

Scanning gate tuning of mesoscopic transport in quantum-ring networks

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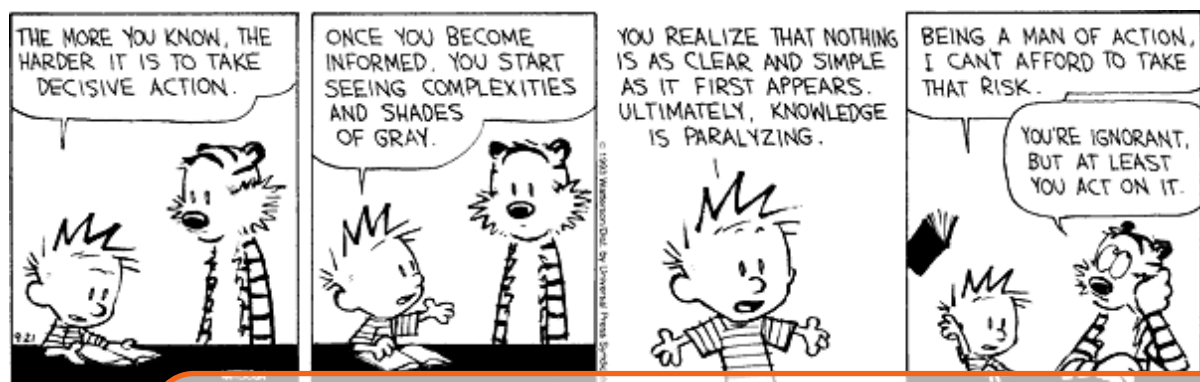
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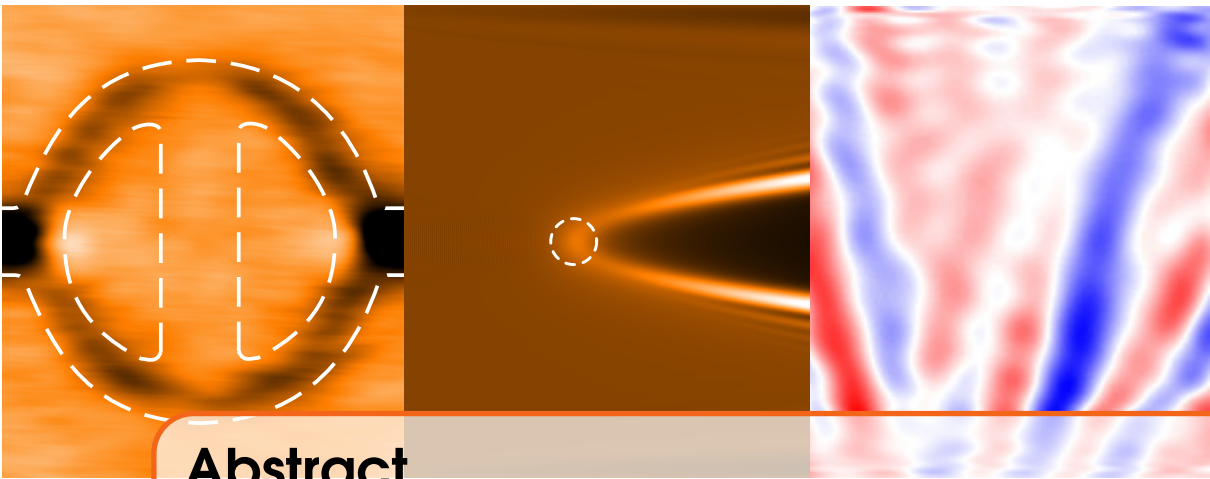
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Abstract

No matter the resistance network, if the resistances of a network are increased, the effective resistance between any two points can only increase. If they are decreased, it can only decrease. This affirmation is known as the Rayleigh's Monotonicity Law. In this thesis, we present two different results showing that this law is no longer valid in a nanoscale resistive network. This work aims to study puzzling electron transport signatures emerging as electrons flow through nanoscale electrically-conducting quantum-ring networks patterned within two-dimensional electron systems (2DES), where charge transport occurs in the ballistic and coherent regime at low temperature. In these so-called mesoscopic networks, quantum mechanical treatment of charge transport is necessary to analyse transport signature that would otherwise *appear* as paradoxical. We pinpointed two deviations from Rayleigh's monotonicity law can be presented as follows :

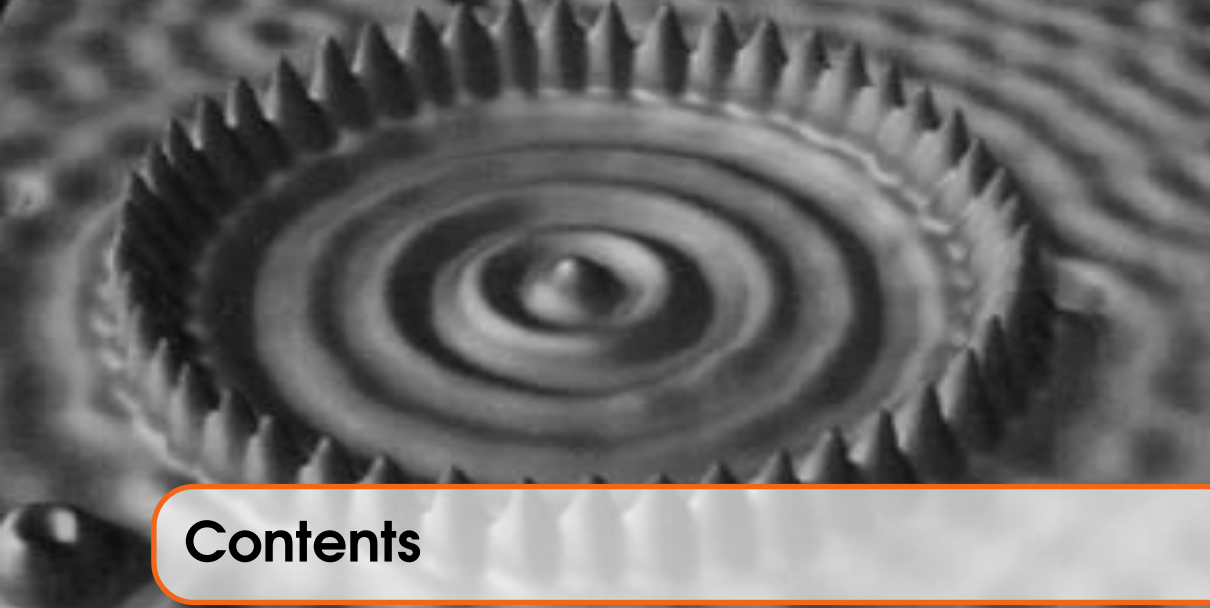
First, puzzling conductance variations analogous to the Braess paradox are encountered in a two-terminal mesoscopic network. The Braess paradox, formalized in game theory goes as follow : The suppression of an axis in a transport network can counter-intuitively improve the overall network transport performances. In analogy with this paradox, we observed that decreasing locally the electron density of one channel of the network paradoxically leads to an increased network electrical conductance. Our low temperature scanning gate microscopy experiments reveal different occurrences of such puzzling

conductance variations, thanks to tip-induced localized modifications of electron flow throughout the network's channels in the ballistic and coherent quantum regime of transport. The robustness of the puzzling behavior is inspected by varying the global 2DES density, magnetic field and the tip-surface distance. Depending on the overall 2DES density, we show that either Coulomb Blockade resonances due to disorder-induced localized states or Fabry-Perot interferences tuned by the tip-induced electrostatic perturbation are at the origin of transport inefficiencies in the network, which are lifted when gradually closing one channel of the network with the tip.

A second violation of monotonicity is proven in the ballistic regime of transport for a peculiar charge injection mechanism. Ballistic injection in a nanodevice is a complex process where electrons can either be transmitted or reflected, thereby introducing deviations from the otherwise quantized conductance. In this context, quantum rings (QRs) appear as model geometries: in a semiclassical view, most electrons bounce against the central QR antidot, which strongly reduces injection efficiency. Thanks to an analogy with Rutherford scattering, we show that a local partial depletion of the QR close to the edge of the antidot can counter-intuitively ease ballistic electron injection. On the contrary, local charge accumulation can focus the semi-classical trajectories on the hard-wall potential and strongly enhance reflection back to the lead. Scanning gate experiments on a ballistic QR, and simulations of the conductance of the same device are consistent, and agree to show that the effect is directly proportional to the ratio between the strength of the perturbation and the Fermi energy. Our observation surprisingly fits the simple Rutherford formalism in two-dimensions in the classical limit.

In addition to these two results, a discussion around various aspects of scanning gate microscopy and scattering by tip-induced potential in 2DES is presented.

In turn, this work provides useful tools in the perspective of building 'electron optics' devices, where a local modulation of the electrostatic potential enables, on one hand, to control Fabry-Pérot electron resonances and, on the other hand, to redirect the electron flow in a similar way as an optical lens curves light rays, allowing to study Rutherford-like ballistic scattering mechanisms.



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1. Introduction

If the specific resistance of any portion of the conductor be changed, that of the remainder being unchanged, the resistance of the whole conductor will be increased if that of the portion is increased, and diminished if that of the portion is diminished. This principle may be regarded as self-evident... James C. Maxwell in "Treatise on Electricity and Magnetism" ([1], p. 427)

1.1 Downscaling conductive networks

The resistance between arbitrary nodes in an electrically conductive network is a subject that is almost as old as the science of electricity itself. This simple problem is the opening door of many introductory physics courses through the teaching of Kirchhoff's node law [2]. Such an electrical network is pictured as nodes connected with variable resistors (see Fig. 1.1a). If two arbitrary nodes are connected to an external voltage source, the equivalent resistance of the network determines the current flow. For arbitrary network topologies, the equivalent resistance $R_{\alpha\beta}$ between two pairs of nodes (α and β) can become a really complicated expression. A large number of authors grappled with this question and tried to find the appropriate formalism to tackle the problem. The Laplacian matrix L of the network, also known as the Kirchhoff matrix emerged as the most convenient tool to study resistor networks [3]. The resistance $R_{\alpha\beta}$ is elegantly expressed with the eigenvectors and eigenvalues of L (two

point resistance theorem). This theory comes with interesting insights about electricity, especially its probabilistic interpretations. The Laplace matrix plays a central role in many domains of mathematics, e.g. graph theory [4], game theory [5], probability etc. This connection between equivalent resistance and probability leads naturally to the concept of "drunkard's" electron walk in a network. The connection between random walks and electrical networks has been recognized for some time [6, 7]. Nonetheless many textbooks considered random walks (e.g. in a unidimensional chain of identical resistors (R)) where electrons could go left *or* right. In such situation the probabilities of escaping such 1D network are ruled by simple relations of the type :

$$p_i = \frac{1}{2}p_{i+1} + \frac{1}{2}p_{i-1} \quad (1.1)$$

with appropriate boundary conditions (we considered that the probability to go left is the same as the one to go right). Downscaling such network would force electron *at some scale* to satisfy Schrödinger equation and solving such simple relation would not lead to the right probabilities. The *scale* we think of will be detailed in a forthcoming section. Nevertheless, ultimately one would have to deal with *probability amplitudes* in the network. For illustration purposes here is the recurrence relation (see Eq.1.2) ruling the electron presence probability amplitude c_i on a node i of a translation invariant 1D network. Hopping from site to site is also a process with a given probability amplitude (here referred to as t).

$$c_i = tc_{i-1} - 2tc_i + tc_{i+1} \quad (1.2)$$

Again appropriate boundary conditions have to be chosen to fix the solution but this fundamental difference between classical random walk recurrence relations and those *at small scale* is very instructive. Indeed our *macroscopically-driven* intuition could not remain valid and we should expect some counter-intuitive behaviors. In this spirit, let us consider something that may appear as self-evident from the Rayleigh's Monotonicity Law : *No matter the resistance network, if the resistances of a network are increased, the effective resistance between any two points can only increase. If they are decreased, it can only decrease* [7].

The validity of this law is going to be inspected in the present manuscript in *mesoscopic* networks. Deviation from monotonicity is always a source of astonishment for scientists because it is often a signature of an unknown process,

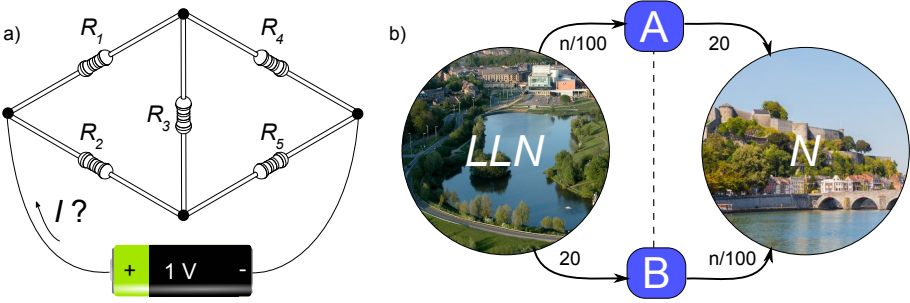


Figure 1.1: a) Illustration of a resistor network. For an arbitrarily-shaped network, the two-point resistance when connecting a power supply (here a one volt battery) to an arbitrary pair of nodes was a long-standing physical problem. b) Illustration of a fictitious route between two cities *LLN* and *N* where a Braess paradox occurs when adding a new road between *A* and *B* with zero travel time. For 2000 people crossing from *LLN* to *N*, the travel time is 30 min without the new road (because $n = 1000$ on each path) and 40 min with the new road (because the 2000 travellers take now the same path - *LLN* to *A* and then *B* to *N*).

and is therefore a powerful driving-force to progress in the understanding of subtle counter-intuitive mechanisms. Although Rayleigh's monotonicity law was mathematically demonstrated in classical resistance network [7], there are other examples in everyday systems where monotonicity is not present.

One of the most interesting example is the so-called *Braess paradox*, well known in graph/game theory [8]. Let us briefly illustrate the mechanism. Consider two cities *Louvain – la – Neuve* (*LLN*) and *Namur* (*N*) and 2000 people that reach everyday *N* from *LLN* with two intermediate gas stations *A* and *B* (nodes) (see Fig. 1.1b). The time spent from *LLN* to *A* is proportional to the number of drivers n according to $t = n/100$ min. From *LLN* to *B* we have 20 min. From *A* to *N* we spend 20 min and from *B* to *N*, $n/100$ min. Given a symmetric network, drivers distribute uniformly from *LLN* to *N*. Years later, politics decide to connect *A* to *B* with a speed way with the hope of decreasing transit time which is 30 min up to now (because the 2000 travellers distribute uniformly in the two paths as it changes nothing from their point of view). This speed way is extremely efficient and the time spend on this one can be considered as negligible. Here is how a selfish driver modifies its route. 'I will go from *LLN* to *A* because I will spent at most 20 min against 20 min for sure.

Then I take the speedway to reach B to go from B to N with at most 20 min'. Unfortunately, each driver thinks this way and thus everybody spends 40 min for the trip, that is 33% increase for the new network configuration. Real world manifestations of this paradox were already reported [9]. This paradox inspired many scientists to find analogous counter-intuitive mechanism in their own field [10–12].

Although no Braess paradox can occur in classical resistor networks, *at small scale* the probability amplitudes might be tuned by appropriately selecting a network geometry and using local probe tools to selectively control transport through the network branches. In that case, an *analogous* Braess paradox would occur. This line of idea lead to the work presented in Ref. 13 where quantum simulations of a nanoscale network and scanning-probe experiments were combined, revealing that an analog of the Braess paradox can occur in mesoscopic semiconductor networks, where electron transport is governed by quantum mechanics. The starting point of this work [13] is presented in Fig.1.2. The conductance G of the network presented in Fig.1.2b presents a counter-intuitive behavior as the width of the central channel is varied. Indeed, going from a finite value for W towards 0 (closing the central conducting channel) can yield an increased value for the conductance of this two-terminal network, when considering the range $W < 200$ nm - hence the analogy with the Braess paradox.

Although pointing clearly towards a possible violation of Rayleigh's monotonicity law in a mesoscopic electronic network, the physical origins of the mechanism remained not clear. Possible physical explanations, based on theoretical investigations of the paradox, were then proposed by Sousa *et al.* [14] and Macucci *et al.* [15].

We propose here first to investigate experimentally the paradox and try to pinpoint experimental conditions where the violation of Rayleigh's monotonicity law is manifest. Understanding these peculiar conditions is unavoidable if one wants to establish guidelines for designing intuition-driven devices down to the smallest scale. Managing electron flow in nano-scale networks could come into play regarding original conceptions of transistors. Inspiring tomorrow's transistor circuit architecture by highlighting interesting network's properties could be one of the targeted applications of this thesis.

In this spirit, integration of carbon nanotubes (CNTs) with silicon technologies is of considerable interest for next-generation nano-electronics [16]. These unidimensional systems are suitable for building CNTs-based networks

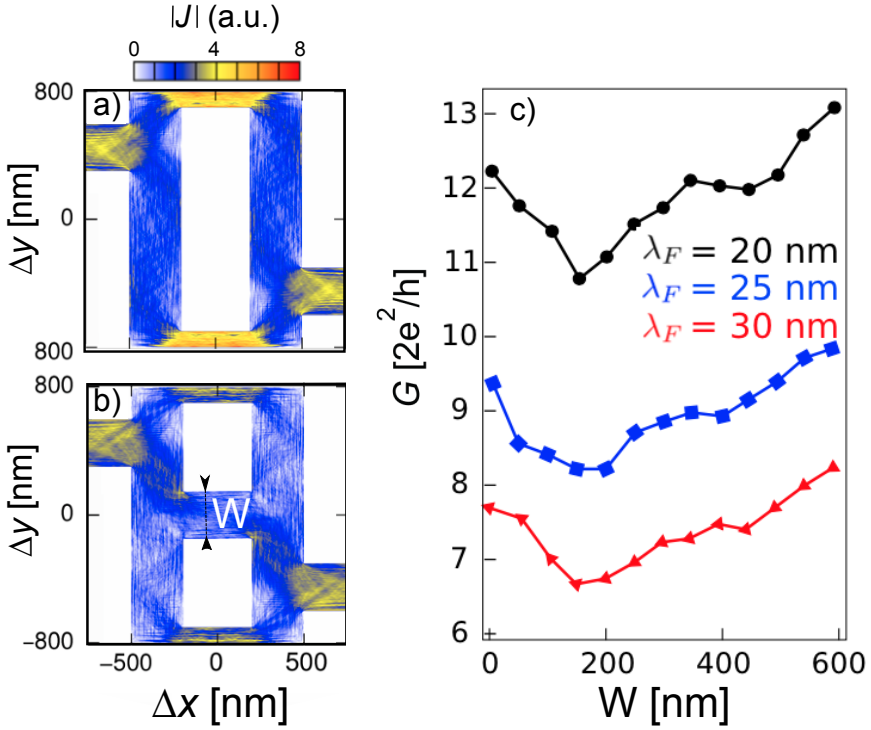


Figure 1.2: a) and b) Contour plots of $|\mathbf{J}|(x,y)$ (arbitrary units) in a rectangular corral supplied with an additional central branch of increasing width W . $W = 0$ in a) and 300 nm in b). c) Conductance G of this two-terminal mesoscopic network for several values of W . The counter-intuitive behavior is that the conductance drops although an additional branch is added up to a certain threshold $W \sim 180$ nm. These data are extracted from [13].

architecture [17]. An interesting conceptual element brought by nanotubes is the following complexity : in nanoscale networks, the *classical* network picture is no more valid as the physics of the junctions between channels (previously considered as abstract nodes) and the physics inside a channel can be both determinant for transport properties [18, 19].

The two main results presented in this manuscript (see chapters 5 and 6) are supporting this message that in nanoscale network, electron transport can *appear* as paradoxical if we neglect emergent properties arising when connecting 1D items. As paradoxical as it may sound, we hope that the present manuscript clarifies that no true anomaly (*i.e.* a behaviour that would go against basic laws of physics) is encountered in the studied networks. Although the word stays appropriate - from the greek paradoxos meaning contrary to our common sense.

A brief comment also about the fact that ultimately, transport properties of conducting network are going to be of high relevance to keep increasing computer's computational power. Indeed as it becomes more and more challenging to reduce the area of single processing units (transistors)[20], one needs to find alternative ways to organise these processing units. A common humanity's reflex is to look at Nature¹, and try to mimic its highly efficient equivalent method of doing what we are trying to do. For its extremely low power consumption in a finite volume, the human brain is probably the most relevant candidate. The fundamental building blocks of its complex architecture are the neurons, organised in a complex network. A vast research field is already at play to try to reproduce neurons with solid-state equivalent elements [21, 22] and use them to implement circuits that outperform the current most efficient CMOS+MRAM technologies [23] in different tasks. Neurons-like conducting networks are thus also a vast field of research where one could use or prevent emergent counter-intuitive phenomena, arising from their networking, in the process of building tomorrow's computer.

1.1.1 Transport at small scale

It is now time to detail the *small scale* we are referring to. The length scale is the one where we can study mesoscopic effects. Actually there is no precise criterion to identify the experimental conditions where mesoscopic physics is at play. This discipline of physics is intermediate between the quantum and classical regimes. In mesoscopic physics, it makes sense to use equations from

¹not the scientific journal

both field of physics (quantum and classical) as guiding tools to explain the observations and as predictive tools to design experiments and test hypothesis. In this manuscript, mesoscopic phenomena are needed to apprehend the complexity of electron transport through nano-scale networks. Among the plethora of phenomena that can be at play in electron transport, the most relevant in this manuscript are : quantization effects, single charging events, stationary electron wave interference patterns and ballistic effects. All these four phenomena, *well-established* in mesoscopic physics, are going to be discussed in the manuscript.

We mentioned interference patterns and ballistic transport. The wave/particle duality is central to explain some of the mechanisms presented below. In this context, we have to think in term of probability amplitudes to measure electrons in a given quantum state and hopefully, these probability amplitudes are prescribed up to the framework of relativistic quantum field theory by Dirac equation. No relativistic effects are discussed here so the non-relativistic Schrödinger equation is sufficient. This first principle equation allows us to get numerical estimates through simulations. We found it to be important to enlighten our experimental results by testing if they were at least compatible *in principle* with state-of-the-art computations (serving as guidance). Based on this first principle equation we use an implementation of two formalisms : Keldysh-Green and scattering (both equivalent [24]). The first code is currently developed by the team of Marco Pala (Keldysh-Green) and the second one is called Kwant [25] (briefly described in appendix C).

1.2 Mesoscopic transport

From planetary motion to random walk, the physical laws governing the motion of object are a fascinating paradigm. Even more fascinating are the principles withstanding these laws. A motion is modelled by a law formally derived from a principle that no one can justify. Although the quest of an unique set of principles governing the whole reality has been proven to be unreachable [26], an interesting game has to be played by physicists to describe at least part of the reality : state a principle, derive a formalism (suitable algebra), express laws and formulas and check that they are valid. In physics, the ultimate test for the validity lies in the accordance between the prediction (theory) of observables and the observations (experiment). During the past century, this game was successfully played as scientists came up with principles [27] different from

those governing classical mechanics : quantum mechanics was born. This shift of paradigm teaches us a valuable message. On the one hand, we know a principle leading to classical laws of motions (least action principle) in the macroscopic world and, on the other hand, we have a set of principles valid at atomic scale (tenths of nanometers - $0.1 \cdot 10^{-9}$ m). Inevitably there must exist a scale ² at which our first set of principles leaves *continuously* the floor to the second : the mesoscopic scale. This means that at some intermediate scale, one can observe peculiar motions where some features of the motions are described by classical laws and others by quantum laws. There is no sharp transition, based on *a priori* criteria, for which we could tell : we are in the mesoscopic domain. But we know that there exist an experimental domain where both principles set can serve as a compass to characterise what is happening down there.

In the manuscript we focus on electron transport in condensed matter systems in the mesoscopic regime and try to describe some features of their motion when transporting through a conducting devices.

What distinguishes the motion of electrons in vacuum from the one in condensed matter ? This question was asked by Drude in the beginning of the past century and is often a starting point to understand electrical conduction [28, 29]. The Drude model is based on the possibility to use the rules applied to ideal gases to the motion of electrons in the solid state. We start by considering electrons that are supposedly *free*, *i.e.* moving in a flat electrostatic potential. In such case, when applying a potential difference to a piece of metal, the acceleration of charges would be constant along the conductor. The velocity would keep rising and this model is not compatible with the observed relation between the applied electric field E and the current density J (classical Ohm's law $J = \sigma E$). Instead, Drude proposed that in addition to the Force caused by the electric field, an additional effective drag force was at play, limiting the *average* velocity ($\langle v \rangle$) gained by electrons. This force was due to collisions with the way heavier, stationary crystal ions. Considering a constant uniform electric field E turned on at time $t = t^*$, the equation of motion was then :

$$m_e \frac{d\langle v \rangle}{dt} = eE\theta(t - t^*) - \frac{m_e \langle v \rangle}{\tau}. \quad (1.3)$$

For a particle starting at rest, we obtain :

$$\langle v \rangle(t) = v_d(1 - \exp(\frac{-t}{\tau})) \quad (1.4)$$

²this reasoning can be done at even smaller or larger scale

with $v_d = \frac{eE\tau}{m_e} = \mu_e E$ where $\mu_e = \frac{e\tau}{m_e}$ is the electron mobility. This is a central quantity - discussed further in the manuscript - when considering nanoelectronics and mesoscopic transport.

The current density is then given by :

$$J = nev_d = \frac{ne^2\tau}{m_e} E \quad (1.5)$$

where n is the *free* electron density. Identifying with Ohm's law, we get :

$$\sigma = \frac{ne^2\tau}{m_e}. \quad (1.6)$$

In the case of copper (one free electron per atom) at 20°C we measure $\sigma = 5.96 \times 10^7 (S/m)$. With a molar mass of 63.54 g/mol and a volumic mass of 8.94 g/cm³, we have an electron density of $8.47 \times 10^{28} \frac{e}{m^3}$ leading to a τ of 2.4×10^{-14} second. This model predicts that after a time of the order of τ a collision randomises the electron initial momentum value.

