2003).
The first steady-state profile defined by ε_s, V_s, P, $V_{m,0}$ above, gives the initial conditions of the simulation. A small variation is done on the input U of the system, in order to match the second steady-state profile given by $\varepsilon_{s,a}$,$V_{s,a}$ P_a, V_a. Let recall that the system is in open loop, and that no controller is implemented. The simulations are intended to validate the behaviour of the model for $t \simeq 6h$ in figure 6 and $t \simeq 3h$ in figure 7.

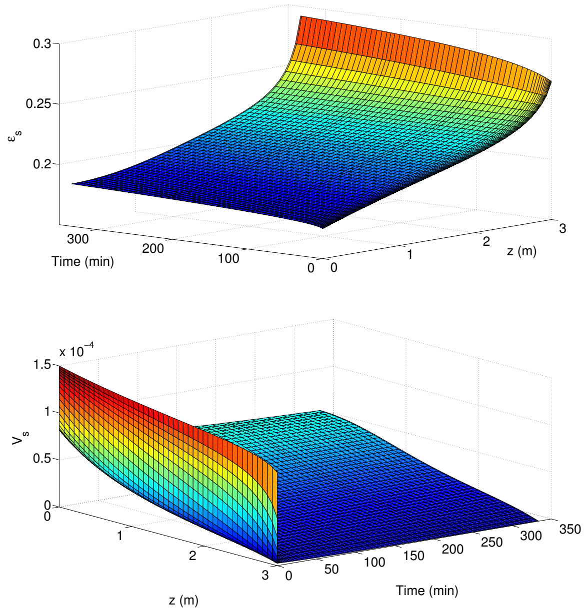

Figure 6. 3D simulation of solid phase volume fraction ε_s and solid particles velocity V_s.

If the behaviour is satisfactory, one can observe that the second steady-state profile is not reached after six hours in Fig. 6. It shows that the settling tank system is a very slow system, with delay phenomena. In the following figures,

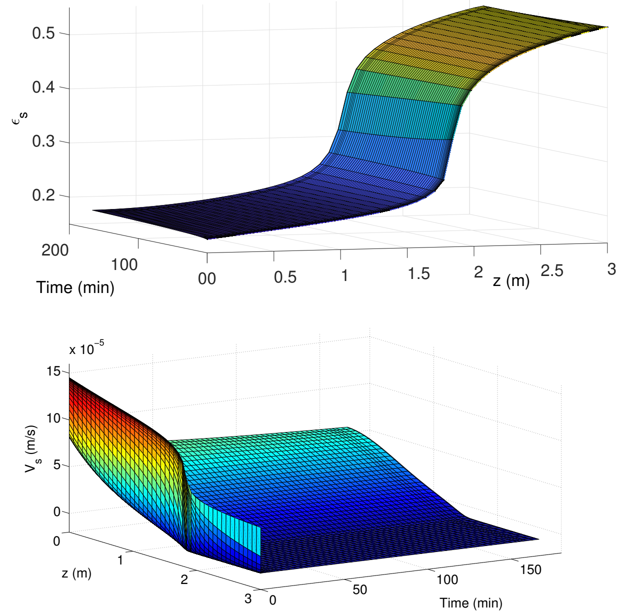

Figure 7. 3D simulation of ε_S and V_s

the initial and desired steady-state profile are changed (Fig

5) to show that the model developed takes into account more complicated cases (Fig. 7). Indeed the interface takes place at depths between $1.75m < z_c(t) < 2,25m$. Figure 7 shows that, below the interface, compaction of the sludge bed is more important than in the previous case (Fig. 6). The simulation performs well.

6. CONCLUSIONS AND PERSPECTIVES

In this paper, a hybrid partial differential non linear equations (Hybrid NL-PDE) representation of a continuous settling tank has been presented. It takes into account the moving interface between a bottom zone with compressed solid particles which form a porous network, and an upper zone with a lower concentration of solid particles.

Our goal is to apply this structured Hybrid NL-PDE representation of a settling tank to the secondary settler involved in wastewater treatment plants. Indeed in practice, settling tanks are still largely used but they may undergo dysfunction due to gravity settling problems or to the quantity and the quality of sludge. It is also often due to undesirable bacteria, which cause a phenomenon called sludge swelling (development of filamentous bacteria). Besides new technologies like membrane filtration are often proposed as alternatives to settling tanks but are not yet up to expectations. Therefore it is still interesting to model and optimize the behaviour of existing wastewater treatment unitary equipment's and to propose control strategies for a more efficient and compact installation. The variables and parameters ranges are very different from the treatment of tailings from mines (solid particles concentration and solid phase density are much lower, etc) which may change the model (especially the constitutive equations).

7. ACKNOWLEDGMENTS

The authors thank the LAGEPP DYCOP team for valuable discussions on these topics, especially F. Couenne, and the French Persyval program, with MODSIM-DENSE project, for financial support.

8. NOTATIONS

Index i stands for liquid phase or solid phase (particles).

A_T (m^2)	cylindric settling tank (thickener) surface
A_f (m^2)	Feeding pipe surface
A_u (m^2)	concentrated sludge output pipe surface
z_b (m)	cylindric settling tank depth
z_f (m)	feeding depth (fixed)
$\varepsilon_i(z,t)$	solid (liquid) phase volume fraction.
ε_c	solid volume fraction percolation (compression) threshold, sludge blanket level
ρ_i (kg/m^3)	solid (liquid) phase density
$\Delta\rho$ (kg/m^3)	solid and liquid density difference, $\Delta\rho = \rho_s - \rho_l$
$C_i(z,t)$ (kg/m^3)	solid (liquid) phase mass concentration, $C_i(z,t) = \rho_i \varepsilon_i(z,t)$
$C_f(t)$ (kg/m^3)	solid phase mass concentration of the feeding mixture
$V_i(z,t)$ (m/s)	solid (liquid) phase eulerian average velocity
$V_m(t)$ (m/s)	mixture (bulk) average velocity
$Q_e(t)$ (m^3/s)	clarified water effluent volume flow
$Q_u(t)$ (m^3/s)	compressed sludge discharge volume flow (it will be determined by the control system whose design is in progress)
$Q_f(t)$ (m^3/s)	sludge feeding volume flow
$P_i(z,t)$ (Pa)	solid (liquid) phase presssure
$P(z,t)$ (Pa)	excess pore pressure
$Patm$ (Pa)	atmospheric pressure, $Patm = 101325Pa$
$\sigma_e(\varepsilon)$	effective solid stress function (Pa)
r $(kgm^{-3}s^{-1})$	resistance coefficient of the drag force proposed by Darcy and Gersevanov in their semi-empirical expression used in the two-phase model

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Manuscript 1178 submitted to 21st IFAC World Congress, 2020.
Received November 8, 2019.