In the framework of Drude's model, the electron can be considered in an effective *linear*³ motion with a drift velocity v_d . Nevertheless, between two collisions, the typical distance travelled by the electron is :

$$l_\mu = v_m \tau. \quad (1.7)$$

For the weak electric fields, we can consider that the velocity v_m is the mean velocity computed from the velocity probability density function $f(v)$ derived by Maxwell and Boltzmann :

$$f(v) = \left(\frac{m_e}{2\pi k_B T} \right) 4\pi v^2 e^{-\frac{mv^2}{2k_B T}}. \quad (1.8)$$

We have :

$$\frac{m_e v_m^2}{2} = \frac{3}{2} k_B T. \quad (1.9)$$

This equation is really informative as it states clearly that classical mechanics is not a sufficient tool to understand the motion of electrons in any experimental condition. Indeed, as temperature is going towards the absolute

³where distance scales linearly with time. It is different from free fall $\propto t$, or diffusive motion $\propto \sqrt{t}$

zero, the velocity should be going to zero as well. However, metals keep conducting electricity at low temperature. This means that Maxwell-Boltzmann statistics is not valid at low temperature. The problem comes from the fact that as we keep decreasing temperature, the ideal gas statistics keeps getting less and less valid. Indeed as Louis de Broglie showed in his "Recherche sur la théorie des quanta"[30], electrons are some fuzzy objects, wave-like objects whose wave nature was experimentally shown with an attributed wavelength $\lambda = \frac{h}{p}$ where p is the particle momentum. We can then state that as the average spacing between electrons is way bigger than this wavelength, electrons can be statistically described from a classical point of view. Otherwise, their quantum nature cannot be ignored. We write this criterion in a 2D space as we are going to study 2D electron systems in the remaining of this manuscript.

The average separation is $\sqrt{\frac{1}{n_{2D}}}$ with n_{2D} the 2D electron density and $\lambda = \frac{h}{p}$ with p the average momentum arising from thermal agitation defined by $\frac{p^2}{2m_e} = k_B T$. Our criterion write :

$$\frac{h\sqrt{n_{2D}}}{\sqrt{2m_e k_B T}} \ll 1. \quad (1.10)$$

When this criterion is not met, a quantum treatment is required to understand the electrons in condensed matter. For example, Maxwell-Boltzmann statistics is no more valid and we need the so-called *Fermi-Dirac* statistics given by

$$f(E, T) = \frac{1}{1 + e^{\frac{E - \mu}{k_B T}}}, \quad (1.11)$$

where μ is called the chemical potential of the electrons. As temperature goes to zero, the chemical potential is also referred to as the Fermi energy $E_F = \lim_{T \rightarrow 0} \mu(T)$.

This statistics is of central relevance as it is a direct consequence of the electron nature. According the spin-statistics theorem⁴, particles distinguish by their *spin* given strong constrain on their wave functions : the wave function of identical integer-spin (called *bosons*) particles is symmetric under real space particle swap - asymmetric for half integer-spin (called *fermions*). As electrons spin- $\frac{1}{2}$ are fermions, they obey the aforementioned Fermi-Dirac statistics.

⁴although central to quantum mechanics, this theorem is rarely presented due to its complex demonstration exploiting the relation between SO(3) and SU(2) in group theory - famously illustrated by Dirac's belt trick

According to FD statistic, even at 0 K, electrons occupy energy states up to an energy E_F : the Fermi energy. In near-equilibrium transport measurement ($V_{bias} \leq \frac{k_B T}{e}$), E_F is central and the associated velocity v_F and wavelength λ_F are of great importance for the remaining of the manuscript.

We are now going to motivate the use of cryogenic temperatures for our measurements. At room temperature, many mechanisms scatter electrons at a high rate from their original state to a different one (with distinct momentum, energy, spin, etc). Among these mechanisms, we recall (extracted from [31]):

- optical phonon scattering (dominant at high temperatures)
- acoustic phonon scattering (deformation potential scattering)
- piezoelectric scattering originating from acoustic phonons in piezoelectric semiconductors (e.g., III-V semiconductors or II-VI semiconductors, as a result of the lack of bulk inversion symmetry of the crystal lattice)
- ionized impurity scattering (undesired background doping)
- ionized donor scattering (leading to disorder potential detailed in section 2.1.3)
- scattering from neutral defects or impurities
- alloy scattering in ternary semiconductors, e.g., in InGaAs heterostructures
- surface roughness scattering

All these mechanisms are responsible for the limited value of τ (the scattering time). The temperature dependency of the mobilities (and thus τ) associated to each of the aforementioned scattering mechanisms has to be inspected. Indeed, according to Matthiessen's rule, the smallest time is the one limiting the total scattering time. In the quest of the highest mobilities, we could think that $\text{In}_{0.75}\text{Ga}_{0.25}\text{As}$ would be a better candidate than GaAs, owing its lower effective mass (0.036 for $\text{In}_{0.75}\text{Ga}_{0.25}\text{As}$ against 0.063 for GaAs)[32]. It is even lower than $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ where $m_e = 0.041$. Nevertheless, at temperature close to 0K, for rather pure materials, the electron mobility in InGaAs is always limited by the alloy scattering, absent in GaAs, explaining the mobility records measured in GaAs heterostructure. For alloy scattering to be limiting, the scattering by residual impurities and the silicon dopants need to be negligible. Only the dopant induced disorder was inspected in this work (see chapter 2) and the scattering time related to this disorder was estimated to 40 ps at Fermi energies of 55 meV (see Fig. 2.4). This scattering time leads to a mobility of 195 (m^2/Vs) which is one order of magnitude higher than what was measured in our Hall bar (see Fig. 2.1d). The scattering mechanism limiting the mobility

in our case is then most probably alloy scattering [33]. In GaAs, no alloy scattering being present, the limiting scattering mechanism at low density comes from dopant induced disorder. In GaAs the dopant density is limited so that to reduce disorder scattering and this is how the record values for the mobilities are obtained in GaAs heterostructures. Reducing (if possible) the influence of all these mechanisms leads to an increased electron mobility μ_e and thus mean free path ($l_\mu = v_F \tau$). For a device built with characteristic dimensions comparable to this mean free path, electrons are supposed to transit across the device with a conserved momentum and thus energy. In this transport regime, called *ballistic*, the device conductance depends strongly on its geometry. Indeed, electrons experience elastic bounces against the edges of the device. Also, in near-equilibrium measurements, as electrons preserve their energy E_F along their path, we can study transport equation solutions at E_F , which is a great of simplification.

Reaching experimental conditions in which electrons present a large mean free path (high μ_e) is a challenge achieved with : (1) the fabrication of 'defect-free' devices, (2) precisely controlled fabrication processes (see appendix B about the fabrication processes) and (3) measurements at low temperature.

Let us briefly detail two of these three points.

First, nowadays high quality semiconducting samples are grown by *Molecular Beam Epitaxy* (MBE)[34]. Thanks to MBE we can grow heterostructures on the surface of monocrystalline substrates and obtain high quality surfaces, both well characterized in terms of composition and spatial arrangement. Effusion cells are heated behind shutters. We can then open or close the shutters following a programmed sequence. The shutters allow to control composition of the deposited elements. The process is monitored with a fluorescent screen that gives the needed informations on the ordering through the analysis of the diffracted electron beam hitting the growing surface with a grazing incidence (Reflection high-energy electron diffraction - RHEED). This process allows to grow ternary compound allays such as InGaAs, InAlAs etc. on monocrystalline substrate e.g. InP. The different heterostructure layers selected to grow heterostructures are carefully chosen (both in composition and size) in light of crystal growth theory [35] and band-gap engineering [34].

These techniques allowed to grow heterostructures hosting high-mobility 2D electron systems [36]. Such high mobilities are measured down to the lowest temperatures. This leads us to the second aspect : decreasing temperature.

As temperature keeps going down, the contribution of most of the scattering

mechanisms, present at room temperature, become less and less important. The temperature dependence of each scattering mechanism must be inspected separately [31]. These mechanisms often present a power law dependency, and the mobility finally saturates at the level fixed by temperature independent ones (impurities, disorder). An heuristic argument can be delivered for acoustic and optical phonons modes⁵. As they are *bosons*, they obey the Bose-Einstein statistics given by :

$$n(\omega, T) = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} \quad (1.12)$$

where $n(\omega, T)$ is the average number of phonons occupying a quantum state of energy $\hbar\omega$ at a temperature T . As temperature approaches the absolute zero, only low energy, low momentum phonons are remaining in the material. These are really inefficient to scatter electrons from their initial state and thus do not limit any more the scattering time τ .

Also, as we decrease temperature, for a given electron density, we can see that the criterion, presented in Eq.(1.10), gets progressively less valid. For a 2DES with $n_{2D} = 10^{16} \text{ (m}^{-2}\text{)}$ and $T = 20 \text{ mK}$ (experimental conditions that we will reach in the rest of the manuscript) the criterion (see relation 1.10) gets strongly violated. A quantum treatment becomes inevitable and other *characteristic lengths* need to be introduced.

The quantum nature of electrons implies that they do not only behave as particles but also as waves. This wavy nature allows electrons to produce standing-wave interference patterns. Such interferences modify the probability amplitudes of finding electrons at a given location. Phase relations between electron wavefunctions are of capital importance when considering transport in the mesoscopic regime of transport. In analogy with electron momentum, the phase associated to the wavefunction can be randomised by inelastic scattering events, including electron-phonon, electron-electron interaction and scattering by impurities with an internal degree of freedom, *e.g.* magnetic scattering. Phase can be considered as conserved on a timescale τ_ϕ - the phase coherence time. Depending on the transport regime, this time can be associated with a phase coherence length l_ϕ . In the ballistic regime of transport, we have :

$$l_\phi = v_F \tau_\phi. \quad (1.13)$$

⁵the actual phonon dispersion relation must be computed for the material hosting the 2DES to be quantitative

In 2D systems, at low T, a static disorder potential can limit the scattering time τ without randomising the phase. This leads to the following relation in the diffusive regime :

$$l_\phi = \sqrt{D\tau_\phi} \quad (1.14)$$

with D the diffusion constant $\frac{v_F^2 \tau}{2}$. The coherence time of the electron phase is still a current source of interest in mesoscopic devices, both theoretically and experimentally [37]. In general the coherence time follows a Matthiessen's rule, giving the scattering rates as :

$$\frac{1}{\tau_\phi} = \sum_i \frac{1}{\tau_i} \quad (1.15)$$

where the sum is taken over all the possible scattering mechanisms : electron-phonons, electron-electron, ... whose temperature dependency must be inspected. In most cases, however, the temperature dependency of τ_ϕ follows a power law of the type $\tau_\phi \propto T^{-p}$. Magneto-transport measurements constitutes a powerful probe of the dephasing time τ_ϕ [38].

As we probe charge transport in devices whose characteristic lengths are made smaller than (or comparable to) l_ϕ , transport is said to be *coherent*.

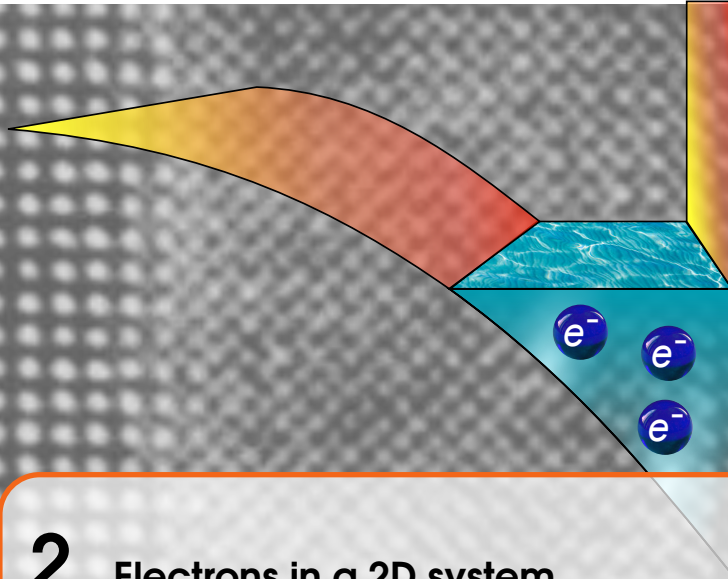
In the 2D electron system we study in this manuscript, with geometries whose characteristic dimensions are $\sim 2 \mu m$, as temperature goes from 0.02 to 300 K we probe three different regimes of transport. Based on experiments realised in comparable structures [39], we can expect phase *coherent* signatures in transport measurements between the base temperature and approximately 10-30K. From 30 to 100K, *ballistic* transport is expected. Eventually, above 100K, transport is essentially *diffusive*.

1.3 Structure of the thesis

This thesis is organized as follows :

- Chapter 2 introduces the basic concepts relevant to describe electron systems confined in 2D, including the approach that we followed to take the disorder potential into account.
- Chapter 3 concerns the formalism allowing to describe, predict (through simulations) and understand electron transport in complex nanoscale devices, taking into account scattering inside the device.

- Chapter 4 reviews earlier scanning gate microscopy studies and emphasizes the role of key concepts such as screening of the tip-induced potential.
- Chapter 5 presents our experimental investigation of the Braess paradox in a model nano-network. We examine in particular the conditions of occurrence of the paradoxical behaviour, and propose an interpretation based on experimental observations, which relies on coherence and on Coulomb blockade, depending on the regime of electron transport within the network.
- Chapter 6 describes our combined experimental/simulation effort to evidence a paradoxical situation where a depleting (resp. accumulating) tip potential induces an increased (resp. decreased) device conductance. Our simulations show that the observed phenomena are caused by a tip-perturbation-induced modification of the current density distribution in the device. The current is then either forward scattered or redirected towards a hard wall which increases backscattering.
- Chapter 7 concludes this thesis and discusses the open questions that are left for future work, including *e.g.* physics of graphene-based nano-networks.



2. Electrons in a 2D system

As mentioned in the introduction, at the mesoscopic scale, conductive networks are expected to present behaviors described both by quantum and classical mechanisms. In such conditions, the validity of *Rayleigh's Monotonicity Law* has to be reinspected. To do so, networks have to be fabricated and probed at the *small scale*. Historically, two approaches can be used : the *bottom-up* or the *top-down*. The bottom-up approach to build networks would consist in picking 1D items (such as carbon nanotubes) and assemble them to form a network. We rather opted for the second, the top-down approach. The heterostructure - a macroscopic wafer - hosts a 2D electron system (2DES) at the interface between two layers. The idea is then to remove specific locations of the 2DES by chemically etching - and/or electrostatically depleting - some part of it in order to define an arbitrarily shaped conductive network. The fabrication process, as well as some critical device parameters related to device processing (such as depletion at the edges, Fermi level pinning at surfaces, etc.) are fully detailed in appendix B. But before digging into the specific transport properties of networks, let us briefly detail some general aspects of the 2DES from which they are fabricated.

2.1 2D electron system (2DES) : the underlying physics

Since the seventies, 2D electrons systems are among the fundamental building blocks to unveil fundamental physical mechanisms in condensed matter and lead to very rich and interesting discoveries (yielding three Nobel prices [40–42]). Understanding the physics of 2DES studied at low temperature and under high magnetic field was one of the unended quest of physicists in the end of the past century. To get insights on how 2DES are a great opening door to a wealth of subtle and remarkable mechanisms, the Nobel lecture concerning the fractional quantum hall effect is really worth reading [41].

In a first approximation, let us consider the 2DES as an infinite plane where the probability amplitude to find an electron is ruled by the spin-degenerate non-relativistic Schrödinger equation. No many body effects are discussed in a first place and we consider non-interacting electrons. Let us also assume that the potential within the 2DEG is uniform (set to zero for simplicity). In this situation, the time independent simplified effective mass Schrödinger equation reads :

$$\frac{\hat{p}^2}{2m^*} \psi(\rho) = E \psi(\rho), \quad (2.1)$$

with $\hat{p} = -i\hbar\nabla$, m^* the isotrope effective mass, E the energy and ρ , the in-plane coordinate. The solution to such equation is of the type :

$$\psi(\rho) = A e^{i\mathbf{k} \cdot \rho} \quad (2.2)$$

for all $\mathbf{k}$. Let us consider a 2D box of length L_x and width L_y (with surface $S = L_x L_y$) in the 2DEG and ask the question : how many states are in that box? If there is at least one, it must be normalized, meaning $A = 1/\sqrt{S}$. Also, due to translation invariance, if there is a state in our box, an exact same states must exist in a neighbouring identical box. This leads to the following periodic boundary conditions :

$$\psi(\rho + \mathbf{R}) = \psi(\rho) \quad (2.3)$$

with $\mathbf{R} = mL_x + nL_y$ where (m,n) are integers. Rewriting the condition presented in (2.3), we have : $\exp(i\mathbf{k}\mathbf{R}) = 1$. This brings restrictions on the admissible values of vector $\mathbf{k} = (m\frac{2\pi}{L_x}, n\frac{2\pi}{L_y})$. The artificial trick to consider a 2D box allows us to count the admissible quantum states in an arbitrary sized box. Since we are dealing with finite energy systems, it can be useful to know the number of

available states below a given energy E . From equation 2.1 we know that a state in $\mathbf{k}$ space (referred to as $\mathbf{k}_{m,n}$) corresponds to an energy $E_{m,n} = \frac{\hbar^2}{2m^*}(k_m^2 + k_n^2)$ with $k_m = m \frac{2\pi}{L_x}$ and $k_n = n \frac{2\pi}{L_y}$. The number of states in our box below an energy E is :

$$N(E) = 2 \sum_{\mathbf{k}_{m,n}, E_{m,n} < E} 1 \quad (2.4)$$

The factor two takes into account the spin degeneracy. To go further, it can be useful to note that the discrete $\mathbf{k}$ -space can be approximated as a continuum with an associated constant density $\frac{S}{4\pi^2}$. Considering polar coordinates in $\mathbf{k}$ -space, $N(E)$ can be approximated by :

$$N(E) \approx \int_0^{2\pi} \int_0^{k^*} d\theta dk \frac{S}{4\pi^2} = \frac{m^*}{\pi \hbar^2} ES. \quad (2.5)$$

2.1.1 The 2DES electron density

As it was shown, the electron density n_{2D} is a central quantity that is directly proportional to the Fermi energy. Measuring this quantity is the fundamental starting point of this work. A typical measurement that can be done is the Hall experiment. The experimental scheme, based on Fig. 2.1a, is the following : a current is injected in the lefter pad and collected at the grounded pad while all other contact pads are floating. The longitudinal resistance is measured by measuring the difference of potential as presented in Fig. 2.1a. The difference of potential between pads facing on each side of the Hall bar is also measured, leading to the transverse resistance. As a magnetic field orthogonal to the 2DES is swept (Lorentz force applied on the charge carrier), the evolution of the longitudinal and transverse resistance are recorded. Such traces are shown in Fig. 2.1b (longitudinal resistance in red and transverse in blue). A simpler illustration of the Hall bar is presented in the inset of Fig. 2.1b. Usually, the slope of the blue trace in Fig. 2.1b is used to extract the carrier density (classical Hall effect). Nevertheless, due to a peculiar feature of our Hall bar layout (presented in appendix B), we do not have access to the Hall resistance. We rather exploited another feature of the longitudinal resistance measurement that are Shubnikov-de Haas (SdH) oscillations [29]. These oscillations in the longitudinal resistance (see the blue trace in Fig. 2.1b) are consecutive to the formation of Landau levels that eventually lead to the integer quantum Hall effect [40]. The successive values B_1 and B_2 of the magnetic field at which

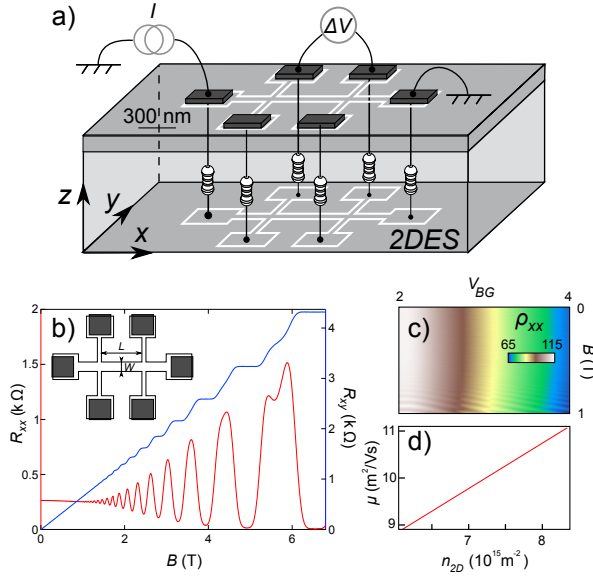


Figure 2.1: a) Illustration of a typical Hall bar geometry. The 2DES (bottom of the parallelepiped) is located approx. 45 nm beneath the surface of the heterostructure where metallic pads (dark gray) of $300 \mu\text{m}^2$ are deposited, realising ohmic contacts (illustrated by the white resistances - details about the ohmic contacts can be found in appendix B). The bold white line represents the location of the etched regions (for details about the actual Hall bar geometry, see appendix B). A known current (I) is imposed in the left contact and collected in the grounded right contact. The difference of potentials (ΔV) is measured as depicted. We then compute their ratio to get the resistance ($R = \Delta V/I$). The resistivity (ohms/square) is obtained with the geometric parameter L and W shown in b (inset) ($L/W = 580 \mu\text{m}/150 \mu\text{m}$). b) Curves of the longitudinal (red) and transverse (blue) resistance of our Hall bar as a function of the magnetic field (B). Around 1 T, we can identify the onset of Shubnikov-de Haas (SdH) oscillations[29, 43]. The typical signature of the integer quantum Hall effect [40] is revealed at high magnetic field ($R_{xx} = 0$). SdH oscillations allow to extract the 2DES density n_{2D} thanks to relation (2.6). Also the resistance at vanishing magnetic field allows to get the mobility (μ). Measuring the longitudinal resistivity ρ_{xx} as a function of B for different backgate values V_{BG} (see c) allows to get n_{2D} and μ when varying V_{BG} . The horizontal axis of c) and d) are matched to give a correspondence between V_{BG} and n_{2D} . Note that the mapping of SdH oscillation was not realized at fields higher than 1 T

the oscillating longitudinal resistance peaks appear are characterised by the following formula :

$$n_{2D} = \frac{2e}{h} \left(\frac{1}{B_2} - \frac{1}{B_1} \right)^{-1} \quad (2.6)$$

where n_{2D} is the electron density and the two takes into account the spin degeneracy. SdH oscillations depend thus on the electron density and this is extremely helpful. Indeed an interesting feature of our heterostructure is the presence of a back-gate (doped InP layer - see Fig. 2.2a). This back-gate gives us the opportunity to tune the 2DES electron density by applying a back-gate potential V_{BG} (V). From a classical point of view, the system can be approximated as a simple capacitor between the grounded 2DES and the polarized back-gate (V_{BG}). The different layer between the 2DES and the back-gate are detailed in Fig. 2.2a. The capacitance per unit area is given by

$$\frac{dn_{2D}}{dV_{BG}} = \frac{\epsilon_0}{e} \left[\frac{d_{InAlAs}}{\epsilon_{InAlAs}} + \frac{d_{InAlAs}}{\epsilon_{SbAlAs}} \right]^{-1}, \quad (2.7)$$

which is equal to $1.121 \times 10^{15} (e/m^2V)$.

Practically, the system is much more complicated and measuring this capacitance per unit area is much more easy. To do so, the longitudinal resistivity (ρ_{xx} in $\Omega/\square$) is mapped as a function of the magnetic field B (T) and the back-gate voltage V_{BG} (V) (see Fig. 2.1c). This way, applying formula (2.6), we can extract n_{2D} at any V_{BG} . The mobility is extracted from the value of the resistivity at $\rho_{xx} = 0$. In summary, from the value of n_{2D} and $1/\rho_{xx}$, we have the value of μ_e from which we can get the mean free path :

$$l_\mu = \frac{h}{e} \frac{\mu_e}{\lambda_F} \quad (2.8)$$

with $\lambda_F = \sqrt{\frac{2\pi}{n_{2D}}}$. The correspondence between n_{2D} and μ_e with respect to V_{BG} is given in Fig. 2.1d.

Our heterostructure is actually fairly well described by this *Jellium* model. Nevertheless the actual 2DES is embedded in a 3D space and the confinement along the z -axis leads to several subbands. Indeed, in the potential well defined by the profile of the bottom of the conduction band E_c (see Fig. 2.2b) the first subband refers to the lowest energy state. The probability distribution to find electrons within the quantum well (15 nm 2DES domain presented in

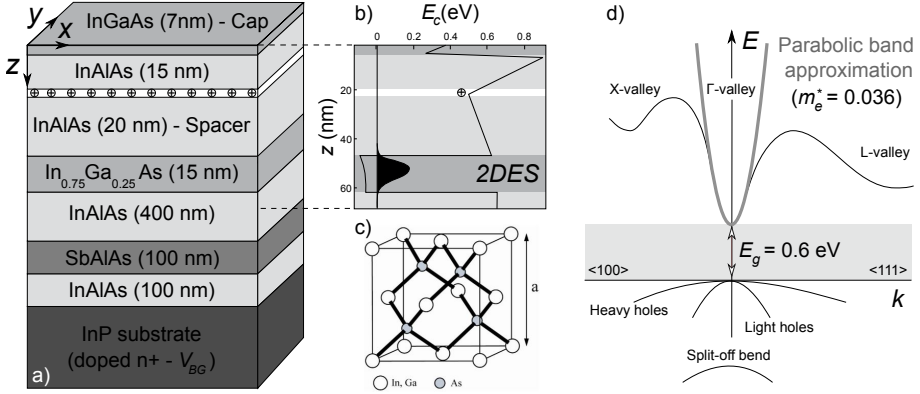


Figure 2.2: a) Illustration (not at scale) of the heterostructure from which the nano-networks studied in this manuscript were fabricated. This scheme presents the different layers. The particularity of this structure is the highly doped InP substrate serving as a backgate allowing to tune the electron density at the level of the 2DES. The singly ionised silicon dopants are located randomly in a *plane* (this peculiar doping is called δ -doping) 20 nm above the layer hosting the 2DES (forming at the interface between the InAlAs spacer and the InGaAs layer). The dopants are sketched by the plus signs. b) Profile of the bottom of the conduction band (E_c) as a function of z . This profile is extracted from 3D Poisson-Schrödinger self-consistent simulations allowing to extract the presence probability distribution (in black) of the first occupied subband in the 2DES. Given the InGaAs zinc-blende structure presented in c), one can compute the charge dispersion relation in the first Brillouin zone (d). One can approximate the conduction band with a parabola (that can be considered isotropic). This illustrates that the approximate parabolic dispersion relation is approximately valid up to few hundreds of meV above the bottom of the conduction band. The effective mass approximation is then valid with an effective mass m_e^* of $0.036m_e^0$. Additional informations about the heterostructure can be found in appendix B

see Fig. 2.2b) is now going to be inspected with the help of the Fang-Howard variational approach. Indeed, the total energy of an electron is a sum of several energies. Roughly, one can consider that, with respect to a free electron in vacuum, electrons in a 2DES are characterised by the following energies : the electron affinity in InGaAs, the thermal energy ($k_B T$), the confinement energy within the quantum well and their kinetic energy. The Fang-Howard approach helps us in estimating the confinement energy within the quantum well.

2.1.2 Fang-Howard variational approach

To find the energies associated with the quantum well (QW) many textbook use a Fang-Howard variational approach [31]. The usual Fang-Howard (FH) variational wave function for the ground state in a triangular well (see Fig. 2.3) is :

$$\phi_0(z) = \sqrt{\frac{b^3}{2}} z e^{-bz/2} \quad (2.9)$$

where

$$b = \left(\frac{33m^* e n_{2D}}{8\hbar^2 \epsilon_0 \epsilon_R} \right)^{1/3}. \quad (2.10)$$

However as shown in Fig. 2.2, our heterostructure presents a finite depth potential well (see Fig. 2.2b). This means that the FH wave function cannot be used because it could not fulfil the boundary conditions imposed by the well.

For an electron in the 2DES, we can write the Hamiltonian within the self-consistent Hartree approximation.

$$\left[\frac{-\hbar^2}{2m^*} \Delta + V_H(z) \right] \psi_{n\mathbf{k}} = E_n(\mathbf{k}) \psi_{n\mathbf{k}} \quad (2.11)$$

with

$$V_H(z) = \sum_{n\mathbf{k}} \int d^3r G(\mathbf{r}, \mathbf{r}') |\psi_{n\mathbf{k}}(\mathbf{r}')|^2. \quad (2.12)$$

Summing over all occupied states at a temperature T of zero Kelvin. $G(\mathbf{r}, \mathbf{r}')$ is taken as the Green's function for Poisson's equation with the appropriate boundary conditions. This Hamiltonian is separable in the x , y and z coordinates

and so an *Ansatz* for the wave vector expressed in suitable polar coordinates (ρ, z) is :

$$\psi_{n\mathbf{k}} = \frac{1}{\sqrt{S}} e^{i\mathbf{k}\cdot\rho} \phi_n(z) \quad (2.13)$$

with n labelling the subband order. The energy associated to a state is given by :

$$E_n(\mathbf{k}) = E_n + \frac{\hbar^2 \mathbf{k}}{2m^*} \quad (2.14)$$

with E_n satisfying the following equation :

$$\left[\frac{-\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + V_H(z) \right] \phi_n(z) = E_n \phi_n(z). \quad (2.15)$$

This problem can be solved self-consistently with nowadays reasonable computing resources. The Hartree potential has to satisfy the following Poisson's equation :

$$\frac{d^2 V_H}{dz^2} = -\frac{\rho(z)}{\epsilon_0 \epsilon_R} = -\frac{-en_{2D} |\phi_0(z)|^2}{\epsilon_0 \epsilon_R} \quad (2.16)$$

with the following boundary conditions :

$$V_H(0) = V_H(15 \text{ nm}) = 0 \quad (2.17)$$

where 15 nm is the width of the InGaAs quantum well.

The following tentative wave function can be used [44]:

$$\phi_0(z) = \begin{cases} Nz(e^{bz/2} - e^{bw/2}) & \text{for } 0 \leq z \leq w \\ 0 & \text{otherwise} \end{cases} \quad (2.18)$$

where N is the normalisation factor and b the variational parameter.

The ground state energy E_0 is the one for which b gives the minimal energy for the following expression :

$$E_0 = \langle \hat{T} \rangle + \langle V_H \rangle = \frac{\hbar^2}{2m^*} \int_0^w dz \phi_0(z) \left(\frac{d^2}{dz^2} \phi_0(z) \right) + e \int_0^w dz V_H(z) |\phi_0(z)|^2. \quad (2.19)$$

Unfortunately, no elegant analytical solution can be found and the problem is worth solving numerically. Solving this problem is extremely relevant because

the ground state wave function is the key function when wondering how does the 2DES local density looks like. Indeed, up to now, we considered the 2DES as translationally invariant in x and y . Nevertheless, the delta-doped silicon layer is breaking this invariance (see Fig. 2.2a). Indeed, the presence of randomly located charge donor creates a varying potential in the 2DES : the so-called *disorder potential*.

This fluctuating potential (intrinsic to the heterostructure) causes local potential gradient leading to forces causing the appearance of an inhomogeneous charge density within the 2DES at equilibrium. The induced charge density n_{ind} is characterised by an intrinsic property of the 2DES, its polarizability $P(\rho, z, \rho', z')$ [45], defined by the following relation :

$$n_{ind}(\rho, z) = \int d\rho dz P(\rho, z, \rho', z') V_{tot}(\rho, z). \quad (2.20)$$

The derivation of this non-linear, non-local quantity is rather complicated but hopefully, in the Thomas-Fermi approximation, with only one occupied subband, the polarizability becomes :

$$P(\rho, z, \rho', z') = -\frac{m^*}{\pi\hbar^2} \delta(\rho - \rho') |\phi_0(z)|^2 |\phi_0(z')|^2, \quad (2.21)$$

which leads us to :

$$n_{ind}(\rho, z) = -\frac{m^*}{\pi\hbar^2} |\phi_0(z)|^2 \int dz' |\phi_0(z')|^2 V_{tot}(\rho, z'). \quad (2.22)$$

The relevance of the ground state wave function becomes clearer : *The density along the z axis for a given ρ in the quantum well is a simple homothecy of the ground-state wave function modulus.*

The polarizability of the 2DES leads to an interesting phenomenon known as linear screening [46]. The idea is that under an external potential, the induced free charges reorganise and lead to a modified or induced effective potential. The potential distribution (V_{tot}) in which electron will flow is determined as the superposition of the external potential ($V_{ext}(\rho, z)$) and the induced potential ($V_{ind}(\rho, z)$). Let us proceed sequentially. Under an external potential, we have an induced charge density :

$$n_{ind}^0(\rho, z) = \int d\rho dz P(\rho, z, \rho', z') V_{ext}(\rho, z). \quad (2.23)$$

This n_{ind}^0 is a source term for the Poisson equation of the induced potential. In integral form, we have :

$$V_{ind}^0(\rho, z) = e^2 \int d\rho' dz' G(\rho, z; \rho', z') n_{ind}^0(\rho', z'). \quad (2.24)$$

The total potential :

$$V_{tot}^0 = V_{ext} + V_{ind}^0 \quad (2.25)$$

is responsible for a different induced density via 2.20. This is the scheme for the self-consistent loop leading to the determination of the total potential V_{tot}^∞ .

Going one step further, Lindhard expressed the link between the external and total potential via the Lindhard's non-local dielectric function $\epsilon(\rho, z; \rho', z')$:

$$V_{ext}(\rho, z) = \int d\rho' dz' \epsilon(\rho, z; \rho', z') V_{tot}(\rho', z'). \quad (2.26)$$

The exact expression of the Lindhard's dielectric function is rather complicated but hopefully is really easy under the Thomas-Fermi approximation, as it is shown later.

To go further, it is more convenient to consider the in plane Fourier transform of the external and total potential for a given z : $V_{tot}(q, z)$ and $V_{ext}(q, z)$. To get rid of the finite width of the quantum well, let us consider the effective potential associated with only one subband :

$$\langle V_{tot}(q) \rangle = \int dz |\phi_0|^2 V_{tot}(q, z). \quad (2.27)$$

Doing the same for the external potential, we have :

$$\langle V_{tot}(q) \rangle = \frac{\langle V_{ext}(q) \rangle}{\epsilon(q)}. \quad (2.28)$$

This equation is really the heart of Thomas-Fermi approximation because the Fourier transform of the Lindhard's dielectric function is considered to be independent of the Fermi energy and temperature and is given by :

$$\epsilon_{TF}(q) = 1 + \frac{2}{qa_B^*} = 1 + \frac{q_{TF}}{q} \quad (2.29)$$

where a_B^* is the effective Bohr radius equal to $\frac{\epsilon_R}{m^*} a_0$ (with a_0 the Bohr radius $\sim 0.53 \text{ \AA}$) and $q_{TF} = \frac{2}{a_0^*}$. In the case of $In_{0.75}Ga_{0.25}As$, we have [32] $a_B^* = 20 \text{ nm}$ with $m^* = 0.036$ and $\epsilon_R = 14$.

As presented in the next section, we are now able to estimate quantitatively one important ingredient needed in this work : the disorder potential $\phi_d(x, y)$.

2.1.3 The disorder potential

Up to now, we neglected one important feature of the heterostructure : the delta-doped silicon layer. Indeed, during the MBE growth of the heterostructure, the silicon dopants are randomly distributed in a plane located at a distance s (of 20 nm) above the 2DES layer. The dopants and the 2DES are separated by the so-called *spacer* layer. A large spacer thickness increases the mobility (shown later) but decreases the 2DES electron density. The maximum thickness of the spacer is limited due to the lattice mismatch between InGaAs and InAlAs : elastic energy stored through the strain can yield threading dislocations, which are detrimental for the properties of the heterostructure.

Let us now go back to electrical considerations. Within a finite window (L_x, L_y) of our heterostructure, the singly ionized silicon dopants density can be expressed as :

$$n_{2d}^{Si}(\rho, z) = \sum_{i=1}^N \delta(\rho - \rho_i) \delta(z + s) \quad (2.30)$$

where N is the number of dopants in the window and ρ_i the location of the i^{th} dopant. The frame of reference is fixed in Fig. 2.4a, illustrating the random distribution of dopants and the plane (dashed lines) within the 2DES where the disorder potential $\phi_d(x, y)$ has to be estimated in order to (later in this manuscript) perform 2D transport simulations in an accurate potential landscape.

Let us first consider the potential distribution associated with a single dopant V_d , e.g. the one in white in Fig. 2.4 a, we have :

$$V_d(\rho, z) = -\frac{e}{4\pi\epsilon_0\epsilon_r} \frac{1}{\sqrt{(\rho - \rho_i)^2 + (z + s)^2}}. \quad (2.31)$$

In order to find the external potential leading to disorder we have to subtract from the total dopant density n_{2d}^{Si} , the average density $\bar{n}_d = \frac{N}{L_x L_y}$ that we already took into account to solve the bottom of the conduction band profile. Indeed, only deviations $\tilde{C}$ from this total density are responsible for the disorder profile. We have :

$$\tilde{C}(\rho, z) = \left[\sum_{i=1}^N \delta(\rho - \rho_i) - \bar{n}_d \right] \delta(z + s) = C(\rho, z) \delta(z + s). \quad (2.32)$$

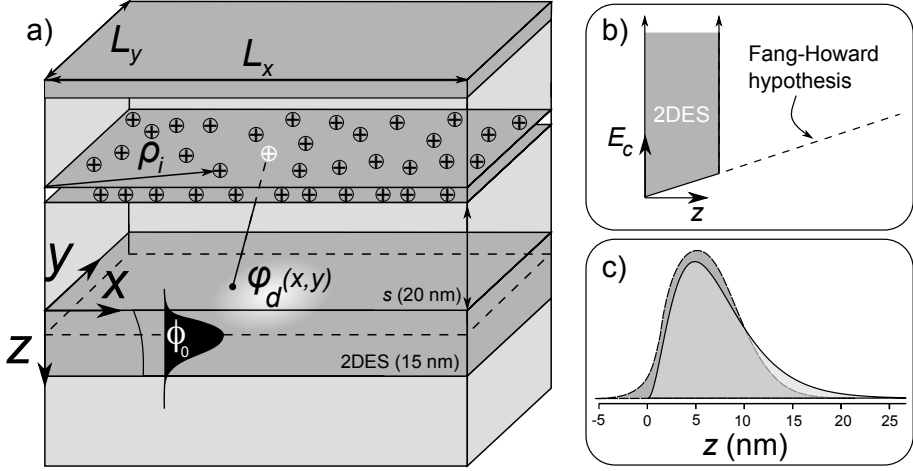


Figure 2.3: a) Illustration of the heterostructure retrieved from Fig.2.2a where the dopants are illustrated by circled '+'. They are randomly located in a plane s nm above the 2DES. Their location is given by the stochastic variable ρ_i . The white dopant induces a potential illustrated in white at the level of the 2DES. The disorder potential ϕ_d depends is taken as a superposition of the potential induced by each dopant. ϕ_d is estimated at the location where the probability distribution along the z -axis for the first subband is maximal. b) Comparison between the tentative conduction band profile E_c in the Fang-Howard hypothesis (semi-infinite quantum well) and in our specific situation (finite width quantum well). c) Comparison between the module squared of the first subband wavefunction along the z -axis for the Fang-Howard model (light grey) and for our specific situation (3D Poisson-Schrödinger simulations).

The external potential induced by deviations from the average dopants density is a 2D convolution product between V_d and C . Working in the Fourier q -space is much more convenient as the convolution becomes a product.

$$\varphi_d^{ext}(q, z) = C(q)V_d(q, z) \quad (2.33)$$

where $V_d(q, z) = 2\pi e^{-q(z+s)}/q$.

At this point we have to get rid of the third dimension in order to discuss 2D transport in the rest of the manuscript. Nevertheless, as mentioned earlier, the effective potential 2.27 requires the knowledge of the ground state wave function $\phi_0(z)$.

In order to go on analytically, we assumed that the ground state wave function of our 2DES was correctly approximated by the Fang-Howard wave function. The Fang-Howard wave function is given for a triangular semi-infinite quantum well [28] see Fig. 2.3 b.

The effective external potential (integrated along the z axis) caused by the delta-doped silicon layer is :

$$\langle V_{ext}(\mathbf{q}) \rangle = \frac{-e}{4\pi\epsilon_0\epsilon_r} C(\mathbf{q}) \frac{2\pi e^{-qs}}{q} \int_0^\infty dz e^{-qz} |\phi_0(z)|^2 \stackrel{\text{FH}}{=} \frac{-e}{2\epsilon_0\epsilon_r} C(\mathbf{q}) \frac{e^{-qs}}{q} \left(\frac{b}{q+b} \right)^3 \quad (2.34)$$

where $b = \frac{1}{a_B^*} \left(\frac{33\pi}{2} n_{2D} a_B^{*2} \right)^{\frac{1}{3}} \sim 0.41 \text{ nm}^{-1}$ for our quantum well.

In order to get the Fourier coefficient of the screened total disorder potential $\varphi_d(\mathbf{q})$ we have to divide by the Thomas-Fermi dielectric function ϵ_{TF} (see Eq.(2.29)) which yields :

$$\varphi_d(\mathbf{q}) = -4\pi E_{Ry}^* a_B^* \frac{e^{-qs}}{q + q_{TF}} \left(\frac{b}{q+b} \right)^3 C(\mathbf{q}) \quad (2.35)$$

where E_{Ry}^* is the 'effective' Rydberg energy given by $\frac{1}{\epsilon_r} \frac{a_0}{a_B^*} E_{Ry} \sim 2.6 \text{ meV}$ where E_{Ry} is the famous Rydberg energy of 13.6 eV.

We can now return back in real space :

$$\varphi_d(\rho) = \frac{1}{(2\pi)^2} \int d^2\mathbf{q} \varphi_d(\mathbf{q}) e^{-i\mathbf{q}\cdot\rho} = -\frac{E_{Ry}^* a_B^*}{\pi} \int d^2\mathbf{q} \frac{e^{-qs}}{q + q_{TF}} \left(\frac{b}{q+b} \right)^3 C(\mathbf{q}). \quad (2.36)$$

The Fourier coefficients $C(\mathbf{q})$ contains all the statistical informations about the deviation from the average dopant density. The statistical properties of these $C(\mathbf{q})$ lead to :

$$C(\mathbf{q}) = \sqrt{\frac{\bar{n}_d L_x L_y}{2}} r(\mathbf{q}) e^{i\theta(\mathbf{q})} \quad (2.37)$$

where $r(\mathbf{q})$ is a random variable with a χ -probability density function (PDF) with 2 degrees of freedom and $\theta(\mathbf{q})$ has a uniform PDF on the interval 0 to 2π .

Such expression is really not straightforward but few physical insights can be extracted. For example, the role of the *spacer* thickness s becomes clearer. The short wavelengths component of disorder are exponentially damped (e^{-qs}) all the more so as s is made larger.

Also, according to Fermi's golden rule [28], a static potential can scatter a particle from a given initial state (characterized by a wave vector $\mathbf{k}$) to a final state ($\mathbf{k} + \mathbf{q}$) with a *rate* ($1/\tau_q$) given by :

$$\frac{1}{\tau_q} = \frac{1}{A^2} \langle |\varphi(\mathbf{q})|^2 \rangle = (4\pi E_{Ry}^* a_B^*)^2 \frac{e^{-2qs}}{(q + q_{TF})^2} \langle C(\mathbf{q})^2 \rangle. \quad (2.38)$$

Knowing the statistical properties of $C(\mathbf{q})$ we have $\langle C(\mathbf{q})^2 \rangle = 2$. We can now find an extremely useful transport quantity, the scattering time τ defined by the inverse of the sum of all the scattering rates of a particle from its initial states to final states. Each final state has to be taken into account since quantum mechanically, a particle has a non zero probability amplitude to be scattered in any admissible state. We have the general formula :

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \sum_{\mathbf{q}} \frac{1}{A^2} \langle |\varphi(\mathbf{q})|^2 \rangle (\delta(E(\mathbf{k}) - E(\mathbf{k} + \mathbf{q})) + \delta(E(\mathbf{k} + \mathbf{q}) - E(\mathbf{k}))). \quad (2.39)$$

This is a general expression that can be used for any external potential, e.g. as we will move further in the manuscript, this relation can also be used to quantify how the tip potential is scattering electrons (see chapter 4). Let us note that disorder is not supposed to favour any direction for electron scattering. Thus, this expression must only depend on the amplitude of $\mathbf{q}$.

The Dirac deltas in Eq.(2.39) ensures that the energy is conserved under a scattering event. This model considers that no inelastic processes can scatter electrons. We thus have to constrain the possible values of $\mathbf{q}$. Using the Al

Kashi theorem, we have :

$$\frac{\hbar}{\tau} = 4\pi E_{Ry}^* \bar{n}_d \int d\theta \frac{e^{-2q(\theta)s}}{(q(\theta) + q_{TF})^2} \left(\frac{b}{b + q(\theta)} \right)^6 \quad (2.40)$$

with $q(\theta) = \sqrt{\frac{2m^*}{\hbar^2} E(1 - 2\cos(\theta))}$ where E is the energy of the particle. This relation can be derived from the inset of Fig. 2.4 b. Indeed we consider static disorder where no inelastic process can scatter electron so energy must be conserved as electron are scattered from a state $\mathbf{k}$ to a state $\mathbf{k} + \mathbf{q}$ with a scattering rate given by the Fermi golden rule. In Fig. 2.4a, the scattering time τ is estimated for different energies via Eq.(2.40).

The value of the spacer is set to 20 nm. As expected, for a given disorder, an electron is less scattered as its energy increases which follows our intuition. Also, as the spacer keeps getting larger, the τ keeps increasing (see Fig. 2.4b) because the spatially rapidly varying Fourier components of disorder are being exponentially damped. A large spacer would bring us closer to the assumption of a uniform potential value ϕ_m where electrons probability amplitude would be pure Bloch states. Obviously for a really large spacer, no electrons would be present in the quantum well so a trade-off on the value of s must be reached.

It is now time to get numerical estimations of the disorder potential (via Eq. (2.36)). The result is shown in Fig. 2.4d. This map of disorder must fulfil some properties that we can *a priori* expect from disorder. Indeed, no direction can be favoured by disorder. No matter the direction, the potential profile must have the same average and same standard deviation. Doing so we computed an average value around 0.6 meV and the standard deviation around 4.78 meV for the map presented in Fig. 2.4 d. Also disorder is expected to vary spatially with the same correlation length no matter the direction. The auto-correlation must therefore present rotational symmetry. The auto-correlation γ is mapped in Fig. 2.4c in the following way. For any given correlation lag in the (x,y) plane, we compute γ for the overlapping part of the real-space disorder map. Qualitatively, the value of γ quantifies how much two maps are 'alike'. For the disorder map shown in Fig. 2.4d, this value is found to be divided by 2 as a correlation lag $x_{lag} = 70$ nm is introduced along the x direction. As shown in the inset of the same figure, we can see that $\gamma(x_{lag}, y_{lag})$ is indeed rotational invariant. This map of disorder will later be used in numerical simulations of transport inside the nano-devices we studied. In comparison, for an Anderson model [47], this correlation length is zero.

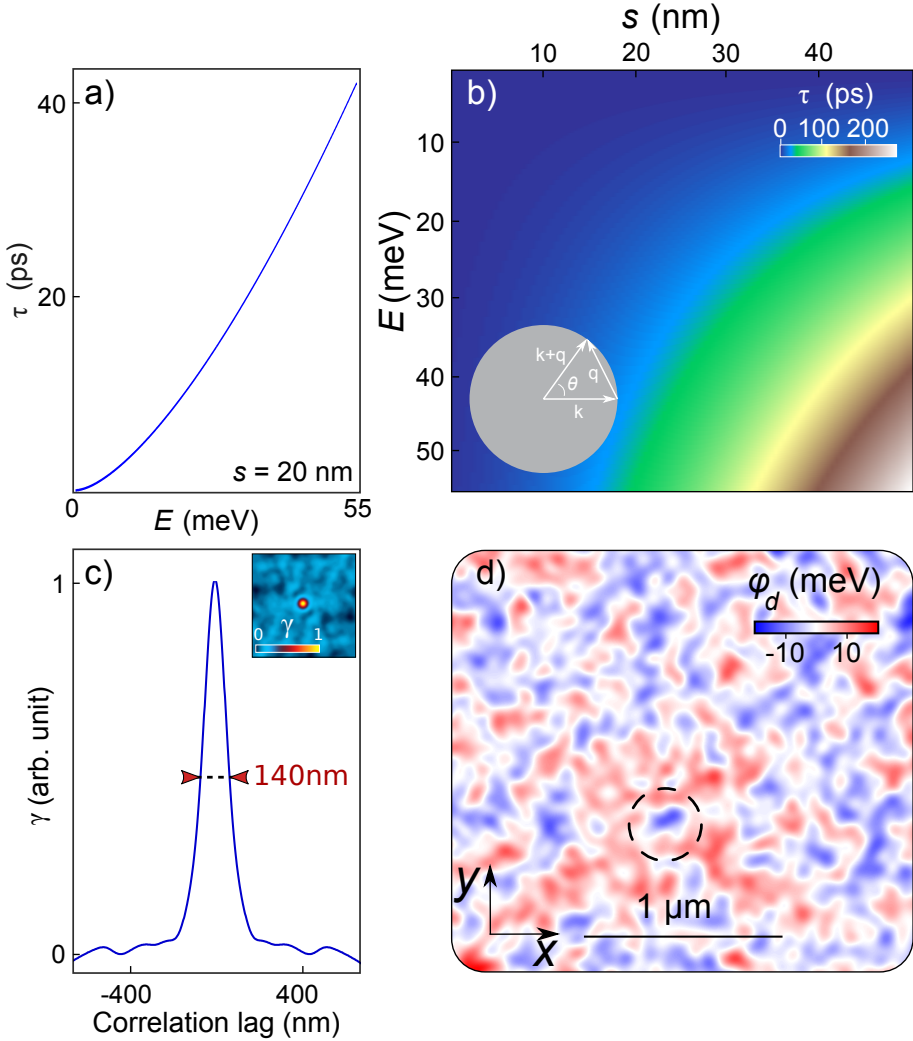
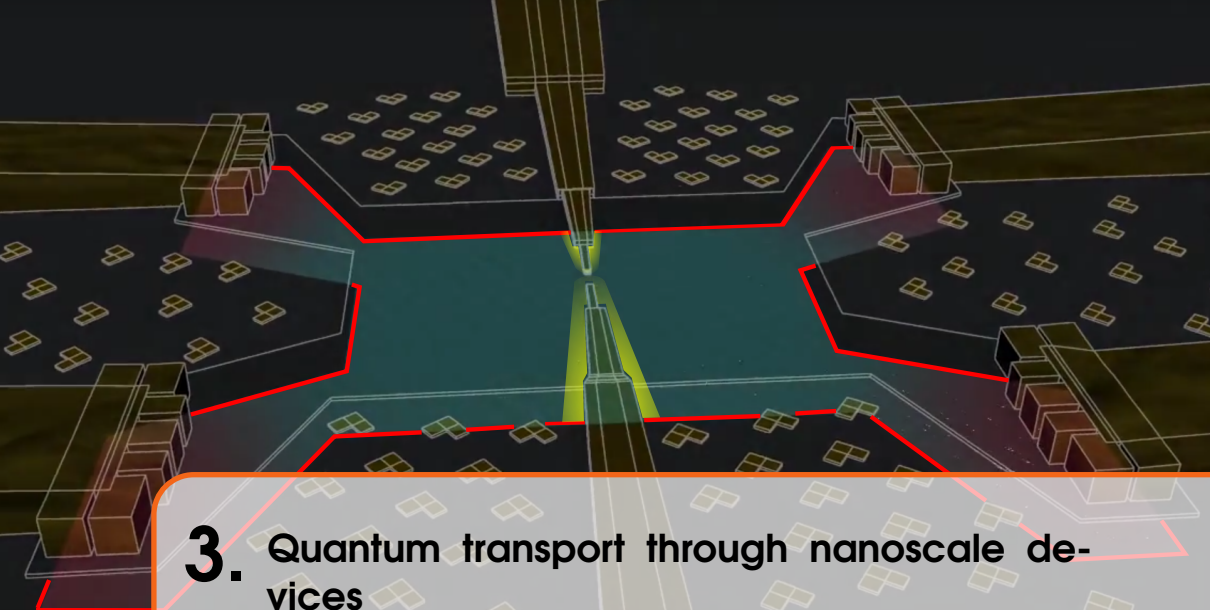


Figure 2.4: a) Scattering time τ as a function of the kinetic energy within a disordered 2DES (where disorder is induced by a distribution of ionized dopants, s nm away from the 2DES). b) The same scattering time mapped as a function of the spacer thickness s . c) Normalised auto-correlation γ as a correlation lag shifts the potential map presented in d) along the x-axis. d) Computed real-space disorder potential (ϕ_d) at the level of the 2DEG that will be used in the forthcoming simulations. The disorder standard deviation (S_d) is 4.78 meV, calculated taking into account a random distribution of Si ionized dopants located 20 nm above the 2DEG (*i.e.* thickness of the InAlAs spacer).



3. Quantum transport through nanoscale devices

The conductance of a device can be viewed as the solution of a transmission problem [48]. In this thesis, we considered two-terminal systems, but the Landauer-Büttiker [49] formula is derived for the general multi-terminal situation. The scattering theory as we describe it in this chapter is expressed considering both infinite electron mean free path and coherence length. Also, the theoretical framework discussed hereafter considers non-interacting particles. In this framework, electrons are only scattered by the specific geometry of the considered device ('bouncing' against the hard-wall of the device).

The general idea is that electron plane waves are scattered by the lithographic boundaries of the device. These waves are *guided* in 1D entities - called leads - that quantize the momentum of electrons in the transverse direction and let them flow in the longitudinal direction. The way a connected device reflects or transmits electrons is a fingerprint of its geometry (for the aforementioned assumptions). Let us start by conceptually dividing the 2DES into 3 regions. An arbitrarily-shaped structure (a ring-like geometry, a quantum billiard, ...) is attached to two leads connected to two electron reservoirs characterized by electron chemical potentials (the left and right chemical potentials are μ_L and μ_R , respectively).

The leads are considered as disorder-free and semi-infinite. We consider for illustration purposes that both leads are of width W . In general, the leads can have different widths. These assumptions imply that an *ansatz* solution

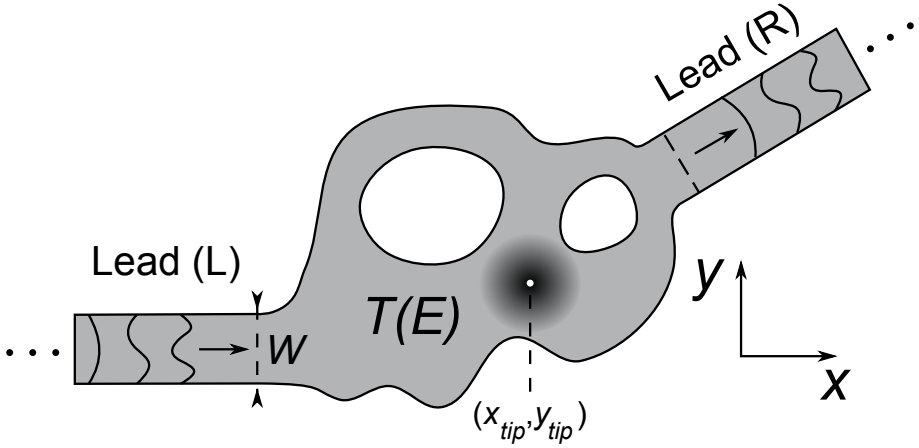


Figure 3.1: Illustration of an arbitrarily shaped geometry. This is a 2D conducting device connected by two leads (left (L) and right (R)). The width of the each leads is W . Depending on the energy E , N transport modes can be present in the leads. The three first modes are illustrated. Solving the scattering problem consists in finding the total transmission $T(E) \in [0, 1]$ (see Eq.(3.12)). As discussed in the rest of the thesis, the transmission can be tuned thanks to a tip induced potential illustrated by the black shaded circle centred in (x_{tip}, y_{tip}) .

to Schrödinger equation in the leads is a product of particle-in-a-box wave-functions (along the confinement direction y) and propagating Bloch wave-functions in the transport direction x . Particle-in-a-box solutions are :

$$\phi_n(y) = \sqrt{\frac{2}{W}} \sin\left(\frac{n\pi y}{W}\right), \quad (3.1)$$

where n is an integer labeling the different solutions called the propagative modes.

Considering a single particle 2D continuous effective-mass Hamiltonian (separable operator), an incoming particle with a given energy E can have different momenta in the longitudinal direction as long as energy is conserved. Thus we have :

$$E = \frac{\hbar^2}{2m^*} \left(\left(\frac{n\pi}{W} \right)^2 + k_n^2 \right). \quad (3.2)$$

From now, we call N the largest integer value of n for which Eq.(3.2) is verified. In such conditions, we say that there are N propagative modes in the leads.

Eventually, the incoming lead-states (labelled with a minus sign (-)) coming from left (L) lead are :

$$\varphi_{L,E,n}^{(-)}(x,y) = \frac{1}{\sqrt{v_n}} e^{+ik_n x} \phi_n(y) \quad (3.3)$$

with $k_n > 0$, the '+' implies a plane wave travelling towards increasing values of x . Also with $(v_n)^{-1/2} = (m^*/\hbar k_n)^{1/2}$ each of these modes carries a unit of current density in each mode. These are the quantum states that are expected in the lead.

The next step is to write what is expected asymptotically (far away from the peculiar situation close to the device) for the *outgoing* states (labelled with a plus sign (+)) - scattered by the central region (see Fig. 3.1). Thanks to the linearity of Schrödinger equation, we can write the scattering states corresponding to an electron incoming from lead 1 with energy E in the mode n as [50]:

$$\Phi_{L,E,n}^{(+)}(\mathbf{r}) = \begin{cases} \varphi_{L,E,n}^{(-)}(\mathbf{r}) + \sum_{m=1}^N r_{mn} \varphi_{L,E,n}^{(+)}(\mathbf{r}) & x < 0 \\ \sum_{m=1}^N t_{mn} \varphi_{R,E,n}^{(+)}(\mathbf{r}) & x > 0 \end{cases} \quad (3.4)$$

$$\Phi_{R,E,n}^{(+)}(\mathbf{r}) = \begin{cases} \varphi_{R,E,n}^{(-)}(\mathbf{r}) + \sum_{m=1}^N r'_{mn} \varphi_{R,E,n}^{(+)}(\mathbf{r}) & x < 0 \\ \sum_{m=1}^N t'_{mn} \varphi_{L,E,n}^{(+)}(\mathbf{r}) & x > 0 \end{cases} \quad (3.5)$$

with $\mathbf{r} = (x, y)$ (see Fig. 3.1).

The reflection (r_{mn}, r'_{mn}) and transmission (t_{mn}, t'_{mn}) coefficients are complex quantities (characterized by an amplitude and a phase, both fingerprint of the scatterer). One can think of straightforward situations to understand such relations. For example, a perfect mirror could reflect a mode and an incoming solution $\phi_{L,E,n}^{(-)}$ would be superimposed with $e^{i\pi} \phi_{L,E,n}^{(+)}$. The outgoing solution in the lead would be identically zero which translate evidently into no current.

When considering transmission, the real space values of the wavefunction are not the most easily interpretable quantities we can consider. Indeed, as we identified "plane waves" carrying unitary current densities, the interesting information is how much of this unit is reflected or transmitted by the scatterer (the device in our case). This is precisely captured by a matrix relation transforming incoming plane waves ($\phi_{L,E,n}^{(-)}$ and $\phi_{R,E,n}^{(-)}$) into outgoing ones ($\phi_{L,E,n}^{(+)}$ and $\phi_{R,E,n}^{(+)}$). This relation can be formally written using a matrix S called the *scattering matrix* or simply *S-matrix*, relating the incoming flux to the outgoing fluxes.

Given the following wave functions :

$$\Psi_L(x, y) = \sum_{i=1}^N a_i \phi_i^{(-)}(x, y) + \sum_{i=1}^N b_i \phi_i^{(+)}(x, y) \quad (3.6)$$

in the left lead and

$$\Psi_R(x, y) = \sum_{i=1}^N c_i \phi_i^{(-)}(x, y) + \sum_{i=1}^N d_i \phi_i^{(+)}(x, y) \quad (3.7)$$

in the right lead, the relation between the 'incoming' coefficient (a_i, c_i) and outgoing coefficient (b_i, d_i) for $i = 1, \dots, N$ is given by :

$$\begin{bmatrix} b_1 \\ \vdots \\ b_n \\ d_1 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} r_{11} & \dots & r_{1N} & r'_{11} & \dots & r'_{1N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ r_{N1} & \dots & r_{NN} & r'_{N1} & \dots & r'_{NN} \\ t_{11} & \dots & t_{1N} & t'_{11} & \dots & t'_{1N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ t_{N1} & \dots & t_{NN} & t'_{N1} & \dots & t'_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \\ c_1 \\ \vdots \\ c_n \end{bmatrix} \quad (3.8)$$

For each mode, the total transmission probability T from left to right lead

is :

$$T = \sum_{n=1}^N \sum_{m=1}^N |t_{nm}|^2 \quad (3.9)$$

The total reflection R is related to T by $T + R = N$.

To lighten the burden of notations, let us write the scattering matrix ($2N \times 2N$) as :

$$b = Sa \quad (3.10)$$

with

$$S = \begin{bmatrix} r & t' \\ t & r' \end{bmatrix} \quad (3.11)$$

and $a=(a_i, c_i)$ and $b=(b_i, d_i)$ are column vectors. r, r' and t, t' are the $N \times N$ reflection and transmission matrices (that we can identify in Eq. (3.8)). Using these notations we can write Eq. 3.9 as :

$$T = \text{tr} \{ t t^\dagger \} \quad (3.12)$$

As proposed by Rolf Landauer, this result is directly connected to the electrical conductance (G) of a nanodevice in the coherent and ballistic regime of transport. Indeed, due to current conservation, we can look at what happens in the leads (see Fig. 3.2a). Let us recall that the transmission depends on the particle energy (as illustrated in Fig. 3.2b). In addition, we know that a lead is conceptually a 1D device where the number of electrons in mode n with an energy between E and $E + dE$ is given by the 1D density of states (for non-interacting particles) $n_{1D,n}(E) = \frac{1}{h v_n(E)}$ with $v_n(E)$ the velocity of a particle with energy E in the n^{th} mode. The current I_{in} for each mode then simply writes : the electron charge times the number of them with energy E times their energy dependent velocity times their availability in the left 2DES electron reservoir (given by the Fermi-Dirac statistics f_L).

$$I_{in}(E) = \sum_{n=1}^N \sum_{m=1}^N e n_{1D,n} v_n |t_{nm}|^2(E) f_L(E, T). \quad (3.13)$$

The net current is then given by the integral in energy of $I_{in} - I_{out}$. Using the above notations, we have :

$$I = \frac{2e}{h} \int_0^\infty dE T(E) [f_L - f_R] \quad (3.14)$$

where the factor 2 takes into account the spin degeneracy.

From this formalism, we can extract two important messages regarding forth-coming experiments and simulations.

First, at vanishing temperatures, the Fermi-Dirac distributions starts to look as a Heavyside distribution $\theta(E - \mu)$ (with μ the the average electrochemical potential of the unbiased system - also written E_F). A further simplification can be made in the small bias linear regime. Thanks to a Taylor development, we have :

$$[f_L - f_R] \sim \frac{\partial(f_L - f_R)}{\partial E} \Big|_{\mu} (\mu_L - \mu_R) \sim e \delta(E - \mu) V_{bias} \quad (3.15)$$

where the differences of chemical potentials is imposed by the voltage bias V_{bias} (see Fig. 3.2). The latter approximation is experimentally valid if we consider a difference of potential smaller than the extrinsic temperature smearing of $\sim k_B T$. This translates into an assumption central to this manuscript :

$$V_{bias} \ll \frac{k_B T}{e}. \quad (3.16)$$

Combining the near-equilibrium measurements and assuming a coherent and ballistic regime of transport, we have, at low temperature :

$$G = \frac{I}{V_{bias}} = \frac{2e^2}{h} T(E_F) \quad (3.17)$$

where $\frac{2e^2}{h}$ is the so-called quantum of conductance. The value $T(E_F)$ is precisely the one simulated through chapter 6 (computed with the Kwant package as discussed in appendix C).

A second important message can be learned from the scattering formalism : it concerns symmetries. Indeed we fabricated geometries presenting real space symmetries (rotational, mirror planes etc. - see the two geometries presented in chapters 5 and 6). In our experiments, a tip induced local electrostatic perturbation presenting a 2D circular symmetry (detailed in chapter 4) is swept e.g. in a plane parallel to the 2DES. A legitimate question is to ask whether or not these geometrical symmetries (defined by the *whole* device - leads *and* nanostructure) can be retrieved in $T(E_F)$ and thus G as the perturbation is swept. To answer that question we have to look at some properties of *S-matrices*.

The scattering formalism is really convenient as many properties of the *S-matrices* can be derived on the basis of fairly simple arguments [29]. For

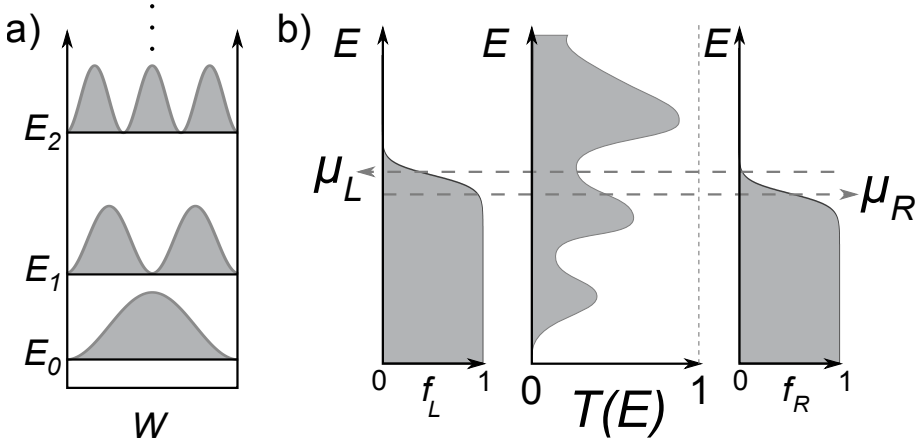


Figure 3.2: a) Energies and module of the wavefunctions of a particle in a box. The box corresponds to the cross section of one lead presented in Fig. 3.1. The energy level positions are proportional to $1/W^2$ and n^2 . As presented in Eq.(3.2), the total number of allowed modes must be such that $E_F = \frac{N^2 \pi^2 \hbar^2}{2m^* W^2}$. For a density of $1 \times 10^{16} \text{ m}^{-2}$ - that is $E_F = 55 \text{ meV}$, and a lead width of 400 nm (that is the width used in chapter 6), we have $N = 29$ modes. b) The total transmission $T(E)$ of an imaginary device geometry as a function of the energy of the particle. In the leads, particles are available according to the Fermi-Dirac statistics whose derivative varies on a energy range given by $k_B T$. In the near-equilibrium condition, the bias $V_{bias} = \frac{\mu_L - \mu_R}{e}$ is smaller or comparable to $k_B T$ ($\sim 3.44 \text{ } \mu\text{eV}$ at 40 mK).

example, with current conservation, it can be shown that S -matrices are *unitary* ($S^\dagger S = \mathbb{1} = SS^\dagger$). When added to this property, time reversal symmetry allows to state that S -matrices are *symmetric* ($S = S^T$). In our measurements, only the magnetic field could break time reversal symmetry [29] (which concerns only few measurements in this manuscript).

A symmetric S -matrix means that $T = \text{tr} \{tt^\dagger\} = \text{tr} \{t't'^\dagger\} = T'$. This means that probing the system with a current from left to right or right to left leads to the same conductance. This is really useful to infer the expected symmetry in an SGM map (see chapter 4).

Of course, for an actual measurement, it is technically impossible to achieve perfect symmetries. For a nanodevice we can cite different sources of asymmetry : intrinsic potential disorder, edge roughness due to etching, etc. For the tip induced potential we have : non homogeneous potential screening¹ (presented in chapters 5 and 6), the actual AFM tip geometry, scans in a plane non perfectly parallel to the 2DES, etc. Despite these deviations from the ideal situation, symmetries can be obtained in SGM maps as illustrated by the simulations in figure 6.3² and experiments in figure 6.4b.

3.1 Solving the transmission problem : the general guidelines

As presented above, $T(E_F)$ is a central quantity in this manuscript and we aim to get numerical estimates. To do so, we realized transport simulations (e.g. see chapter 6) using Kwant [25]. This python package allows to solve the *scattering* problem. Details about the implementation of Kwant can be obtained in [25, 51]. Although the actual implementation to solve the scattering problem is taken care of, there are a few things that we need to pay attention to when discretizing continuous physical equations. First, we want to state clearly that no many body effects are simulated in this work. The single electron model seemed to be suitable to study the transport mechanism analysed in chapter 6.

To get the presence probability amplitude in the 2DES ($\Psi(x, y)$), we have to solve the 2D effective mass time-independent non-relativistic Schrödinger equation :

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial y^2} + U(x, y) \right] \Psi(x, y) = E \Psi(x, y). \quad (3.18)$$

¹indeed as the geometries are etched from the homogeneous 2DES, the etched domains can no more screen the tip-induced potential

²these maps presents the symmetry of the *whole* system in absence of disorder

An important remark on those solutions must be made regarding transport. The (leads - device) system is infinite. Whereas finite system (isolated systems) have a discrete set of eigenvalues and wavefunctions, in an infinite system one can always find an homogeneous solution for the wavefunction at any energy (except naturally in band gaps). Indeed the device connects two electron reservoirs that have a continuous density of states. The transmission as a function of the energy can be tuned at will (with electrostatic gates, fancy geometries, etc.) and could even mimic isolated systems (e.g. quantum dots) but not arbitrarily close to an isolated system.

Finding analytical solutions for an arbitrarily shaped geometry is really complicated. This is why we try to approximate the solution on a discrete mesh (squared mesh with a lattice parameter a along the x and y directions). A site on this lattice is labelled with the indices i and j (along x and y respectively). If a is *small enough* we can approximate $[H\Psi]_{(i,j)}$ with :

$$\left[\frac{\hbar^2}{2m^*a^2} [-\Psi_{i+1,j} + 2\Psi_{i,j} - \Psi_{i-1,j}] + \frac{\hbar^2}{2m^*a^2} [-\Psi_{i,j+1} + 2\Psi_{i,j} - \Psi_{i,j-1}] + U_{i,j} \right]. \quad (3.19)$$

By setting $t = \frac{\hbar^2}{2m^*a^2}$ and introducing the bracket notation, we can write the Hamiltonian as :

$$H = \sum_{i,j} |\Psi_{i,j}\rangle V_{i,j} \langle \Psi_{i,j}| - t \sum_{i,j} [|\Psi_{i,j}\rangle \langle \Psi_{i-1,j}| + |\Psi_{i,j}\rangle \langle \Psi_{i,j-1}| + h.c.] \quad (3.20)$$

where $-t$ is the hopping energy between first neighbour lattice sites and the on site energy is given by $V_{i,j} = U_{i,j} + 4t$.

The idea now is to conciliate the simulation and measurement parameters. The first parameter is the 2D electron density n_{2D} . In a 2DES, we have :

$$n_{2D} = \frac{m^*}{\pi\hbar^2} E_F. \quad (3.21)$$

Inserting the value of t in this equation leads to :

$$E_{simu} = 2\pi t n_{2D} a^2. \quad (3.22)$$

The lattice parameter a is then chosen so that the energies of the discretized system are made arbitrarily close to those of the continuous system. We know the characteristic length scale with which the solution for Ψ in the 2DES varies

: the Fermi wavelength λ_F . In order to correctly sample the solution in the nanostructure, we must at least resolve correctly the solution in the 2DES. This implies :

$$a \ll \lambda_F. \quad (3.23)$$

There is no exact criterion to prescribe the value of a numerical parameter such as a . We have to realize a convergence analysis and see how the solution evolves as we change this parameter. Eventually, we selected a lattice parameter of 2 nm with a Fermi wavelength 25 nm.

We now have to come back to the on site energy U_{ij} which is in our case a superposition of the disorder potential and the tip potential. In many simulation models, the Anderson-type disorder [47] is modelled by a random value on each site (see e.g. [52]). This random value is chosen from a uniform distribution $[-U_0, U_0]$. Depending on the strength of U_0 , we limit the electron mean free path in the device. For this specific model, using Fermi's Golden Rule, we have [53]:

$$l_\mu = 48a \frac{\sqrt{E/t}}{(U_0/t)^2} \quad (3.24)$$

where a is the lattice parameter. We preferred to opt for a more subtle model because in the actual situation, reducing the sampling parameter should not have any impact on the electron mean free path. To model disorder in the sample, we opted for a numerical evaluation of Eq. (2.36) that is developed in section 2.1.3 but evaluating disorder is a vast subject of theoretical investigation [54, 55]. This allowed us to estimate the amplitude of disorder (see 6.2). This also allowed us to evaluate the scattering time τ associated with silicon dopant induced disorder and see that it was not the limiting elastic scattering time for electrons in the 2DES. The scattering time of 40 ps at Fermi energies of 55 meV (see Fig. 2.4a) would leads to a mobility of 195 (m^2/Vs) which is one order of magnitude higher than what was measured - typically 10 (m^2/Vs) (see Fig. 2.1d).

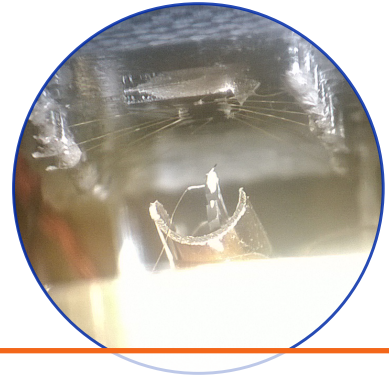
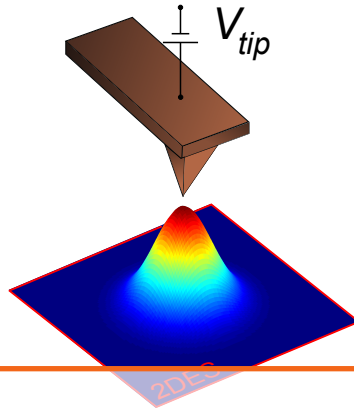
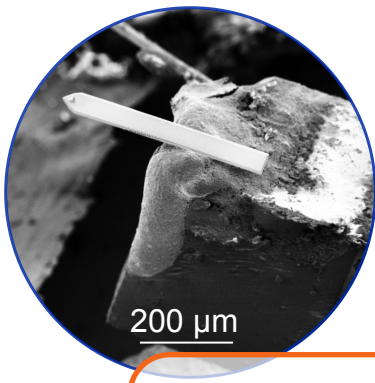
Another contribution to the on-site value comes from the tip induced potential that will be discussed in the next chapter 4.

The central value discussed up to now is the electrical conductance in the framework of the scattering formalism. Nevertheless, this formalism does not allow to directly access *locally* defined quantities e.g. the current density (J) and the local density of states ($LDOS$). In this context, the Keldysh-Green

non equilibrium formalism (referred to as the Non-Equilibrium Green's Functions - NEGF formalism [29, 56–59]) can be used. Thanks to the Fisher-Lee relations [24], the scattering formalism and the NEGF formalism are mathematically equivalent with respect to the Landauer formalism. Nonetheless, the aforementioned *local* quantities can be directly accessed from the Green's functions³. Regarding these mathematical approaches, in particular in the context of scanning gate microscopy, we can cite [60–62].

Up to now, we presented the most relevant 2DES features and the scattering formalism that describe the conductance of nano devices defined from the 2DES. In the next chapter we introduce the central technique used in this thesis : scanning gate microscopy (SGM). SGM is used to modify scattering, hence the conductance, in a conductive nanodevice with a tip induced potential variation (sketched by the black shaded circle in Fig. 3.1 for a tip centred above x_{tip} and y_{tip}).

³that are indeed not functions but rather distributions



4. Scanning gate microscopy

4.1 Working principle

The working principle of scanning gate microscopy (SGM) is illustrated in Fig.4.1. An electrically conductive atomic force microscopy (AFM) tip is voltage-biased so as to modify the electric potential landscape within a nano scale device. Doing so, transport properties (e.g. the electrical conductance G , the Seebeck coefficient...) of the charge carrier system are mapped as a function of the tip-sample relative position, the tip voltage (V_{tip}) and possibly any other parameters specific to the considered experiment (voltage-biased back or top gates, temperature, magnetic field...). We insist on the fact that no current is flowing between the tip and the 2DES which is a fundamental difference to scanning tunneling microscopy - STM. Also, in this work we choose to measure transport properties under near-equilibrium conditions - meaning small source-drain bias ($V_{bias} \ll \frac{k_B T}{e}$). In order to sense the physical transport mechanisms at play in nanodevices, the spatial extent of the tip-induced potential perturbation has to be comparable to the characteristic length scales of some of the aforementioned mechanisms in section 1.2. The heart of this technique consists in identifying and characterising the physical mechanism(s) leading to the features observed in the maps. SGM is an indirect imaging technique in the sense that it allows to pinpoint the position where the mechanism that is modifying the global transport properties is active. An important remark is

that, in this manuscript, SGM was not used with microscopy purposes in mind. Indeed it is used to tune transport through a nanodevice by affecting a physical mechanism at play in the device, so the greatest appeal of SGM is here the scanning gate and not the microscopy. There is no clear limit between : linking tip-induced conductance changes to local properties of the unperturbed system and conductance features caused by tip-induced potential [63]. Nevertheless it is important to distinguish a strongly invasive tip-induced potential from a weakly perturbing one. In chapters 5 and 6, both detailed mechanisms are visible for strongly invasive tip-induced potential.

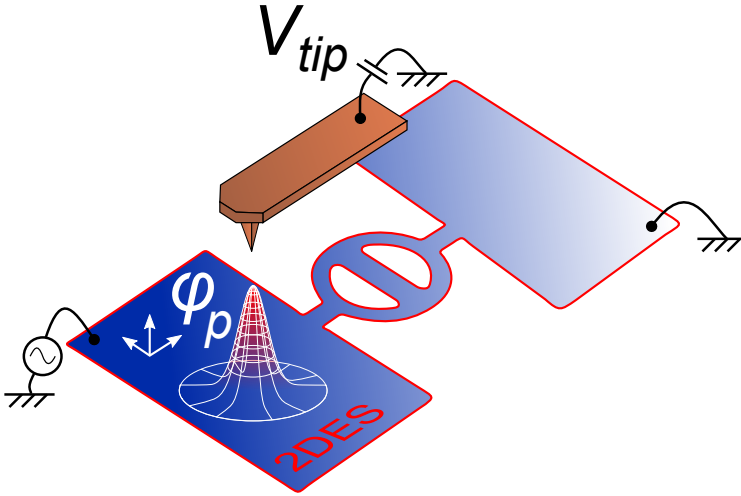


Figure 4.1: Illustration of the scanning gate microscopy working principle. A conductive AFM tip (here biased with a voltage source - V_{tip}) induces a remote electrostatic potential distribution ϕ_p at the level of the 2DES. Here, one side of the ring-like nanodevice is biased with an oscillating source-drain voltage so as to measure - using standard lock-in techniques - the conductance of the device. We can then obtain SGM maps as detailed in this section.

Implementing this working principle requires to:

- develop tools to precisely locate the tip with respect to the device
- understand the amplitude and spatial extent of the tip induced potential perturbation.

An important part of the SGM discipline is to be able to identify *locii*. A *locus* is a set of points whose location satisfies or is determined by one or

more specified conditions. We borrowed that term from geometry. Here is how it is supposed to come into play in SGM. A quantity F will be mapped as a function of external parameters $(a, b, c, \dots)$. The main trick of the SGM game is to identify a *locus* $L(a, b, c, \dots)$ expressed with the external parameters where the quantity F (or a quantity mathematically-derived from F) is quasi-invariant.

4.2 State of the art

Scanning gate microscopy is a technique overlapping many different domains of expertise and that gathered many aspects of both physics and engineering : nano-positioning, cryogenic physics, force detection, low-noise electrical measurements, micro- and nano- fabrication, numerical simulations, ...

Since the eighties and the invention of scanning tunneling microscope (STM)[64], a wealth of scanning probe microscope techniques emerged. These so called *near-field* techniques allow the imaging of surface local fields of various types (e.g. the surface local density of states with STM [65]). Tremendous progresses in the fabrication of piezoelectric actuators gave experimentalists the control of the positioning of a narrow tip, *i.e.* the probe, at a precision better than the atomic scale. With such techniques, one can overcome the *far-field* diffraction limit (characterized by Airy function and Rayleigh criterion) and the resolution is limited by the tip apex characteristic dimension and by the tip-sample distance. Image processing is thus crucial to get the actual size of what has been imaged. For example, in atomic force microscopy (AFM), the image is a dilation of the sample surface topography due to the finite-sized AFM tip (not a δ -like probe) and dilation-erosion [66] algorithm are used for image reconstruction. The image is thus not separable from the probe and morphological image analysis [67] is sometimes needed to reveal the actual characteristic of the probed object. As a sub-part of the vast scanning probe microscopy near-field researches, SGM was developed keeping track of the inspiring and valuable messages carried by advances in other near-field techniques. Without being exhaustive, we would like to name : STM, AFM (and its various modes : EFM, MFM, KPFM, C-AFM, PFM etc.) (as the two precursors in the field), scanning near-field optical microscope (SNOM) [68–70], tip-enhanced Raman scattering (TERS)[71], scanning SQUID-on-tip microscopy (SSM) [72–75], etc.

Regarding SGM, the local induced potential perturbation ϕ_p is the lever to tune or image: the spatial distribution of microscopic currents (4.2.1), resonant

or interference patterns (due to the undulatory nature of charge carrier)(4.2.2), single charge events in traps or quantum dots (4.2.3) etc. Although rather young, this technique inspired a wealth of original ways to address both numerically and experimentally physical puzzles about the nature of transport at the local scale in a wide variety of device geometries - quantum rings, ballistic cavities, quantum point contacts... - as well as in different materials - III-V semiconductor heterostructures, carbon based devices (graphene, carbon nano-tubes, bilayer graphene) etc.

4.2.1 Spatial distribution of microscopic currents

Redistributing charge current at the local scale with an SGM tip-induced potential gives valuable information about the current distribution itself. Indeed this way allows to identify discrepancies between an ideal vision of charge transport and actual transport. For example, important insights were brought by the group of professor Westervelt (birthplace of SGM through the pioneering work presented in Refs. 76, 77) about electrons exiting one of the simplest nanoscale device : the quantum point contact. A quantum point contact (QPC) [78][79] is a narrow constriction where, in the ballistic regime of transport, confinement orthogonal to the transport direction leads to a quantization of transverse momentum in the QPC. This constriction, forming a 1D conductive channel, exhibits peculiar transport properties. Indeed, as predicted by the Landauer formula, as the channel gets narrower, the conductance of the constriction presents conductance plateaus at integer multiples (propagative modes) of the quantum of conductance $G_0 = \frac{2e^2}{h}$ (see Fig. 4.2a)¹. Theoretically, the module of current densities associated with the propagative modes take smooth and simple shapes in the vicinity of the constriction (see Fig. 4.2b and c for the two first theoretical propagative modes). What they showed, by perturbing transport in the vicinity of the QPC with the SGM tip in the strongly perturbative regime (the tip bias creates a moving depletion disk in the 2DES)(see Fig. 4.2d), is that electrons flow out of the QPC coherently along *branches* [80, 81](see Fig. 4.2d). We will spend some time on this result as it shows an instructive situation where SGM lead to a paradigm shift. Indeed this unexpected branched flow is related to the presence of disorder in the 2DES (see chapter 2). An important message emanating from this experiment is that even though *conventional* transport measurements (see Fig. 4.2a) are in agreement with theory, real-space transport microscopy is crucial to grasp the physics of electrons flowing out of QPCs.

¹ which makes this the natural unit to express the measurements and simulation results

A coherent interference mechanism also decorates the data with oscillations separated by half the Fermi wavelength. It can be attributed to a Fabry-Perot interference patterns forming between the tip-induced potential and the QPC.

Another remarkable feature of the curve presented in Fig. 4.2a occurs beneath the last conductance plateau at $0.7G_0$: this is the so-called 0.7 anomaly as it is not predicted by Landauer formula [82]. The peculiarity of this anomaly is that it occurs independently from the material or the QPC geometry. SGM appeared as a suitable tool to study such anomaly since the SGM tip allows the fine tuning of the potential landscape within the QPC. On the verge of pinch-off, the non-interacting electron approximation is no longer valid as the electron potential energy becomes greater than its kinetic counterpart: the single electron picture is no more valid and electrons start to *feel* one another (breakdown from the single electron picture is presented in section 4.2.3). In such case, the experimental conditions for the formation of a Wigner crystal can be met, as demonstrated experimentally both using SGM and length-tunable constrictions [83–85]. As electrons close to the constriction interact with the crystal, it can also lead to rigid phase relation for electrons described by Kondo related-physics, which was evidenced thanks to the SGM tip-induced interferometer [86]. The evolution of this coherent branch flow contrast at different point of a conductance plateau or between plateau keeps generating interest in the SGM community [87, 88].

With similar microscopy purposes in mind, SGM was used to image electron cyclotron orbits in the framework of magnetic focusing [89]. In high charge carrier mobility devices [90], charges travel along cyclotron orbits when applying a magnetic field orthogonal to the 2DES [61, 91]. By scattering electrons away from these classical trajectories with a tip-induced potential, the change in focusing signal is mapped which yields real-space imaging of cyclotron orbits in the ballistic regime of transport.

SGM also offers the opportunity to re-orient topologically protected edge states in the quantum Hall regime in different geometries and infer their spatial distribution. In QPCs, edge states are hence brought close to one another. Inducing a local potential perturbation allows one to selectively back-scatter the so-called integer [92] or even fractional [93] edge states.

A tip induced potential is also used to modify the current distribution in ballistic cavities and quantum stadii [63, 94, 95], revealing electron dynamics inside these complex systems [96].

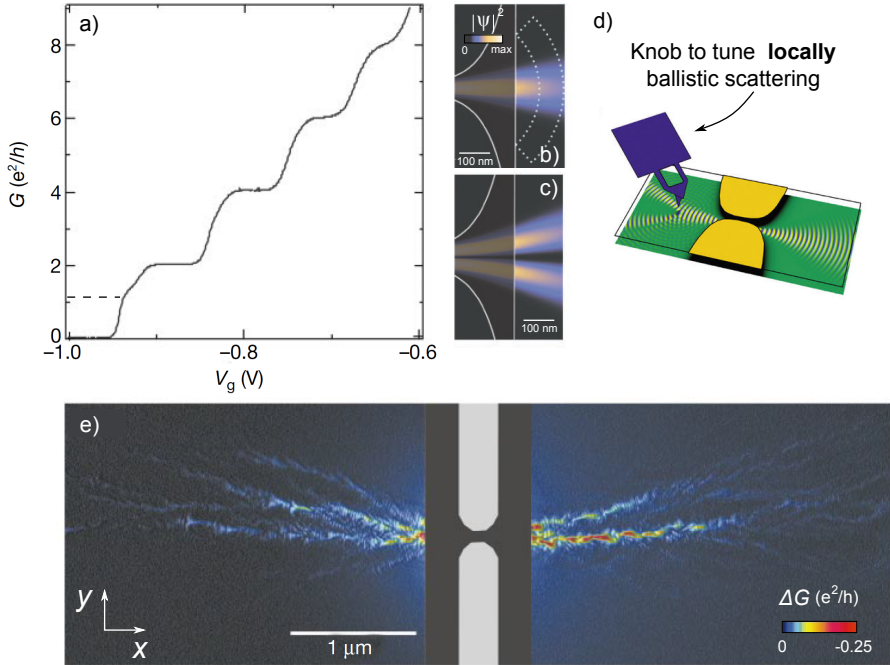


Figure 4.2: Conductance (G) measurement of a quantum point contact as the channel gets narrower thanks to a depleting metallic - negatively biased - top-gated defining a constriction. b) Probability amplitude (squared modulus $|\Psi|^2$) associated to the first (b) and second (c) propagating modes. d) Illustration of how a conductive - voltage biased -AFM tip can be used to tune transport at the local scale. e) Conductance variations (ΔG) as the tip is scanned in the vicinity of the QPC (biased on the first conductance plateau) at a temperature of 1.7K. All the data in this figure were extracted from [81].

4.2.2 Resonant or interference patterns

As mentioned above, SGM can be used to redistribute current lines in a device but other features of electrons transport can be tuned. For example, as the characteristic dimensions of the device are reduced, the local electron density of states (LDOS) presents features that are specific fingerprint of the studied device geometry. These peculiar features, called 'quantum scars' [97], correspond to unstable semi-classical orbits in the vicinity of which charge carriers tends to localize (leading to a higher LDOS value). The general idea that connects these scars and SGM is the following : a tip-induced potential crossing this peculiar features tends to destroy the charge localization and perturb thus the conductance. These conductance variation are in that sense a way to image the LDOS, provided that the tip induced perturbation is small enough [98]. This can be viewed as equivalent to the imaging of LDOS with STM for electronic quantum corrals [65] or SNOM that images the photonic LDOS in confined nanostructures [99].

Mesoscopic devices, such as quantum rings, are prone to exhibit such LDOS features. For example, several works of Prof. Hackens' research team were dedicated to the understanding of the physical origin of different SGM maps features. In a quantum ring (QR), they revealed various phenomena stemming from tip-induced modifications in the potential landscape at the local scale. For example, SGM maps revealed both concentric fringes outside the QR and radial fringes inside the QR in conductance variation maps. These fringes have different physical origins. The radial ones are reminiscent from characteristic LDOS features [98, 100], and have also been imaged in graphene based QRs [101, 102]. They were ascribed to quantum scars, and were found to be recurrent with energy in the case of graphene QRs. The concentric fringes are due to the electrostatic Aharonov-Bohm effect caused by tip-induced electron phase variations [103]. These fringes and transport properties of quantum rings were also studied through simulations in Refs. 104, 105.

Theoretical investigations to map LDOS features in resonant cavities with an SGM probe have also been proposed by [106, 107]. The same group also proposed to use SGM to image the double slit interference pattern [108, 109] for a pair of QPCs [110].

4.2.3 Single charge events : sequential tunneling through quantum dot

In this section ² we exploit the valuable opportunity offered by the technology behind 2DES to keep reducing the dimensionality of the objects we can deal with, down to "zero dimensional objects" (0D). Let us start with electrons flowing in 2D in a Hall bar. In 2D, we encounter some remarkable features, *e.g.* the resistance and resistivity have the same physical dimensions, the quantum Hall effect (integer and fractional) is observed, etc. From the 2DES we can also build 1D systems, *i.e.* the quantum point contacts (detailed in section 4.2.1) where current flowing in the ballistic regime leads to a quantized value of the device conductance. One interesting feature about quantized conductance is that it can be predicted from a non-interacting electron Hamiltonian (except for peculiar features such as the 0.7 anomaly). Electrons simply trade their kinetic energy in potential energy to elastically cross the QPC.

In contrast, there are systems where the electron-electron interactions cannot be neglected : the *so-called* quantum dots (QDs). Quantum dots are 0D systems where electrons are localised in a *small* box, which can be formed, for example, as shown in Fig. 4.3 a, by biasing different metallic finger gates deposited at the surface of the heterostructure hosting a 2DES. Among the variety of ways to define QDs (self assembled QDs [112], QDs in nanowires [113, 114]), Starting from a 2DES, a QD is traditionally defined with lateral electrostatic gates defined using e-beam lithography and a metallization process. The electrostatic gates (in dark grey on Fig. 4.3a) once negatively polarized enable to define depleted region (in white on Fig. 4.3a) defining a *small* island (disk of radius R). Transport through this box presents some really peculiar features that we are going to present now. The general idea is that in order to be transferred from the left to the right electron reservoir (constituted by the 2DES), electrons are *sequentially* going to charge and discharge the QD but this process require an extra amount of energy (compared to the Fermi energy : the electron energy in the reservoir).

In such island we can count the number N of electron using the 2DES density of states. For an island of radius $R = 150\text{nm}$, we have :

$$N = \frac{m^*}{\pi\hbar^2} E_F \pi R^2 \sim 700 \quad (4.1)$$

where E_F is taken as 55 meV ($n_{2D} = 1 \cdot 10^{16} \text{ m}^{-2}$) and $m^*=0.036$. Qualitatively, one can estimate an average spacing between quantum levels due to confine-

²inspired from the following references [31, 111]

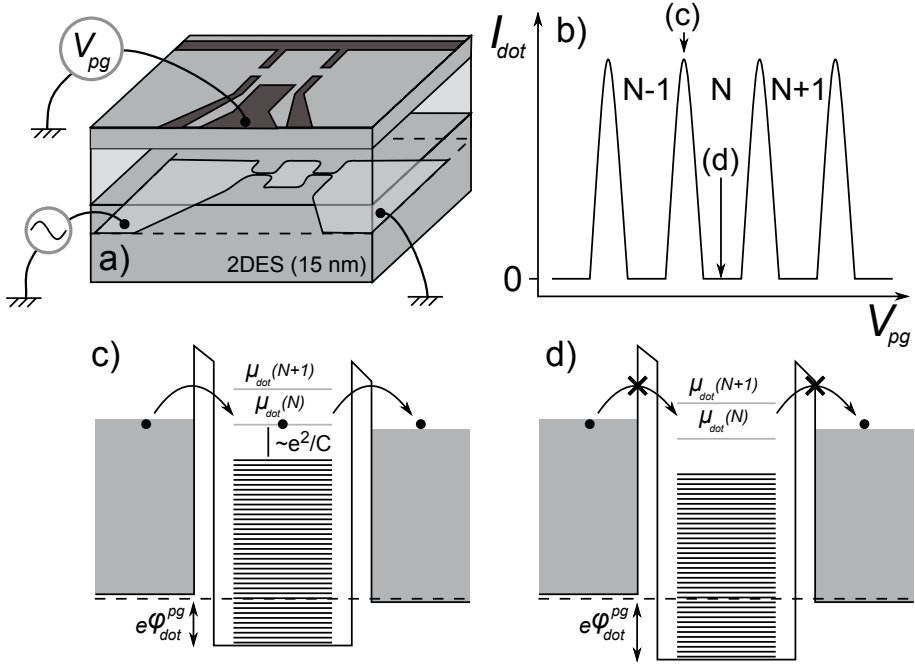


Figure 4.3: a) Illustration of a quantum dot (QD) (central island in light grey) where metallic top gates (dark grey) are biased to deplete the 2DES underneath the gates. b) Evolution of the current measured through the QD (I_{dot}) as a plunger gate is biased (V_{pg}) to *sequentially* empty the number of electron in the dot (going from $N+1$ to N , N to $N-1$ etc.). Transport through the dot occurs in resonant conditions (see c)) that is to say alignment between the chemical potential in the dot $\mu_{dot}(N)$ and in the electron reservoirs - current is flowing (see b)). The average potential ϕ_{dot}^{pg} at the level of the dot induced by the plunger gate is used to charge or discharge the dot and overcome the so-called charging energy (e^2/C). In d), a situation of Coulomb blockade is represented and no current is flowing (see b)).

ment inside the island as $E_F/(N/2)$ (where the 2 takes into account the spin degeneracy) *i.e.* $160 \mu\text{eV}$. One could idealise the potential defining the island by an isotropic 2D harmonic potential characterized by an angular frequency ω_0 fixed by the gate-defined potential. In such situation, the ground state of this non interacting set of particles is a 2D gaussian solution characterised by a standard deviation given by $\sqrt{\frac{\hbar}{m^* \omega_0}}$. The spacing between two successive levels in such situation is $\hbar \omega_0$. If we consider that the spatial extension of the dot is roughly equal to the standard deviation of the ground state, we obtain an energy-level spacing Delta given by :

$$\Delta = \hbar \omega_0 = \frac{\hbar^2}{m^* R^2} = 84 \mu\text{eV} \quad (4.2)$$

which is of the same order of magnitude as the spacing roughly obtained above. Up to now, no electron-electron interaction have been taken into account but let us evaluate Coulomb repulsion between charges confined into the dot.

Considering a metallic island as the one presented in Fig. 4.3a, one might ask how much potential energy represents the fact of gathering N charges into a confined domain (island of surface πR^2). Let us evaluate this energy but before, let us note that such isolated system has a potential energy defined with respect to a conductive hollow sphere of infinite radius, centred on the island. Placing the first charge in the island is energetically costless. The first charge define a conservative potential - in this potential one needs to pay an energy³ of $e^2/(8\epsilon_0\epsilon_r R)$, to place a second charge,⁴ typically R away from the first one. Bringing a second one costs $2e^2/(8\epsilon_0\epsilon_r R)$ etc. The total electrostatic energy $U(N)$ for N charges is given by :

$$U(N) = \frac{e^2}{8\epsilon_0\epsilon_r R} \sum_{i=1}^N i = \frac{e^2}{8\epsilon_0\epsilon_r R} \frac{N(N-1)}{2} \quad (4.3)$$

As the number of charges becomes large (compared to 1), we can see that the electrostatic energy growths as N^2 . This dependency can now be used to define the total energy (electrostatic (classical origin) + confinement (quantum origin)). Let us not forget that singly ionised dopants compensate the charge in

³the denominator $8\epsilon_0\epsilon_r R$, in the forth-coming expression, is the self-capacitance of a disk of radius R . Electrostatic problems, such as the 2D conductive disk, are solved for a wide variety of problems in different dimensions in Ref. 115

⁴which is assumed to come from infinitely far away

the island for a dot defined with lateral gates from a heterostructure 2DES - the dot is thus initially neutral electrically although filled with e.g. ~ 700 electrons. Also, a *so-called* plunger gate⁵ can be used as a lever-arm to modify the number of charges in the island. This *plunger* gate is modelled via a simple capacitive model. This plunger gate can be treated as if it placed a fictive charge Q_0 within the dot. Q_0 can be continuously adjusted via V_{pg} as it is capacitively⁶ coupled to the dot $Q_0 = C_{pg} V_{pg}$.

The total energy in the dot is thus :

$$U_{tot} = \sum_{i=1}^N E_i + \frac{(-en + C_{pg} V_{pg})^2}{2C} \quad (4.4)$$

where $C = 8\epsilon_0\epsilon_r R$, the self-capacitance of the island and $n = N - N_0$ with N , the number of electrons in the dot and N_0 the initial number of electron in the dot at charge neutrality [111]. This model is the so-called "constant interaction model".

What happens during a charging event? To answer this question, let us define the electro-chemical potential μ which is the energy required to add the N^{th} electron to the dot, that is to say :

$$\mu(N) = U(N) - U(N-1) \sim E_N + \frac{(n-1/2)e^2}{C} - e \frac{C_{pg}}{C} V_{pg} \quad (4.5)$$

with n being a discrete parameter and V_{pg} a continuous parameter. $\mu(N)$ is the energy required and in near-equilibrium conditions, E_F is the energy available. As we can apply a bias between the left and right reservoir, let us distinguish the left and right electro-chemical potential μ_L and μ_R , respectively. Let us assume that V_{pg} is compatible with N electrons in the dot. In near-equilibrium conditions, we have :

$$\mu(N) = \mu_L = \mu_R = E_N + \frac{(n-1/2)e^2}{C} - e \frac{C_{pg}}{C} V_{pg}. \quad (4.6)$$

How much is this needed to rise V_{pg} to have a system with $N+1$ electrons? We get the answer by writing the same condition for $N+1$:

$$\mu(N+1) = \mu_L = \mu_R = E_{N+1} + \frac{(n+1-1/2)e^2}{C} - e \frac{C_{pg}}{C} (V_{pg} + \Delta V_{pg}). \quad (4.7)$$

⁵plunger gate is the "grille piston" in french

⁶which is here a mutual capacitance (not self capacitance)

Indeed, for an electron to cross the dot without any energy gain or loss, $\mu(N)$ must be compatible with the available energy in the electron reservoirs for any N . Hence, we have $\mu(N+1, V_{pg} + \Delta V_{pg}) = \mu(N, V_{pg})$ which gives us :

$$\frac{eC_{pg}}{C}\Delta V_{pg} = (E_{N+1} - E_N) + \frac{e^2}{C}. \quad (4.8)$$

This equation is really central to the study of quantum dots. Two comments need to be made. First in Eq. (4.8), we can see that both terms in the righter part can be really different in amplitude. For example, as stated in Eq. (4.2), for a QD laterally defined from a 2DES, the average $E_{N+1} - E_N$ is on the order of tens of μeV . Nevertheless, if we evaluate $\frac{e^2}{C}$, again with $R = 150 \text{ nm}$, we have 1.1 meV which is one order of magnitude larger and is called the *charging energy*. Secondly, the same capacitive model can be developed for any external gate. For example, in chapter 5 we discuss transport signature associated with QDs forming due to potential wells in the disorder potential. These dots are charged and discharged thanks to a tip-induced potential. The SGM tip acting as an external gate can be compared to the plunger gate but the fundamental difference is that the tip-dot capacitance depends obviously on the tip-dot relative position.

Equation (4.8) justifies also the fact that elastic tunnelling is a sequential process. In a resonance situation, $\mu_L \sim \mu_R \sim \mu(N)$, the N th electron tunnels first in the dot and second in the reservoir. While the N th electron is in the dot, no other electron can travel through the dot because it has not enough potential energy. If $\mu_{L,R}$ falls in between $\mu(N)$ and $\mu(N+1)$, transport is blocked : this is the so-called *Coulomb blockade* mechanism (see Fig. 4.3 d).

Many mechanisms can be studied with quantum dots. From electron correlation mechanisms (such as the Kondo effect in QDs [116]) to coupling between multiple dots [117]. Dots are also studied in the context of scanning gate microscopy as the tip is a useful lever-arm to charge and discharge dots [118–120], also in graphene based QDs [121–124]. QDs can also give valuable insights to estimate the tip induced potential in SGM measurements [125, 126]. Using graphene based QDs, it was also shown that the peculiar electrostatic potential screening between the metallic tip and the gates defining the QD was of complete relevance in the interpretation of the SGM maps [127]. Scanning gate microscopy was also used to demonstrate that, in the quantum Hall regime of transport, electrons can tunnel between counterpropagating edge channels via Coulomb islands [128, 129].

Considering a 2DES as the one discussed in chapter 2, QDs can also influence electron transport in nanodevices without defining them with lateral gates. Indeed a potential barrier can coincidentally define an island trapping electrons. The corresponding island area is a place where no current can flow any more except in the conditions of Coulomb resonances. With a 2DES as the one presented in Fig. 2.2, the number of charges in the disorder-related island (circled by the black dashed line in Fig. 2.4) can be varied in two ways. Either we vary the overall density using the backgate V_{BG} , varying the 2DES Fermi energy or the average potential in the dot is varied with the tip-induced potential. The formation of quantum dots in the potential fluctuations associated to disorder were proposed in Ref. 130 and are discussed in chapter 5.

4.3 The tip-2DES interaction : a toy model

Before digging into the task of understanding the capacitive coupling between the polarized tip and the electron system, let us start with a toy model. This consists of a crude approximation of the tip-2DES system but allows one to cover the key characteristics of this complex interacting system. Let us approximate the tip by a conducting sphere of radius a and the 2DES by a conducting plane (see Fig. 4.4). To ease the discussion, we consider that the sphere and the plane are separated by air (absence of dielectric layers). The conductive sphere is biased at V_{tip} with respect to the grounded conductive plane [131]. Finding the potential distribution for fixed value of the potential on boundary domains is called a Dirichlet problem. The solution is found using the method of the images. The idea of the method is to consider point charges located so as to fulfil the boundary conditions (BCs). Once a solution compatible with the BCs is found, we know that the potential distribution is the linear superposition of the well-known point charge potential. This solution verifies Laplace equation and, due to the *Uniqueness Theorem*, we know that this solution is *the one*.

Using this method, we can start by considering an image charge q_0 that is located in the middle of the sphere - that mimics the tip as shown in Fig. 4.4a - and that leads to a potential of V_{tip} on the surface of the sphere. We have :

$$q_0 = 4\pi\epsilon_0 V_{tip} a. \quad (4.9)$$

In order to satisfy the grounded conductive plane condition imposed by the modelled grounded 2DES, a mirroring imaginary charge of opposite sign $-q_0$

has to be placed in $-z_0$. The trick is now to place another imaginary charge q_1 in z_1 inside the sphere to maintain the surface of the sphere at V_{tip} . With this conditions, we can deduce : $q_1 = \frac{aq_0}{2z_0}$ and $z_1 = z_0 - \frac{a^2}{2z_0}$. This charge q_1 generates, in turn, a mirror charge $-q_1$ that needs a charge q_2 and so on... This iterative process is governed by the following recurrence equations :

$$z_i = z_0 - \frac{a^2}{z_0 + z_{i-1}} \quad (4.10)$$

$$q_i = \frac{a}{z_0 + z_{i-1}} q_{i-1}. \quad (4.11)$$

Hopefully, this process is converging. For convenience, it is useful to define $Q_i = \frac{q_i}{q_0}$. The potential expressed in (ρ, θ, z) (cylindrical coordinates) due to a pair $\pm q_i$ is :

$$\varphi_i(\rho, z) = \frac{1}{4\pi\epsilon_0} \left(\frac{q_i}{\sqrt{(z-z_i)^2 + \rho^2}} - \frac{q_i}{\sqrt{(z+z_i)^2 + \rho^2}} \right). \quad (4.12)$$

The expected solution is then given by :

$$\varphi(\rho, z) = \sum_{i=0}^{\infty} \varphi_i(\rho, z). \quad (4.13)$$

The potential due to image charges is only valid outside the sphere $(z - z_0)^2 + \rho^2 \geq a^2$ and above the plane $z \geq 0$.

Formally we have the potential distribution satisfying Laplace equation and the boundary conditions is :

$$\varphi(\rho, z) = \begin{cases} 0 & \text{for } z \leq 0 \\ V_{tip} & \text{inside the sphere} \\ aV_{tip} \sum_{i=0}^{\infty} \left(\frac{Q_i}{\sqrt{(z-z_i)^2 + \rho^2}} - \frac{Q_i}{\sqrt{(z+z_i)^2 + \rho^2}} \right) & \text{otherwise} \end{cases} \quad (4.14)$$

Using this equation, one can simulate the induced charge density σ_{2D} profile in the grounded plane for a tip with $a=20$ nm and a distance d_{tip} of 60 nm (this equation is the result from which we generated Figs. 4.4 and 4.5c). To stay consistent through all the work, d_{tip} refers to the tip surface distance. Actually,

the 2DES is ~ 42 nm below the surface (referred to as d^* in Fig. 4.4a) and the image first image charge q_0 is located in the middle of the sphere. So the total distance between the center of the sphere and the grounded plane is $d_{tip} + d^* + a$. We neglected the dielectrics presents between the tip and the 2DES. This toy model will not be used as a predictive tool but is used to fix the principal order of magnitude when considering scanning gate microscopy.

Looking at the profile presented in Fig. 4.4b, one can see that with a tip potential of -14 V⁷ at $d_{tip} = 60$ nm we can already expect to deplete typical low density electron systems hosted in semiconductor heterostructures. SGM can thus be used fruitfully, with reasonable voltage sources, on electron systems with densities going at most to $10^{15} - 10^{16} \text{m}^{-2}$. As presented in Fig. 4.4b, the spatial extension (d on Fig. 4.4a defined as the full width at half maximum (FWHM)) of the tip-induced charge density profile is of 180 nm with $d_{tip} = 60$ nm and $a = 20$ nm. The evolution of this line profile with d_{tip} and a are presented in Figs. 4.4c and 4.4d keeping V_{tip} at -14 V. As we can see on Fig. 4.4d, σ_{2D}^{max} gets larger as a gets larger so as the FWHM - the technique is naturally less local. The tip geometrical characteristic dimensions have to be selected with respect to the characteristic dimensions of what is going to be probed (d_{tip} and a are somewhat analogous to the aperture of a microscope). As we are going to probe nano-scale networks whose 1D channels are at most 464 nm, a tip presenting an apex radius of curvature (a) of 20 nm is fine⁸. The probes we use trough all this work are characterised by $a \sim 20$ nm and $\theta \sim 30^\circ$ (see Fig. 4.4a). Fig. 4.4 serves as a guiding tool to select, the materials (low density electron systems), the tip and the voltage source used to bias the conductive tip.

From the above expression (4.14), we can derive useful quantities like the electric field $\mathbf{E} = -\nabla\phi(\rho, z)$, the capacitance $C = (\sum_{i=0}^{\infty} q_i)/V_{tip}$, the electrostatic energy $U = \frac{1}{2}CV_{tip}^2$ and the force F that needs to be applied to maintain the sphere at rest in z_0 given by $F = -\frac{\partial U}{\partial z_0}|_{z=z_0}$. The toy-model presented here can be used to roughly estimate the tip-2DES interactions as a biased conductive AFM tip (see Fig. 4.4) is brought in close vicinity to the 2DES. Up to now, as mentioned, the 2DES is modeled as a conductive grounded infinite plane and the tip as a conducting sphere. Details about the experimental setup can be found in appendix A but we find important to present here the outline. A conductive AFM tip is attached to a tuning fork acting as a force sensor [132].

Figure 4.5a presents a zoom on the cantilever glued to the tuning fork.

⁷limit of our laboratory voltage source

⁸even in the event that it could be slightly eroded during a measurement

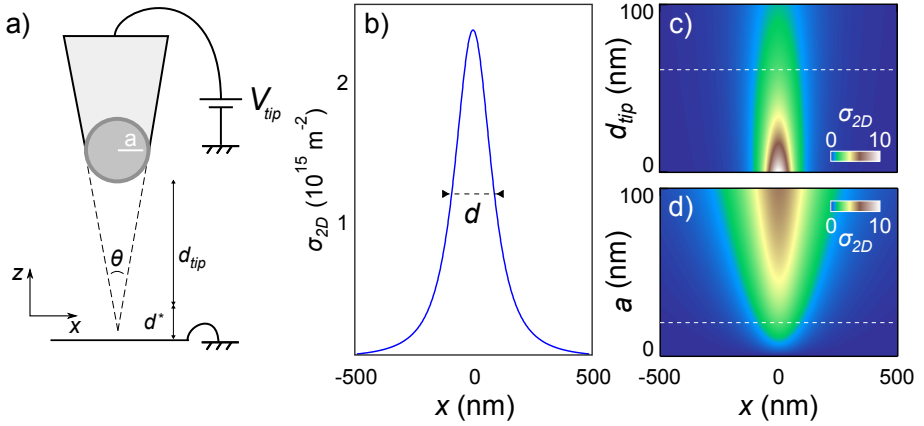


Figure 4.4: a) Cross section of a conductive AFM tip characterised by its apex radius of curvature a and its full tip cone angle θ . The distance d_{tip} is the distance between the sample surface and the tip. An extra-distance of d^* of $\sim 42\text{nm}$ is added because, as explained in chapter 2, the grounded plane (2DES) we talk about is buried under the surface of an heterostructure and is parallel to its surface. b) Local charge density σ_{2D} as a function of x induced by a tip (see in (a)) bias with $V_{tip} = -14 \text{ V}$; $d_{tip} = 60\text{nm}$ and $a = 20\text{nm}$. c) and d) Variation of the profile presented in (b) as a function of d_{tip} (c) and a (d). The white dashed lines in (c) and (d) locate the profile presented in (b) in both mappings. The horizontal axis in (c) and (d) are matched.

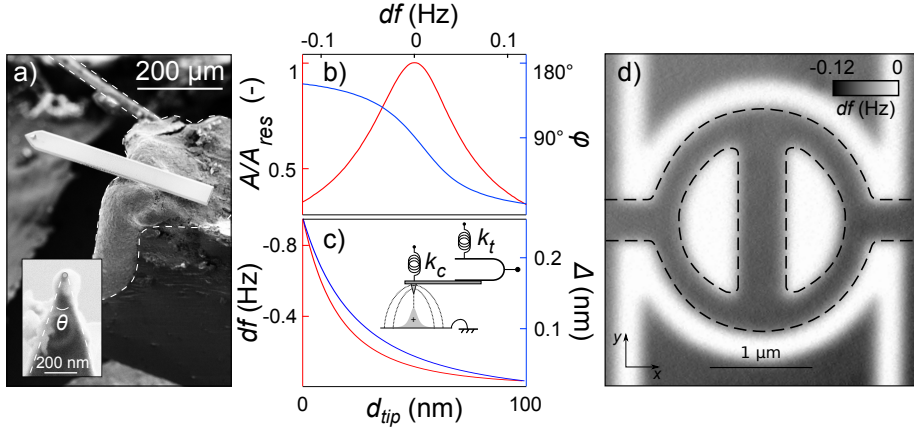


Figure 4.5: a) Scanning electron micrograph (SEM image) of the conductive AFM cantilever that has been glued with conductive epoxy on the end of one arm of the quartz tuning fork. The conductive domain is highlighted by a white dashed line. The golden wire, used to connect this domain, can be seen in the upper part of the image. The inset presents a SEM image, realised at higher magnification, zooming on the apex of the tip showing its good integrity after the SGM measurements have been performed. b) Characteristic tuning fork/tip/gold wire 'resonance curves'. The red curve (left axis) presents the evolution of the tuning fork amplitude of oscillation as frequency is swept (df) around the resonance frequency of f_0 of 32314 Hz, and the blue curve presents the phase ϕ on the right axis on the same frequency domain. This resonance, obtained at 40 mK in high vacuum, is characterised by a quality factor Q of 320000. No surface is in the vicinity of the tip ("free" resonance). c) As the oscillating tip/tuning fork are brought closer (varying d_{tip}) to the grounded 2DES, the frequency shift (red curve - left axis) is simulated in the context of the toy model ($V_{tip} = -14$ V) presented in this section with Eq. (4.15). The blue curve - right axis - presents a numerical estimation of the displacement Δ of the apex of the tip as the tip is brought closer to the 2DES. d) Measurement of the frequency shift df (from the resonance frequency f_0 - see (b)) above one nano network - highlighted with the black dashed line (studied in chapter 6) as the biased tip ($V_{tip} = -14$ V) is scanned in a plane parallel to the 2DES with $d_{tip} = 60$ nm. The location where the 2DES has been etched presents no frequency shift (in white with this color map).

The domain highlighted by the white dashed line is conductive and the tip is coated by continuous films of titanium (first layer) and platinum (second layer)⁹. The inset of Figure 4.5a presents a scanning electron micrograph of the tip used after the experiment presented in chapter 6. Although few residues can be identified on the image¹⁰, the tip integrity is preserved after our scanning gate microscopy measurements. Before performing SGM measurements on a device, the tip/tuning fork system is used as an atomic force microscope (AFM) [133] [134]. This AFM is a bit unconventional as it does not involve the use of the traditional laser/photodiode combination to detect the deflection of the cantilever. Indeed, as we aim to measure our devices at low temperature, no light can be brought inside the experimental setup. In addition, carrier within 2DES can interact with light and this would modify their transport properties. The III-V materials, used in the heterostructure (Fig. 2.2) are also broadly studied for their remarkable light/matter interaction [135]. The tip/surface interaction forces are here measured through shifts in the resonance frequency of the oscillating tuning fork. Before bringing the tip in close vicinity with the sample's surface, the two branches of the quartz¹¹ tuning fork are made to vibrate with an oscillating voltage signal applied to the tuning fork branches (see appendix B). This oscillation causes current to flow back and forth in the tuning fork branches. As the oscillation frequency is swept, a peculiar frequency is of first relevance : the resonance frequency f_0 . At this frequency, the amplitude of vibration is maximal (strongest current measured) and the phase of the current signal is dephased by $\pi/2$ with respect to the off-resonance conditions. The resonance condition is characterised by the measured curves presented in Fig. 4.5b. The red curve in Fig. 4.5b is the RMS oscillation amplitude of the tuning fork and the blue curve present the phase shift as the frequency is swept around the resonance frequency. As the tip is brought closer to the surface, the tip experiences an interaction force that shifts the *free* resonant frequency f_0 . This shift df is related to the derivative of the force F presented in the toy model. The formula, extracted from [136], is :

$$df \sim \frac{\omega_0}{4\pi k_t} \frac{dF}{dz} \Big|_{z_0} \quad (4.15)$$

where k_t is the tuning fork equivalent stiffness (3817 N/m for the tuning fork

⁹A general overview on the tip holder (the 'SGM board') is presented in Fig. A.2

¹⁰probably Poly(methyl methacrylate) (PMMA) residues used throughout the fabrication process (see appendix B)

¹¹quartz is a piezoelectric material

we used [137]). A simulation of the value of df , based on the toy model and Eq. (4.15) is shown in Fig. 4.5c (red curve). The force F is estimated for $V_{tip} = -14$ V for each value of d_{tip} between 0 and 100 nm and the derivative of this curve is injected in Eq. (4.15). For d_{tip} equal to 60 nm, the simulated df is ~ 90 mHz for a force of 175 pN. The detection of such small forces proves that this tip/tuning fork device is truly an open door to the nanoworld¹².

Let us now perform such measurement. Figure 4.5d shows the value of the frequency shift df as the biased tip ($V_{tip} = -14$ V) is swept in a plane parallel to the 2DES at a distance d_{tip} of 60 nm from the heterostructure's surface with an RMS oscillation amplitude of the tuning fork of 5 nm. To understand this data, one should say the nanonetwork we are working with are etched from the pristine heterostructure (see section appendix B on the fabrication processes). The region without a 2DES underneath (etched) presents fewer interactions with the biased tip (the white regions on Fig. 4.5d). When the tip is standing above a region with the 2DES underneath, we can measure a frequency shift of 85 mHz on average (with 4.2 mHz of standard deviation). Such frequency shift is in pretty good agreement with the toy model and show that such simple model can be used as a guiding tool. More importantly, it can be used to locate the nano-network without touching it. Concerning the issue of tip contact with the nano-network as the tip gets more and more biased ($|V_{tip}|$ getting larger), the force felt by the tip (attracted by the charges induced at the level of the 2DES - see the inset of Fig. 4.5c) could deform the cantilever and modify the value of d_{tip} (defined for an unbiased tip). Nevertheless, as already mentioned, we know the force F for each d_{tip} and thus, knowing the cantilever stiffness ($k_c \sim 3.5$ N/m [140]), the displacement Δ for a tip biased with $V_{tip} = -14$ V (see the blue curve presented in Fig. 4.5c). According to the toy model, this displacement Δ is less than 1 Angströms and is thus completely blurred in the tuning fork oscillations.

¹²As a comparison, such small interactions are also encountered when considering the Casimir effect [138, 139]. When two conducting planes are brought in close vicinity, an attracting force called the Casimir force, causes the two planes to be attracted by one another due to the vacuum energy (zero point energy) of the electromagnetic modes confined between the planes. These forces were first measured thanks to the AFM technique where a large sphere (200 μ m of radius of curvature for the apex of the tip) was used to locally approximate the conductive plane. In this measurement, the force felt was of 100 pN for a distance of 100 nm between the sphere and the surface.

4.4 The tip-induced potential : amplitude and shape

The amplitude and shape of the tip induced potential $\phi_p(x, y; x_{tip}, y_{tip})$ is central to this manuscript since this is the tool used to perturb transport through our nano-devices and induce changes of conductance which are recorded to produce SGM mappings. These mappings are obtained measuring the electrical conductance G as one external parameter is swept keeping the others constant. Among these parameters, one has : the tip potential V_{tip} ; the back-gate voltage V_{BG} ; the tip-device relative position in a plane parallel to the 2DES, namely x_{tip} and y_{tip} ; the tip-surface distance d_{tip} . Also, something worth inspecting is how the tip influences transport according to more 'conventional' experimental parameters (such as the temperature T (K) or a magnetic field B (T)).

We will now consider a more complete physical model for the tip-induced potential. At first we note that, a valuable lesson can be remembered from our toy-model (see section 4.3) : the actual tip can be considered (through the method of images) as a discrete distribution of point charges. As distance from the 2DES is increased, we can see that the series of z_i converges extremely fast towards its asymptotic value and that the charged sphere can be considered as an equivalent single point charge Ze (with Z being an integer and e the electron charge). We can thus neglect image charge effects for simplicity. Indeed, for a small ratio between the tip-2DES distance and the tip radius of curvature ($a = 20$ nm), we can consider that the discrete distribution of point charges is well approximated by an unique single charge given by $Ze = 4\pi\epsilon_0 a V_{tip}$.

4.4.1 Screened tip-induced potential in the 2DES

We will now evaluate the effective potential felt by electrons when the tip is biased. Let us start by the well known potential felt in the 2DES for a point charge Ze placed at a distance d from the 2DES :

$$\phi^0(\rho) = \frac{aV_{tip}}{\epsilon_r} \frac{1}{\sqrt{d^2 + \rho^2}} \quad (4.16)$$

where d is the distance between the image charge and the 2DES center of mass for the expectation value of the wave-function $|\phi_0|^2(z)$ which is $(d^* + d_{tip} + a)$. From here, we can follow roughly the same path as with a singly ionised charge in the delta-doping layer. The difference is that we know *a priori* that the tip-induced potential possess a radial symmetry so it is more convenient to work with the so-called Hankel transform of order zero. Indeed, if a 2D function

$U(\mathbf{r})$ possess a radial symmetry then it depends only on the radial coordinates $r = |\mathbf{r}|$ and we have :

$$U(k) = \int_0^\infty dr r U(r) \int_0^{2\pi} d\theta e^{-ikr \cos(\theta)} = 2\pi \int_0^\infty dr r J_0(kr) U(r) \quad (4.17)$$

where J_0 is the so-called bessel function of order 0.

In the Thomas-Fermi approximation, we have :

$$\varphi_p(\mathbf{q}) = \frac{1}{\varepsilon_{TF}(q)} 2\pi \int_0^\infty dr r J_0(qr) \varphi^0(r) = \frac{\varphi^0(\mathbf{q})}{\varepsilon_{TF}(\mathbf{q})} = \frac{aV_{tip}}{\varepsilon_r} \frac{e^{-qd}}{q + q_{TF}}. \quad (4.18)$$

We can now come back in real space. The tip induced potential in real space is given by :

$$\varphi_p(r) = \frac{aV_{tip}}{\varepsilon_r} \int_0^\infty dq q J_0(qr) \frac{e^{-qd}}{q + q_{TF}}. \quad (4.19)$$

Numerical estimates of this expression are extremely useful tools to infer the amplitude and spatial extent of the tip-induced potential. Results are presented in Fig. 4.6.

As we saw in Eq. (4.19), φ_p is directly proportional to V_{tip} so we plotted the data for -1V applied on the tip. Fig. 4.6a shows an illustration of the tip induced potential for $d_{tip} = 60\text{nm}$. A first important thing to note is that if we approximate this profile by a more convenient *Cauchy distribution* (also called Lorentzian distribution) the relative error is always smaller than a few tenths of percent. Thus we have :

$$\varphi_p(x, y; x_{tip}, y_{tip}) = \frac{\varphi_p^{max}(V_{tip}, d_{tip})}{1 + \frac{(x-x_{tip})^2 + (y-y_{tip})^2}{R_{tip}^2}} \quad (4.20)$$

where R_{tip} is the half of the full width at half maximum.

Secondly, as shown in Fig. 4.6b, the influence of screening by the 2DES is crucial. For the unscreened potential, the maximum of $\varphi_p^{unscreened}$ equals 11.7 meV against 0.83 meV for $\varphi_p^{screened}$. Also $R_{tip}^{unscreened}$ is twice bigger than $R_{tip}^{screened} = 100\text{ nm}$.

Third, we simulated the potential profile along the x direction for several values of d_{tip} between 0 and 140 nm and we can highlight two useful findings. First, we can identify a lever-arm $l = \Delta R_{tip} / \Delta d_{tip} \sim 3/4$. This lever arm tells

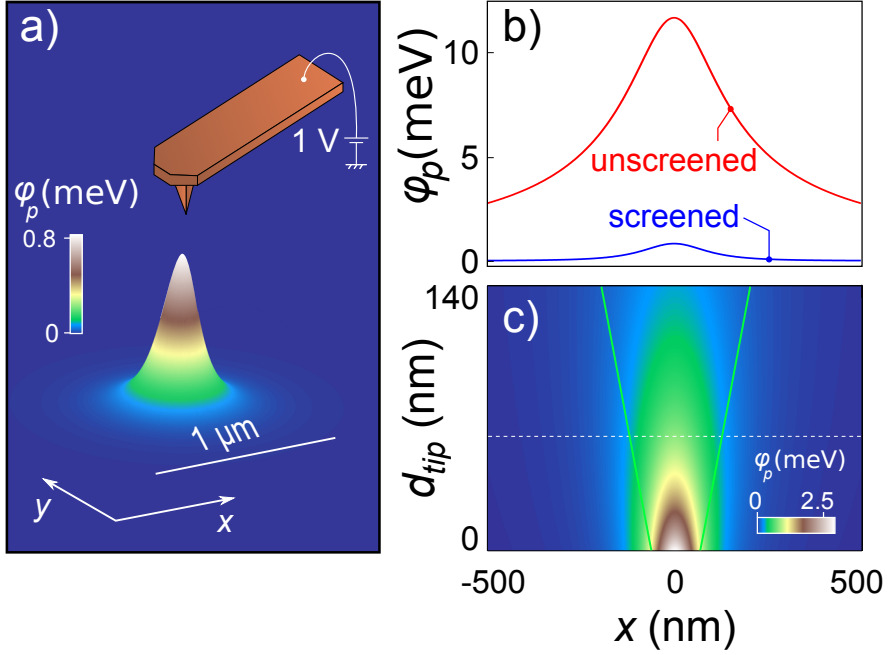


Figure 4.6: a) Real space value of the potential ϕ_p (estimated numerically from (4.19)) for a tip biased with -1V with $d_{tip} = 60$ nm. b) Comparison between two real space cross section of the tip-induced potential ϕ_p obtained in the same conditions as in (a). The red curve corresponds to a tip-induced potential unscreened by the 2DES and the blue takes into account screening of the 2DES according to the Thomas-Fermi model. Both the amplitude and spatial extension of this potential are affected by screening. c) The blue profile is inspected as the tip distance d_{tip} is varied. The white dashed line allows to locate the blue profile presented in (b). The two green lines correspond to the locus of the FWHM. The horizontal axis in (b) and (c) are matched.

us that if we vary d_{tip} by 100 nm, R_{tip} gets 75 nm larger. This value is deduced from the green line on Fig. 4.6c which is the location in x of R_{tip} . Also from $\log\text{-}\log$ profile of $\varphi_p(d_{tip}, x=0)$ we can extract that in the range 50 to 100 nm (where we will be working experimentally) a power law fits extremely well the data and $\varphi_p^{max} \propto d_{tip}^{-0.503 \pm 0.0043}$. These relations can be useful when considering the forthcoming experimental data. We also recall that this is only valid as long as the tip does not deplete the 2DES.

To conclude this theoretical section about SGM, we present a last result combining both quantum transport and electrostatics on the building block of our mesoscopic networks : a nanowire etched from the heterostructure presented in 2.2. To get insights on the interplay between electrostatics and quantum transport, we used a simulation tool developed by the group of Dr. Pala *et al.* solving both the Poisson and Schrödinger equation in a self-consistent way using the Hartree approximation on a 3D mesh¹³ defining the complex tip/heterostructure ensemble.

The results presented in Fig. 4.7b were simulated in order to evaluate the value V_{tip} that needs to be applied to a tip located 50 nm above the surface to reach a transmission of 0 of a 200 nm wide nanowire. This peculiar value of V_{tip} is called the *pinch-off* potential. The nanowire pinch-off occurs for $V_{tip} \sim -15$ V, no matter if we consider a discrete or continuous doping for the δ -doped silicon layer. Considering our model where the tip is approximated by a sphere and the Thomas-Fermi approximation, for $V_{tip} = -15$ V we have $e\varphi_p^{max} = 14.55$ meV. Such tip-induced potential would not be sufficient to deplete the wire presented in Fig. 4.7 where the average density of $8 \times 10^{15} \text{ m}^{-2}$ corresponds to a Fermi energy of 44 meV. This difference of more than a factor of three is really bothering and one needs to dig further as apparently the linear dependency of φ_p^{max} with respect to V_{tip} is not valid for all values of V_{tip} . One needs thus to pay attention to non-linearity for the evolution of the tip-induced potential with respect to V_{tip} for a given d_{tip} . The limits of validity of the Thomas-Fermi model have to be carefully inspected but, as discussed in chapter 5, it seems that the Thomas-Fermi model is a useful guide up to approximately $|V_{tip}| \lesssim 7 \text{ V}$ ¹⁴. Indeed the Thomas-Fermi approximation is independent of the local electron density $n_{2D}(x, y)$ and is probably not suitable to predict the

¹³the mesh is generated using Gmsh [141]

¹⁴Also, the flat potential profile in the wire is reached for $V_{tip} \sim -0.4 \text{ V}$ and not 0. Indeed the difference of work function between platinum and InGaAs is responsible for a tip-induced potential at $V_{tip} = 0 \text{ V}$

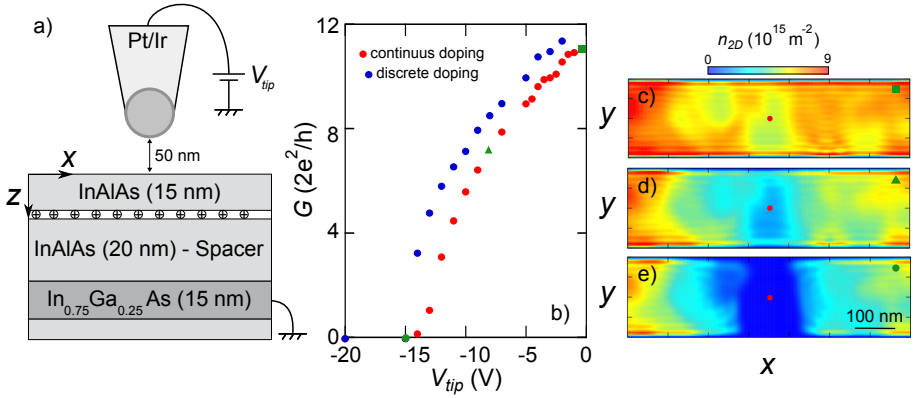


Figure 4.7: a) Schematic side-view of the tip geometry and its position with respect to the 2DES, located in the InGaAs layer. b) Simulated conductance vs V_{tip} in a 200 nm-wide channel patterned in an InGaAs/InAlAs heterostructure, mimicking the central channel in the mesoscopic networks discussed through the whole manuscript, with random dopants inducing disorder in the potential (red data points), or with homogeneous continuous doping (blue data points). We used the same doping concentration as in the experiment. These calculations are based on three-dimensional self-consistent simulations coupling the Poisson equation (in the whole volume of the illustration in a)) and the Keldysh-Green function equations in the wire region [13]. This method allows us to accurately simulate the tip-induced potential experienced by conduction electrons. Noteworthy, depletion occurs for $V_{tip} < -14V$ - which is compatible with the theoretical approach presented in 4. c)-e) Real-space mapping of the electron density within the 200 nm-wide channel with random dopants distribution, for $V_{tip} = -0.5, -8$ and -15 V, respectively - the tip position is given by the red dot in each figure. These images give a direct view of the extension of the tip perturbation inside a channel similar to the central channel of the mesoscopic network studied in the main text.

induced potential on the verge of depletion. Discrepancies can also come from the difference in the actual tip geometry. Indeed the actual tip geometry is taken into account in the 3D finite element mesh used to realise the simulation. Such 3D simulation can be a really useful tool to inspect the validity of simpler model but it becomes quite tricky to disentangle the different factors that contribute to the gaps that we could encounter with analytical models. This is why, as we do in the next section, it is important to develop tools to characterise experimentally the tip-induced potential.

4.5 Experimental characterisations of the tip induced potential

Let us test experimentally our results. To do so, let us present the first SGM map measured on a nano-network whose geometry is highlighted by white dashed lines on Fig. 4.8a. The conductance G is mapped as a function of the tip position in a plane parallel to the 2DES ($d_{tip} = 80\text{nm}$) at a temperature of 40 mK. The tip is polarised at -4V . This negative voltage locally reduces the 2DES density under the tip. This first experiment is presented in a regime where the whole network is on the verge of closure (which explains the low G). This first map is a good starting point as it presents two interesting features. The first one is that, as expected, the tip locations where the conductance drops the more is intuitively above the two entrance leads. Closing a lead is equivalent to closing the network. The second features are those concentric fringes centred on the leads.

The first feature is related to the pinch-off of the nanowire lead. The pinch-off is the point from which the device closes due to a complete depletion. Let us look at what happens on the red line presented on Fig. 4.8a. On a finite Y domain, the conductance drops towards 0. To see how this domain evolves, with the density, we can use our ability to tune the average 2DES density with the back-gate (V_{BG}). The experiment, whose result is presented on Fig. 4.8b, is the following : a conductance profile is measured along the red line in Fig. 4.8a, as a function of V_{BG} , with $V_{tip} = -4\text{V}$ and $d_{tip} = 80\text{nm}$. $Y = 0$ corresponds to the center of the entrance channel. In the blue region of Fig. 4.8b, $G \sim 0$ and the entrance channel is depleted. The edges of this region (white dashed line) correspond to an image of the charge density variation caused by the tip induced potential. This can be used to estimate the shape of the tip-induced electrostatic potential, as experienced by electrons confined in the entrance lead. Indeed, fitting a Lorentzian function to this region yields an amplitude $\Delta V_{BG} = 0.36\text{V}$

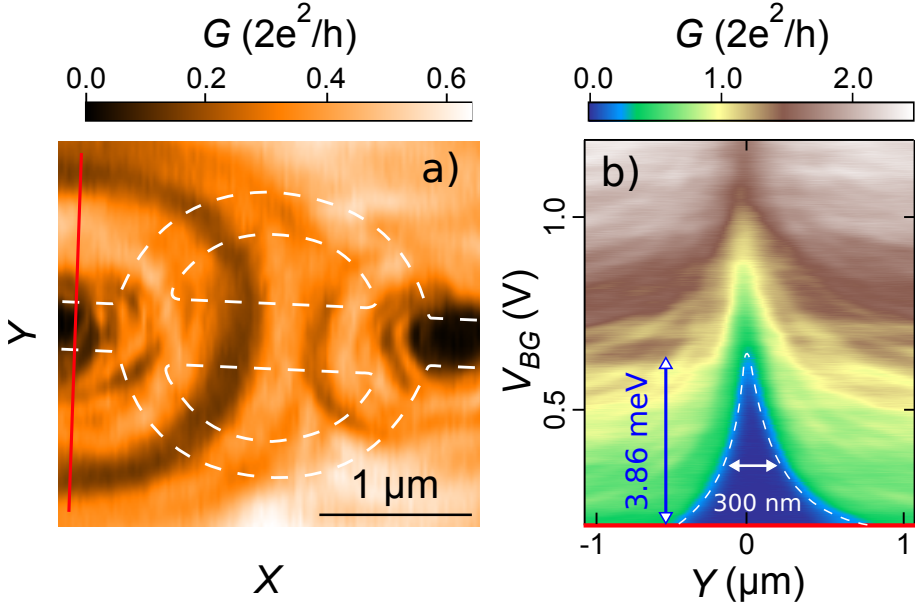


Figure 4.8: a) Scanning gate microscopy conductance map of a ring-like device fabricated next to the device discussed in chapter 6, measured with $V_{BG} = 0.3$ V, $d_{tip} = 80$ nm, and $V_{tip} = -4$ V. White dashed lines in a) delineate the device lithographic layout. The device is close to pinch-off due to the relatively low electron density, which explains the low G , and the presence of concentric rings around the narrowest region of the device, corresponding to Coulomb blockade resonances [130]. b) Conductance profile measured along the red line in a), as a function of V_{BG} , with $V_{tip} = -4$ V and $d_{tip} = 80$ nm. $Y = 0$ corresponds to the center of the entrance channel, which has the same width as the entrance channel of the device discussed in the main text. In the blue region, $G \sim 0$ and the entrance channel is depleted. The edges of this region (dashed white line) correspond to the shape of the tip-induced electrostatic potential, as experienced by electrons confined in the entrance lead. Fitting a lorentzian function to this region yields an amplitude $\Delta V_{BG} = 0.36$ V and full-width at half maximum ~ 300 nm. Thanks to measurements of the electron density in the 2DEG as a function of V_{BG} , one can extract a lever-arm of 10.6 meV/V for the back gate, which allows to convert ΔV_{BG} in $\phi_p^{max} = 3.86$ meV (as indicated on the figure). To calculate ϕ_p^{max} for other values of d_{tip} and V_{tip} , we assume that ϕ_p^{max} scales quadratically with d_{tip} , and linearly with V_{tip} .

and FWHM of about 300 nm. Thanks to measurements of the electron density in the 2DES as a function of V_{BG} , one can extract a lever-arm of 10.6 meV/V for the back-gate, which allows to evaluate the maximum tip-induced perturbation as 3.86 meV for this V_{BG} range (as indicated on the figure). According to Fig. 4.6c, this (d_{tip}, V_{tip}) configuration corresponds to a Lorentzian distribution whose parameters are $R_{tip} = 116$ nm and $\phi_p^{max} = 2.5$ meV. The discrepancies between the Lorentzian parameters are more than likely due to the peculiar screening occurring for such a nanowire geometry. Nevertheless, this relatively good agreement is sufficient for us to consider that the simulated data presented in Fig. 4.6c are quite reliable. As previously discussed, we consider from now that ϕ_p^{max} is proportional to V_{tip} and is inversely proportional to the square of d_{tip} on the range 50 to 100 nm (in agreement with the tip-induced model presented in section 4.4).

The second feature is the concentric fringes centred on the leads that we will discuss further in the manuscript. They are signatures of single charge events (see 4.2.3) and will be detailed in chapter 5.

4.6 SGM as a scattering experiment

A standard question in physics is: how do we deduce the properties of something that is too tiny, remote, transient or inaccessible to be observed directly? A common answer is: by observing the way that it scatters (original text from [142]).

As presented in chapter 2, electrons in a 2DES are described by 2D plane waves (see Eq. (2.13)). We showed that among many scattering phenomena, disorder potential scatters electrons from their initial state to a final state. In this section we plan to deal with the scattering of electrons caused by the static tip induced potential. We will characterise the probability that the tip induced potential ϕ_p scatters an electron - incoming plane wave - to an angle θ with respect to its initial orientation (see Fig. 4.9). Note that the denomination plane wave is abusive in 2D, we should rather use the term line wave because iso-phase domains are here lines and not planes.

Anyway, the time-independent solution for the presence probability amplitude Ψ_{tot} for an electron elastically scattered by a conservative potential converges *asymptotically* ($\rho \rightarrow \infty$) towards a function of the type :

$$\Psi_{tot} \propto \Psi_{inc} + \Psi_{sc} \quad (4.21)$$

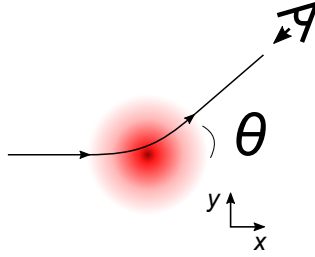


Figure 4.9: Illustration of a 2D Rutherford experiment where the probability amplitude to be scattered in a new quantum state along the angle θ depends on the ratio between the perturbing potential and the energy of the particles in the beam.

where Ψ_{inc} is the incident wave function of the type $e^{i\mathbf{k}\cdot\boldsymbol{\rho}}$ and Ψ_{sc} is the scattered wave that we consider of the type $f(k, \theta) \frac{e^{i\mathbf{k}\cdot\boldsymbol{\rho}}}{\sqrt{\rho}}$.

Ψ_{sc} is an outgoing circular wave pondered by the complex function $f(k, \theta)$ called the *scattering amplitude*. The square-root dependency of Ψ_{sc} ensures its normalisability.

In terms of physical observables, we now detail the fundamental role of $f(k, \theta)$. Indeed, an incident 'plane' wave carries a current density $\mathbf{j}$ anywhere in space given by :

$$\mathbf{j}_{inc} = \frac{\hbar}{2m^*i} [\Psi_{inc}^* \nabla \Psi_{inc} - \Psi_{inc} \nabla \Psi_{inc}^*] = \frac{\hbar \mathbf{k}}{m^*}. \quad (4.22)$$

The current density can also be evaluated for the scattered wave Ψ_{sc} for any angle θ knowing that, in 2D polar coordinates, the gradient expresses as :

$$\nabla = \frac{\partial}{\partial \rho} \hat{e}_\rho + \frac{1}{\rho} \frac{\partial}{\partial \theta} \hat{e}_\theta. \quad (4.23)$$

Let us note that the $\hat{e}_\theta$ component vanishes as $\rho \rightarrow \infty$ - which is relevant since we consider the asymptotic expression for Ψ_{tot} .

This simplifies the evaluation of $\mathbf{j}_{sc}$. After a few lines of algebra, we get :

$$\mathbf{j}_{sc} = \frac{\hbar}{2m^*i} \left[2f f^* \frac{i\mathbf{k}}{\rho} \right] = \frac{\hbar}{m^*} |f(k, \theta)|^2 \frac{\mathbf{k}}{\rho}. \quad (4.24)$$

Let us now consider an observer placed at an angle θ with respect to the incident wave (see Fig. 4.9). The ratio between the amount of flux observed in

a infinitesimal angular window $\rho d\theta$ and the incoming flux is :

$$d\sigma = \frac{\frac{\hbar}{m^*} |f|^2 \frac{|\mathbf{k}|}{\rho} \rho d\theta}{\frac{\hbar |\mathbf{k}|}{m^*}}. \quad (4.25)$$

This formula leads to the so-called differential cross-section [143] expressed by :

$$\frac{d\sigma}{d\theta} = |f(\mathbf{k}, \theta)|^2. \quad (4.26)$$

This expression proves the central role played by f in our interpretation and the importance to find its analytic expression. f has the physical dimension of $m^{-\frac{1}{2}}$. Thus, the total scattering cross-section σ is given by :

$$\sigma(\mathbf{k}) = \int_0^{2\pi} d\theta |f(\mathbf{k}, \theta)|^2 \quad (4.27)$$

This expression can be related to the physical characteristic *length*¹⁵ of the tip induced potential φ_p that leads to scattering. This total scattering cross section was in this chapter roughly approximated by $2 * R_{tip}$ (see Eq. (4.20)). This approximation neglects the asymptotic spatial dependency of φ_p . This asymptotic long range dependency of φ_p ($\rho^{-\frac{3}{2}}$ according to [31, 130]) is really important for the scattering in an infinite 2D space. Nevertheless, as with SGM we focus on scattering in nanodevices, the boundary conditions associated to the finite size of the device makes this problem much more complex and the asymptotic dependency is less crucial.

Let us come back to the problem of finding the scattering amplitude f . As detailed in this section, and more peculiarly in Eq. (4.19), the tip induced potential scales linearly with V_{tip} and so the time-independent Schrödinger equation can take the generic form :

$$\frac{-\hbar^2}{2m^*} \nabla^2 \Psi_{tot} + \lambda \varphi_p \Psi_{tot} = E \Psi_{tot} \quad (4.28)$$

where λ quantifies the strength of the applied potential. As detailed by Eq. (3.17), in near equilibrium transport measurements, we are interested by solution

¹⁵ σ can also be found with the help of the *optical theorem* for scattering in two-dimensional space [143]

at the Fermi energy E_F . We can rewrite the above equation as :

$$\nabla^2 \Psi_{tot} + \frac{2m^* E_F}{\hbar^2} \Psi_{tot} = \lambda U(\rho) \Psi_{tot} \quad (4.29)$$

where $U(\rho) = \frac{2m^*}{\hbar^2} \phi_p$. In front of such equation, the strategy proposed by Lippmann and Schwinger [144] is to consider that the right hand side is known and expresses the solution *formally* [145] with the help of the Green function associated to the left hand side operator $\nabla^2 + k_F^2$. This operator defines the well known Helmholtz equation (elliptic differential equation). The solution Ψ_{tot} writes in the form of an Lippmann-Schwinger integral equation as a superposition of the complementary function $e^{-ik \cdot \rho}$ and the particular integral defined by the Green function associated to the Helmholtz operator G_{2D} :

$$\Psi_{tot}(\rho) = e^{ik \cdot \rho} + \lambda \int_0^\infty d^2 \rho' G_{2D}(\rho, \rho') U(\rho') \Psi_{tot}(\rho') \quad (4.30)$$

where G_{2D} is given by :

$$G_{2D}(\rho, \rho') = \frac{im^*}{2\hbar^2} H_0^{(1)}(k|\rho - \rho'|) \quad (4.31)$$

where $H_0^{(1)}$ stands for the zero-order Hankel function of the first kind.

This expression Eq. (4.30) is formally similar in 3 dimensions, with the Green function associated to the Helmholtz equation in 3D given by :

$$G_{3D}(\mathbf{r}, \mathbf{r}') = -\frac{1}{4\pi} \frac{e^{ik|r-\mathbf{r}'|}}{|r-\mathbf{r}'|}. \quad (4.32)$$

Note that for $k = 0$, the Helmholtz operator becomes a Laplace operator (used widely in the electrostatic description of the 2DES-tip system - $\nabla^2 G_{3D}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$).

This theoretical treatment illustrates the fundamental difference of quantum scattering in 2 or 3 dimensions. Note also that we consider the tip induced potential ϕ_p as a result of a 3D Poisson equation used in the 2D version of the Lippmann-Schwinger equation (it is worth remembering that a Coulomb interaction in a strictly genuine 2D space does not present the typical $\frac{1}{r}$ dependency but rather a $\ln(r)$ dependency). The appeal of a 2DES embedded in a 3D world is our opportunity to exploit a theorem valid in another dimensionality than the 3D world (it is also true for 1 or 0D systems or even system exhibiting properties of non integer dimensions [146]).

As detailed in [143], we can go further with G_{2D} when $k|\rho - \rho'|$ goes to infinity. We have :

$$G_{2D}(\rho, \rho') \sim \frac{im^*}{\hbar^2} e^{-i\pi/4} \sqrt{\frac{2}{\pi k|\rho - \rho'|}} e^{ik|\rho - \rho'|}. \quad (4.33)$$

And in the asymptotic region $|\rho| \gg |\rho'|$:

$$G_{2D}(\rho, \rho') \sim \frac{m^* e^{-i\pi/4}}{\hbar^2 \sqrt{2\pi k}} \frac{e^{ik\rho}}{\sqrt{\rho}} e^{-i\mathbf{k}_\rho \cdot \rho'} \quad (4.34)$$

where $\mathbf{k}_\rho$ stands for $k\hat{e}_\rho$.

We can now compare both expressions 4.30 in the asymptotic limit and 4.21 to get :

$$f(k, \theta) = \frac{m^* e^{-i\pi/4}}{\hbar^2 \sqrt{2\pi k}} \int d\rho' e^{-i\mathbf{k}_\rho \cdot \rho'} U(\rho') \Psi_{tot}(\rho'). \quad (4.35)$$

Before digging any further, one has to wonder about the relevance of the asymptotic solution of such problem. Indeed, the scattering amplitude would be obtained for Ψ -field presenting an infinite coherence length for each of its Fourier series component. In addition, one has to find the solution for Ψ_{tot} and hence solve Eq. (4.30). A frequently used approach is to consider the so-called *first Born approximation*. This approximation consists in considering that the field $\Psi_{tot}(\rho')$ (in Eq. (4.30)) can be approximated by the incoming plane wave $e^{-i\mathbf{k} \cdot \rho'}$ everywhere in space. This first Born approximation leads naturally to an iterative process of the following type :

$$\Psi_{tot}^{(n+1)}(\rho) = e^{i\mathbf{k} \cdot \rho} + \lambda \int_0^\infty d^2\rho' G_{2D}(\rho, \rho') U(\rho') \Psi_{tot}^{(n)}(\rho') \quad (4.36)$$

where $\Psi_{tot}^{(0)}$ is taken as $e^{-i\mathbf{k} \cdot \rho'}$.

The set of $\{\Psi_{tot}^{(n)}\}$ is called a Born expansion or Born series. The convergence properties of such series must be carefully inspected. Indeed, it depends on the incoming particle energy and the amplitude and spatial extension of the tip induced potential. In our case, one would like to solve the problem for a tip-induced potential that would be comparable to the incoming energy (in chapter 6 we will study situations where $\phi_p^{max} \sim E_F$) and for a spatial extension that is larger compared to the incoming wavelength ($\lambda_F \ll R_{tip}$). In such situation,

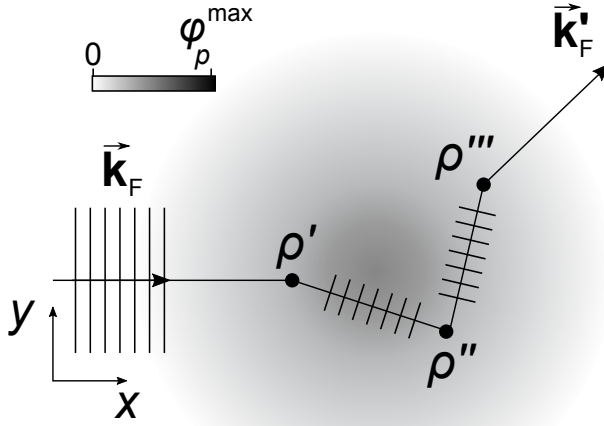


Figure 4.10: Illustration of how a tip induced potential in grey scatters a 2D plane wave. Multiple scattering events in ρ' , ρ'' and ρ''' are taken into account all the more so as the order of the last term in the Born series is high. This figure is inspired from [147]. The results of the multiple scattering events are illustrated by the fact that an electron in a state $\mathbf{k}_F$ gets scattered after three scattering events in $\mathbf{k}'_F$. The momentum is conserved when the electron is scattered from a conservative potential (elastic scattering event).

the first Born approximation is not expected to be converging. To understand why, let us picture the situation in Fig. 4.10. A plane wave characterised by the Fermi wave vector $\mathbf{k}_F$ encounters the potential distribution ϕ_p and electrons are scattered by this potential in a state characterised by $\mathbf{k}'_F$. Note that $|\mathbf{k}_F| = |\mathbf{k}'_F|$ because the collision is elastic. The value of Ψ_{tot} in ρ''' depends on what has been scattered from all other locations. For the particular path illustrated in Fig. 4.10, the value in ρ''' depends on scattering processes in ρ' and ρ'' . The distribution of potential can be viewed as a superposition of δ -like distributions of the type $\phi_p(\rho') \times \delta(\rho - \rho')$. Each of these act as sources from which a solution given by the 2D Green function 'radiates'. The local value of the field Ψ_{tot} also matters as it can be seen in the convolution presented in the righter term of Eq. (4.30). Intuitively, each time you go from order (n) to order (n+1) in the Born series, you take into account one more interaction. A more rigorous approach can be found in [147].

So, as we can presume, the plane wave approximation is not really a good candidate and indeed, an indicative convergence criterion for the Born series is found in [148] for $2k_F R_{tip} > 1$ ¹⁶:

$$\frac{m^*}{\hbar^2} |\phi_p^{max}| R_{tip} \frac{1}{k_F} \ll 1. \quad (4.37)$$

This criterion evaluated for $\phi_p^{max} = 1$ meV, $E_F = 60$ meV and $R_{tip} = 150$ nm is not respected ($313 \not\ll 1$) which indicates that the first Born approximation is most probably not going to converge.

Nevertheless, one can have the following strategy. Let us start with a small value for $\phi_p^{max,0}$, all other parameter kept unchanged. We then realise a Born series that stops when convergence is reached (that is for $||\Psi_{tot}^{(n+1)} - \Psi_{tot}^{(n)}|| / ||\Psi_{tot}^{(n)}|| < 0.001$). $\phi_p^{max,0}$ is then slightly increased to $\phi_p^{max,1}$ and the initial solution for this problem is the last element $\Psi_{tot}^{(n+1)}$ of the series corresponding to $\phi_p^{max,0}$. We can follow such procedure to reach a reasonable approximate value for Ψ_{tot} corresponding to the desired ϕ_p^{max} . This way we found a solution of the Lippmann-Schwinger equation (see Eq. 4.30) for values of $|\phi_p^{max}| = \frac{1}{4} E_F$ (with reasonable nowadays computer resources). The closer one gets to E_F , the more iterations one needs to reach convergence (if it can be reached).

The results are presented in Fig. 4.11 for plane waves impinging on a Lorentzian distribution characterised by $\phi_p^{max} = \frac{1}{4} E_F$ and $R_{tip} = 150$ nm (see Eq. 4.20). Fig. 4.11 presents a real-space map of the modulus squared of Ψ_{tot} for a depleting potential (a and b) and an accumulating potential (c and d). The potential distribution is centred in the middle of the map (at the origin of the frame of reference) and the white dashed circle has a radius of R_{tip} . In order to see the evolution as the density varies, Figs. 4.11a and c are evaluated for $E_F = 10$ meV and Figs. 4.11b and d, for $E_F = 60$ meV.

Two features are interesting to note on these maps.

First, for the depleting potential (Figs. 4.11a and 4.11b), $|\Psi|^2$ drops as expected behind the tip-induced potential but quite unexpectedly, $|\Psi|^2$ reaches the unperturbed value of 1¹⁷ in a non-monotonous way, when considering a vertical line profile behind the tip perturbation : in particular, $|\Psi|^2$ presents global maxima, located on parabola whose position is quite insensitive to the

¹⁶which is fulfilled for our typical situation where $\lambda_F \cong 25$ nm and $R_{tip} \cong 150$ nm

¹⁷corresponding to the unmodified plane wave

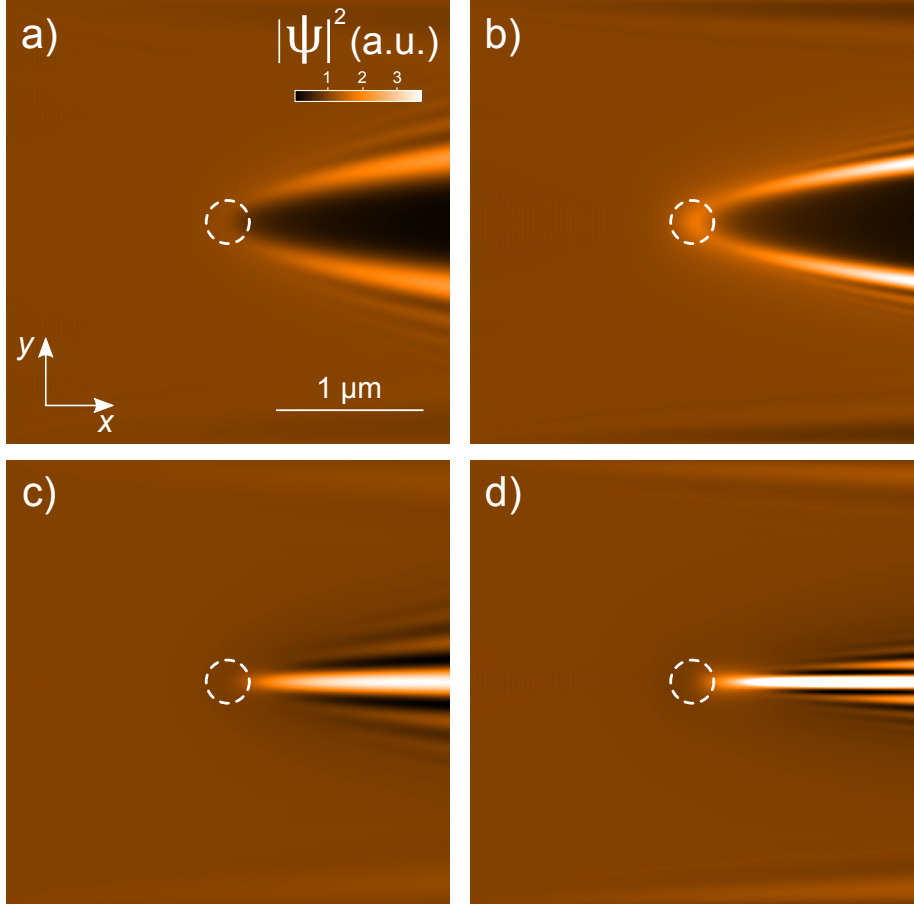


Figure 4.11: All figures present the modulus squared $|\Psi|^2(x, y)$ of the wavefunction mapped in real space. An incoming plane wave $e^{-i\mathbf{k}_F \cdot \boldsymbol{\rho}}$ is impinging on a Lorentzian distribution characterised by $R_{tip} = 150$ nm and $|\varphi_p^{max}| = \frac{1}{4}E_F$ (see formula (4.20)). The Fermi energy $E_F = 10$ meV (a) and (c) and $E_F = 60$ meV in (b) and (d). The tip-induced potential depletes under the tip in (a) and (b) and accumulates in (c) and (d). Both the horizontal and vertical axis are matched. The white dashed circle presents a circle of radius R_{tip} .

value of E_F (for tested values of E_F ranging from 10 to 60 meV) as long as $\frac{\phi_p^{max}}{E_F}$ is maintained constant. Nevertheless, other computations tends to show that this parabola tends to get wider and wider as the ratio ϕ_p/E_F is getting bigger and bigger.

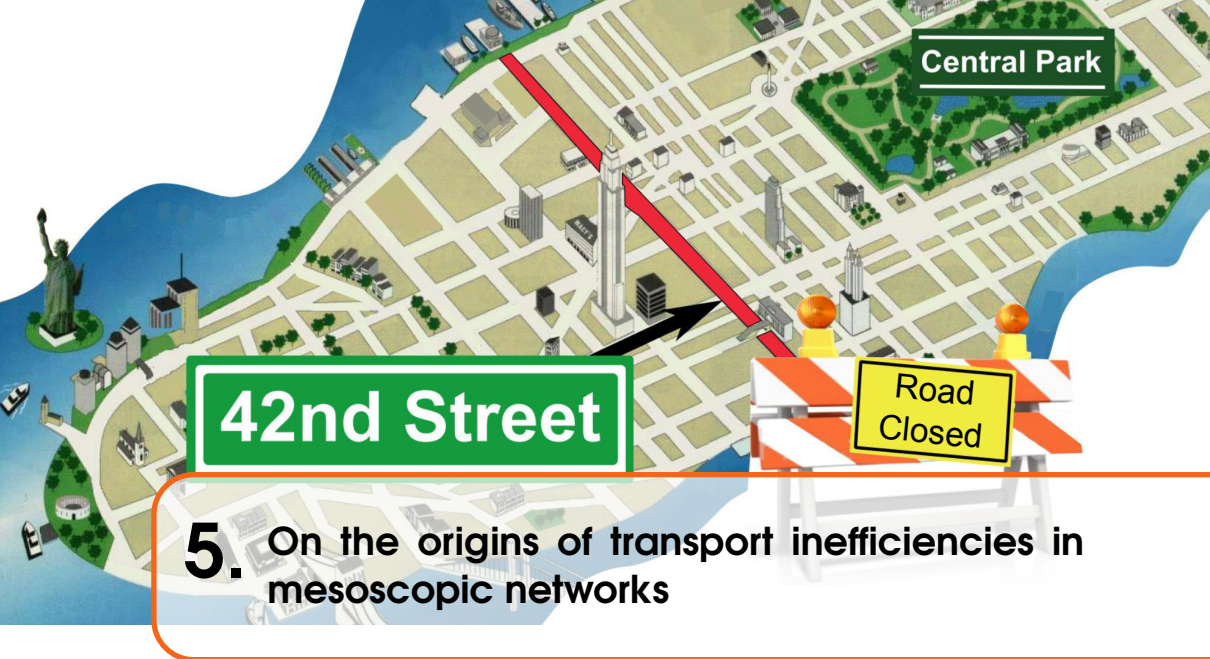
Second, for the accumulating potential, $|\Psi|^2$ presents a global maximum along the line corresponding to $y = 0$ (that is $\theta = 0$ in the schematics of Fig. 4.9).

These two features support the general mechanism presented in chapter 6.

As a conclusive remark, one can say that studying scattering at the local scale can lead to valuable informations. Scattering patterns in 2D systems such as those presented in Fig. 4.11 can be studied in different materials [149, 150], *e.g.* scattering in graphene or bilayer graphene is significantly different from the 2DES presented in this manuscript [151, 152]. Also, the computation time and the limit of validity of Born expansion can be overcome by inspecting other formalisms and semi-classical approximations. To name a few, one can consider the method of partial wave (useful for rotationally-invariant problems)[143, 153], Monte Carlo simulations[154] and the eikonal approximation (very closely related to the Wentzel-Kramers-Brillouin (WKB) approximation)[147, 153, 155, 156]. The eikonal approximation is a semi-classical approximation that is for example used in optics for ray tracing application. In the case of scattered electrons, this connects electron motion with the common picture of ballistic transport with classical trajectories. Scattering of electron plane waves impinging on objects different from a tip-induced potential is also studied, *e.g.* scattering on n-p interface in graphene based devices [154, 157]. The approach presented in Refs. 154, 157 strengthens the connection between nano-electronics and nano-optics. There are also more exotic scattering study of electron wave impinging on a fluid mechanical 'black hole analogue', known as the draining bathtub vortex causing a modified Aharonov-Bohm effect [142].

Counter-intuitive electron transport

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5. On the origins of transport inefficiencies in mesoscopic networks

The results presented in this chapter confirm that deviations from the *Rayleigh's Monotonicity Law* (see chapter 1) occur in nanoscale networks. This deviation is caused by a peculiar interference pattern that we describe below. This chapter is a complemented version of the work we presented in Ref. 158. The motivation of this work is the following. A peculiar transport signature occurring in a nanoscale network (see Fig. 1.2) was identified and presented in Ref. 13. Both simulation and experimental results were presented in the latter work. Nevertheless, the physical origin of the mechanism was not clarified and the present chapter aims to reproduce the observations made in [13] and to collect more experimental data to reach a more complete description of the phenomenon. The philosophy of this work was first to rule out several well-established mesoscopic effects that could occur in such kind of network. In the end, various scanning gate-assisted analysis are performed, demonstrating that two mechanisms could have been at play in the observations of Ref. 13 : Coulomb blockade and Fabry-Pérot electron resonance. The first one is related to the disorder in the 2DES confining potential and is therefore not related to the specific geometry and only the second mechanism is probably associated with the peculiar network geometry. With respect to our initial motivation, we have to clearly point out that no paradox was found in the investigated transport regimes.

5.1 Introduction

The suppression of an axis in a classical transport network can improve the overall network performances. This counterintuitive behavior - initially highlighted in road networks - is known as the Braess paradox [8], formalized in the framework of game theory. In short, the combination of selfish behaviors and non-linear response of traffic roads paradoxically increase transit time when new roads are added. The Braess paradox was later evidenced in electrical, hydraulics and mechanical networks.[10] In the latter cases, selfishness is obviously not at stake, but non-linear response is. In the linear classical regime, however, no physical system is expected to exhibit such paradox. The question is then: Is quantum mechanics a game changer?

This question was first addressed by means of simulations and experiments on an electronic network [13]. Downscaling such a network, one can reach the mesoscopic regime of transport where quantum behaviors such as energy quantization, quantum wave interferences and single charge effects become prominent. Performing a transport experiment on a mesoscopic network fabricated from InGaAs/InAlAs heterostructure, Pala et al.[13] revealed a behavior analogous to the Braess paradox [13]: the network electrical conductance G was found to increase under depletion of its central axis. However, neither selfishness nor non-linear response can be invoked to explain what is - at first sight - perceived as a striking anomaly. While simulations based on the Keldysh-Green formalism predict such paradoxical behavior[159], its possible physical origins are not yet experimentally clarified. Simulations by Sousa et al. [14] and Macucci et al. [15] both conclude that different scenarios based on electron wave interferences can indeed yield conductance increase upon transport axis closures. The questions are now: can we pinpoint experimental conditions where interferences induce conductance improvement, and, are there other physical mechanisms able to induce such phenomenon?

This part of the manuscript is dedicated to these questions. For this purpose we perform conductance measurements on a mesoscopic network while tuning the local electron density by means of a moving nanoscopic scanning gate. We first confirm the existence of a paradoxical behavior in the quantum regime of transport, and then identify two distinct mechanisms that can be at its origin.

We study the sample shown in Fig. 5.1a, similar in size and shape to the one studied in Ref. 13 and patterned using electron beam lithography and wet etching in an InGaAs/InAlAs heterostructure (see appendix B on the fabrication process). A two dimensional electron system (2DES) is confined 45 nm below

the surface of the heterostructure that was grown by molecular beam epitaxy on a degenerately doped InP substrate (InP n+) serving as a backgate to tune the global 2DES density. The experiments were performed at $T = 4.2$ K where the electron density n_{2D} - obtained by analyzing Shubnikov-de Haas (SdH) oscillations on a neighboring Hall bar - varies from 6 to $8.4 \times 10^{11} \text{cm}^{-2}$ as the backgate potential V_{BG} goes from 2 to 4 V. Simultaneously the mobility varies from 9 to $11 \frac{\text{m}^2}{\text{Vs}}$. The device conductance G is measured using a low-frequency (28.5 Hz) standard lock-in technique, with the source-drain voltage across the device always being less than $\frac{k_B T}{e}$ to ensure quasi-equilibrium conditions.

In order to selectively perturb transport in the network channels, we used the scanning gate microscopy (SGM) [95, 160, 161] technique that allows us to locally change the electrostatic potential landscape thanks to a polarised conductive atomic force microscope tip of apex radius smaller than 20 nm. The tip is attached to a tuning fork used as a force sensor to map the topography (see Fig. 5.1b) of the sample and to precisely determine the relative tip-network position.[162] The conducting AFM tip is then withdrawn at a distance d_{tip} from the sample surface - typically a few tens of nanometers - and scanned in a plane parallel to the 2DES. All the following parameters, the tip position x_{tip}, y_{tip} , height d_{tip} and polarization V_{tip} change the electrostatic potential landscape within the network and lead to variations of the device electrical conductance. The conductance is mapped as a function of $x_{tip}, y_{tip}, V_{BG}, V_{tip}, d_{tip}$ and magnetic field B , which will generate all the data presented hereafter.

5.2 Results and discussion

As a first experiment, we measure the device conductance as the biased tip is scanned along the dashed line on Fig. 5.1a, 80 nm above the sample surface. As V_{tip} becomes more negative, we observe an anomalous conductance behavior when the tip is located above the central channel (along the red curve in Fig. 5.1d). On a particular V_{tip} domain, the conductance rises as the negative V_{tip} progressively depletes the 2DES underneath the tip (see the blue domain in Fig. 5.1d). Fig. 5.1c shows that this peculiar behavior occurs only when the tip is located right above the central branch, as reported in Ref. 13. Beyond reproducing comparable results, we now investigate their robustness while varying the average charge carrier density in the whole network. This is done in Fig. 5.1e where the conductance is mapped in the $V_{tip} - V_{BG}$ plane with the tip located above the middle of the central branch. The black-dotted isoconductance

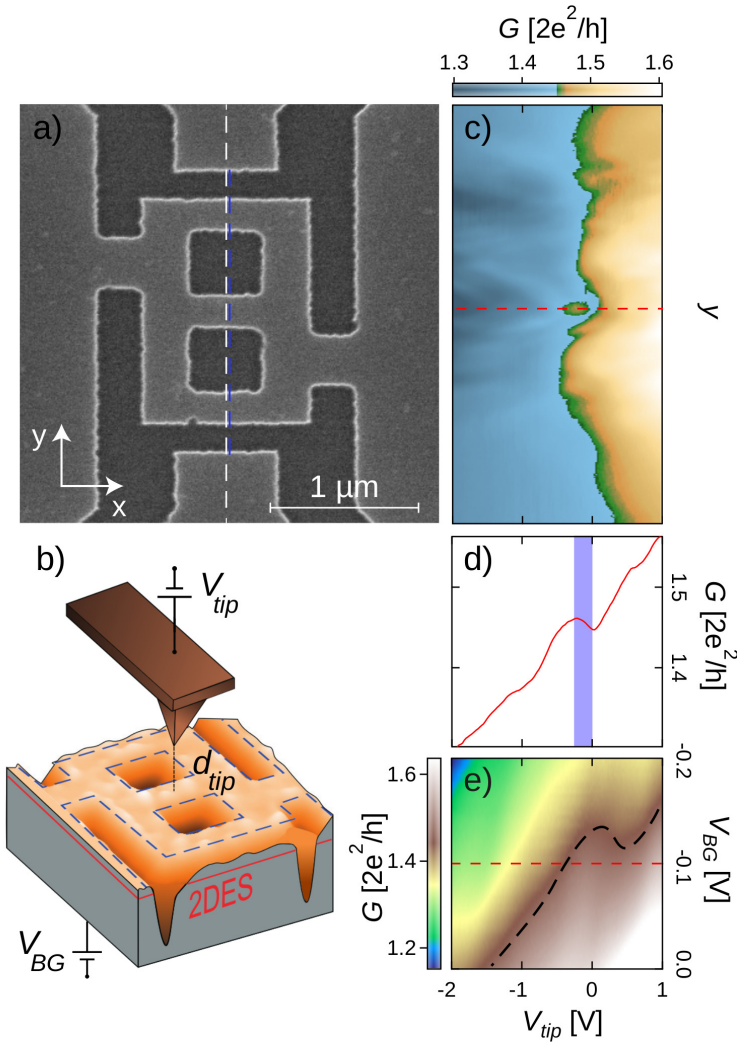


Figure 5.1: a) Scanning electron micrograph of the network whose conductance (G) is measured. b) AFM topography of the network embedded in an artist's view of the tip-network system. The different parameters present in an SGM experiment are shown: the tip potential (V_{tip}), the tip-sample surface distance (d_{tip}) and the backgate potential (V_{BG}). c) Conductance measurement as a function of V_{tip} with the tip scanned along the dashed white line in a), for $d_{tip} = 80\ \text{nm}$ and $V_{BG} = -0.1\ \text{V}$. The vertical axes of a) and c) are matched. d) G measurement along the dashed red line in c) and e). Note that the anomaly in c) and d) occurs when the tip is right above the central branch of the network. e) G mapping as a function of V_{tip} and V_{BG} with the tip located 80 nm above the middle of the central channel. The black dashed line follows an iso-conductance line.

line gives a different perspective to the anomalous behavior we are focused on. Indeed, the anomaly could either be evidenced by a negative $\frac{dG}{dV_{tip}}|_{V_{BG}}$ or by a negative $\frac{dG}{dV_{BG}}|_{V_{tip}}$. One novelty revealed by the present experiment is that the anomalous behaviour can be observed both for negative or positive tip potentials, which was not expected from earlier works [13]. For instance, Fig. 5.1e reveals that the anomaly drifts towards positive V_{tip} when the 2DES density is lowered ($V_{BG} < 0$). This fact is indeed worth emphasizing as it shows that a local depletion is not necessarily a key ingredient for the observed anomaly. Figure 5.2a shows a line scan analogous to Fig. 5.1c, but for a positive value of V_{BG} . Consistently with the trend observed in Fig. 5.1, the anomalous bump is now observed at more negative values of V_{tip} , and remains located above the central branch.

Observing the evolution of the amplitude of transport phenomena with temperature is often key to pinpoint their origin. Indeed, the characteristic times governing transport mechanisms follow different temperature dependences. For example, the phase coherence time τ_ϕ increases in most cases when lowering the temperature, which leads to an enhanced amplitude of electron interference phenomena. In this experiment, we observe strong qualitative changes when lowering the temperature : in the line scan data obtained at 50 mK (Fig. 5.2b) in the same conditions as Fig. 5.2a, the contrast is dominated by strong conductance fluctuations, and the anomalous behaviour observed at 4.2K over the central branch is no longer visible. The large amplitude random fluctuations evidenced at the lowest temperature are a hallmark of Universal Conductance Fluctuations (UCFs), a phenomenon stemming from interferences between a large number of different semiclassical electron paths [163]. The enhancement of UCFs contributions is consistent with the increased phase coherence time. However, if the anomalous bump of Fig. 5.2a was governed by the same phase coherence time as the one determining the UCFs' amplitude, it should be more visible than the random fluctuations attributed to UCFs in Fig. 5.2b. Hence, data in Fig. 5.2 advocate against UCFs as the origin of the anomalous behaviour. In the remainder of this chapter, we will focus on the data collected at 4.2K, where the anomalous behaviour is more clearly visible, and we will come back to the interpretation of data in Fig. 5.2 at the end of the discussion. Note also that additional thermal cycles have been performed and confirmed the robustness of the effect.

We now investigate the conditions required for the emergence of anomalous conductance domains when sweeping both V_{tip} and V_{BG} with the tip sitting

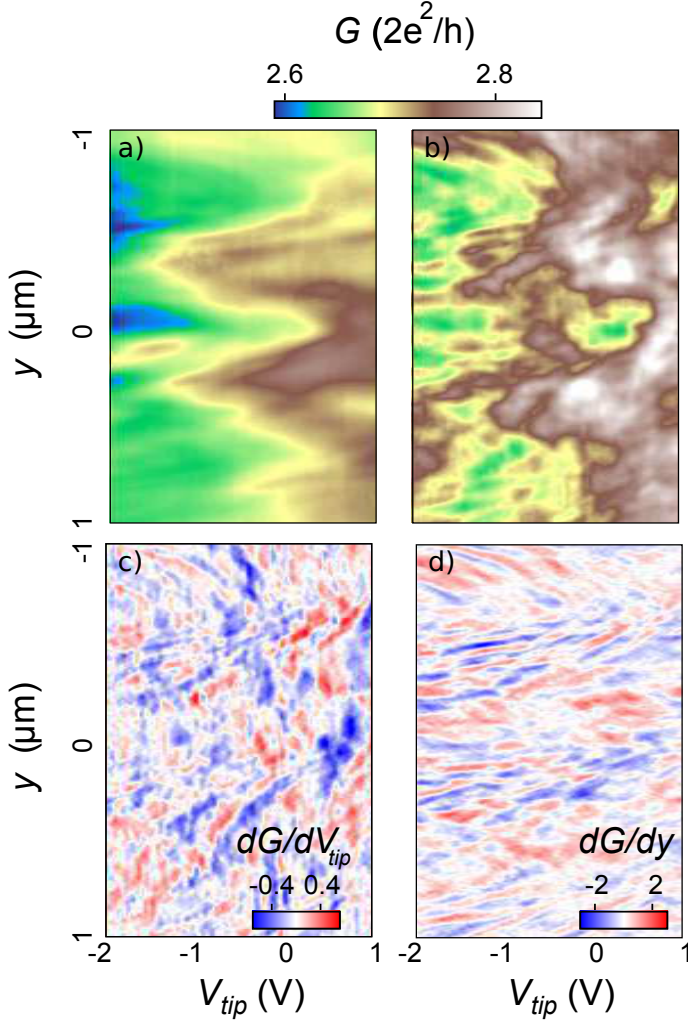


Figure 5.2: a) Mapping of the SGM linescan along the blue dashed line in Fig. 5.1a, as a function of V_{tip} at 4.2 K, exhibiting an anomalous and robust increase of conductance when the tip scans above the central arm, for $-2 V < V_{tip} < -1 V$. b) The same mapping is obtained at 50 mK. $V_{BG} = 0.8 V$ for both mappings. The mappings of the derivative of data of Fig. 5.2b are presented with respect to V_{tip} (c) and y (d), respectively). The horizontal axes are matched.

at a constant distance above the middle of the central channel. The result is presented in Fig. 5.3 that provides a deeper insight into the density dependence of the studied behavior. The two sets of data differ by the doubling of d_{tip} and a different range in V_{BG} , and hence 2DES density ranges. From the conductance maps shown in Figs. 5.3b and e, we extracted two conductance plots at two specific V_{BG} (Figs. 5.3a and d) where the targeted anomalies are emphasized by blue domains. The detection of conductance variations in the conductance maps is made easier by plotting $\frac{dG}{dV_{tip}}|_{V_{BG}}$ as a function of V_{BG} in Figs. 5.3c and f, since the signature of the paradox is a negative sign in this derivative (blue color).

A major piece of information resides in Figs. 5.3c and f, where the presence of two sets of stripes - with significantly different slopes - is the main feature of the data. The phenomenology of stripes, i.e. a gradual and regular shift of periodic oscillations with respect to V_{tip} and V_{BG} , does not correspond at all to that of UCFs, which are random by nature [163], and therefore undergo a chaotic evolution when external parameters such a gate voltage and a magnetic field are varied. As already mentioned, UCFs are necessarily present since transport is in the coherent regime - as evidenced by the Aharonov-Bohm (AB) oscillations [164] and l_ϕ estimates discussed later in the paper - but, at 4.2 K, they are dominated by two other effects that we will now investigate. In contrast, when temperature is lowered down to 50 mK, UCFs become dominant, as shown on Fig. 5.2b. One therefore has two clear signs that the phenomena revealed in Fig. 5.3 are different from UCFs.

The so-labelled slopes 1 and 2 characterize the presence of conductance anomalies which appear in blue in Fig. 5.3. Slope 1 characterizes isoconductance lines in Fig. 5.3b which are mainly governed by the backgate which tunes the overall electron density in the network. Mainly, but not completely, which means that the tip does not only have a local effect on the electron potential in the network, but also affects the global electron density. Expressed in terms of coupling capacitance, that to the back gate is 50 times larger than that of the tip. In short, slope 1 anomalies are directly related to the overall electron density as mainly tuned by V_{BG} , or marginally by V_{tip} . By the way, we also note that slope 1 features progressively leave the floor to slope 2 at higher conductances *i.e.* higher electron densities. The latter point may be reminiscent of single charge effects, but we will need SGM imaging data (presented later) to tell more about the physical mechanism behind slope 1.

Slope 2, on the other hand, does not follow isoconductance profiles and is

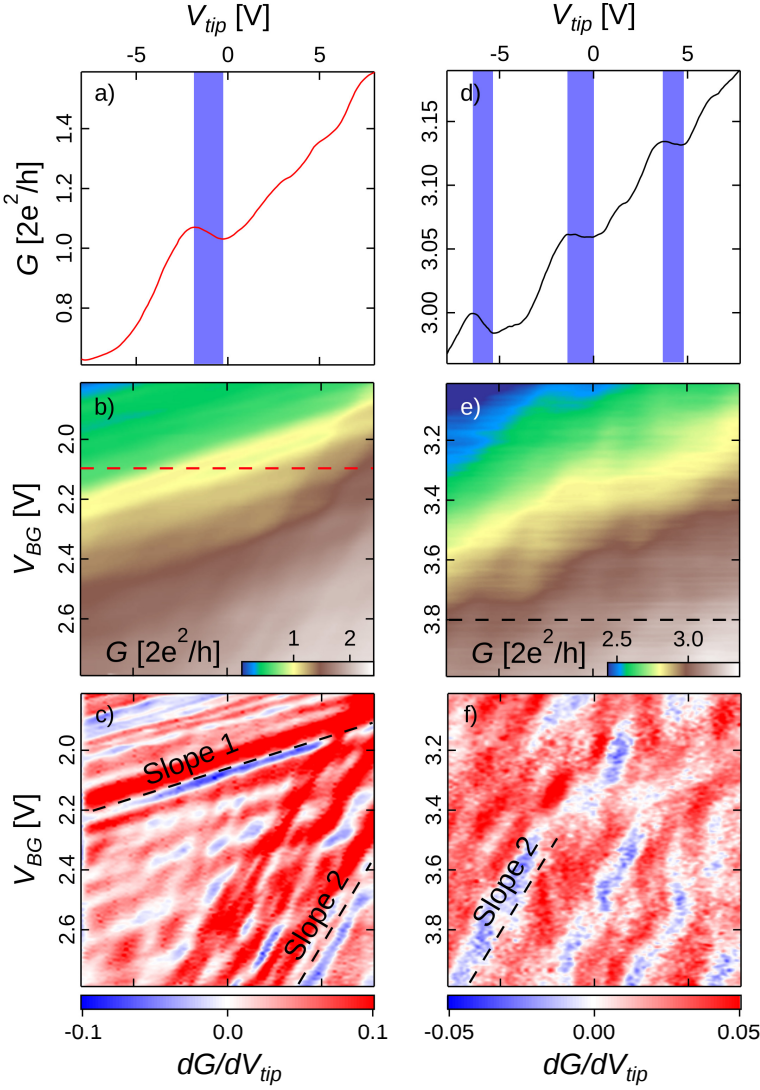


Figure 5.3: a) and d) Conductance measurement when varying V_{tip} along the dashed lines in b) and e) corresponding to different ranges of electron densities. b) and e) Conductance measurement as V_{tip} and V_{BG} are swept with $d_{tip}=50$ nm for b) and 100 nm for e) above the middle of the central channel (see Fig. 5.1). c) and f) present the derivative of G with respect to V_{tip} at constant V_{BG} associated to the G mappings presented in b) and e). All V_{tip} horizontal axis are matched.

much steeper than slope 1, which clearly points towards a mechanism where the local impact of the tip is more prominent.

At this stage, it seems clear that slope 1 and 2 sign two distinct mechanisms and this will be further evidenced by SGM imaging data. Slope 2 is not vertical, which means that the second mechanism is affected by the local potential in the central arm, but also depends on the overall density in the network. Slope 2 also spreads equally in both positive and negative V_{tip} and remains unaffected on a large range of conductance. Slope 2 additionally remains roughly unchanged in Fig. 5.3c and f.

In order to differentiate the two mechanisms, we use the ability of the SGM technique to yield real space insight into the network. The proximity of a polarised tip perturbs the potential landscape along the electron flow, which affects the macroscopic value of G . The measurement of $G(x_{tip}, y_{tip})$ when scanning the tip in a plane - at a few tens of nanometers - parallel to the sample surface allows us to get local information in the form of SGM conductance maps. Let us recall that in SGM, it is generally cumbersome to obtain local information in a system through a conductance measurement that is inherently non-local, particularly in these networks.

Figs. 5.4d and e show SGM maps measured in the range where slope 1 is dominant (Figs. 5.4a to c). Taken at the two closeby values of V_{tip} pointed on Fig. 5.4a, the two SGM images (Figs. 5.4d and e) have clear resemblances. In particular, we note a strong spot that marks a position where the tip brings the conductance close to zero. The mechanism behind slope 1 should thus be able to "close" all 3 arms while the tip sits above one specific region (note that in this experiment, we do not have precise information on the conductance of each parallel arm within the device). To go further in our understanding of Figs. 5.4d and e, it is useful to realise a subtraction of the two SGM images (Fig. 5.4f) as it highlights the small variations atop of a strong varying background. This data allows to identify sets of concentric fringes originating from different locations in the network, and is clearly reminiscent of the data reported by Liu et al. in a similar device [130], as well as SGM data obtained on patterned quantum dots in the Coulomb blockade regime[165] (note that the evolution of the concentric fringe pattern is shown as a function of V_{tip} in Figs. 5.5 c-j). This observation tells us that the mechanism behind slope 1 is most probably related to single charge effects [166–170]. Even though the low conductance spot is not atop the central arm, the long-range contribution of the tip to the electron potential landscape produces strong conductance variations when V_{tip}

varies while the tip sits above the central arm. The fact that the so-called "slope 1" anomalies locate along iso-conductance lines is also consistent with charging effects which are directly related to the long-range potential perturbation in the network. The suppression of slope 1 features at higher densities also advocates for single charging effects of disorder-induced quantum dots (QDs), which naturally disappear as the bottom of the conduction band is lowered. In our device, such QDs are therefore present along all three branches. The presence of QDs within a 2DES drastically affects the conductance of the device whenever transport occurs through the QD regions and when the coincidence of the dots energy levels and the Fermi energy is tuned by an external gate. Here, varying the backgate or the tip voltage changes the number of trapped charges within each QD, which induces Coulomb blockade (CB) dips in the device conductance. Since these CB dips are located on concentric fringes in SGM images, without particular spatial symmetry, one way to practically see a clear signature of Coulomb blockade is to subtract two SGM images realised for closeby tip voltages. This is exactly the purpose filled by Fig. 5.4f.

As evidenced in Ref. 130, discharging events of disorder-induced QD occur when the negatively biased tip approaches the dot location. Pinpointing the actual dot locations is not straightforward. Indeed both the tip geometry and complex anisotropic screening in the network lead to a tip induced potential perturbation difficult to evaluate. A priori, the potential perturbation and the QD present no particular symmetries. Nevertheless, QDs locations can be roughly estimated within the network near the centers of the concentric fringes presented in Fig. 5.4f, *i.e.* in the top branch and near the right exit of the network. Fig. 5.4f also reveals that few conductance variations are observed inside the innermost fringes. This can be attributed either to a completely emptied dot or to potential barriers becoming too high for significant tunneling events to occur. This innermost region naturally expands as the tip is polarised more negatively. [130] Visually this leads to the expansion of concentric fringes as closeby- V_{tip} SGM images are subtracted for more negative tip potential pairs. This expansion is at the heart of the non monotonous conductance behavior observed in Fig. 5.4a where the tip is standing above the middle of the central channel. Indeed, as demonstrated in figure 5.5, a white concentric fringe (evidenced in Fig. 5.4f), corresponding to $\frac{dG}{dV_{tip}} = 0$, crosses the central branch and a clear correspondence is established between the conductance maximum in Fig. 5.4a and the movement of this fringe over the middle of the central channel. In this picture, the average network density is the main parameter for the slope

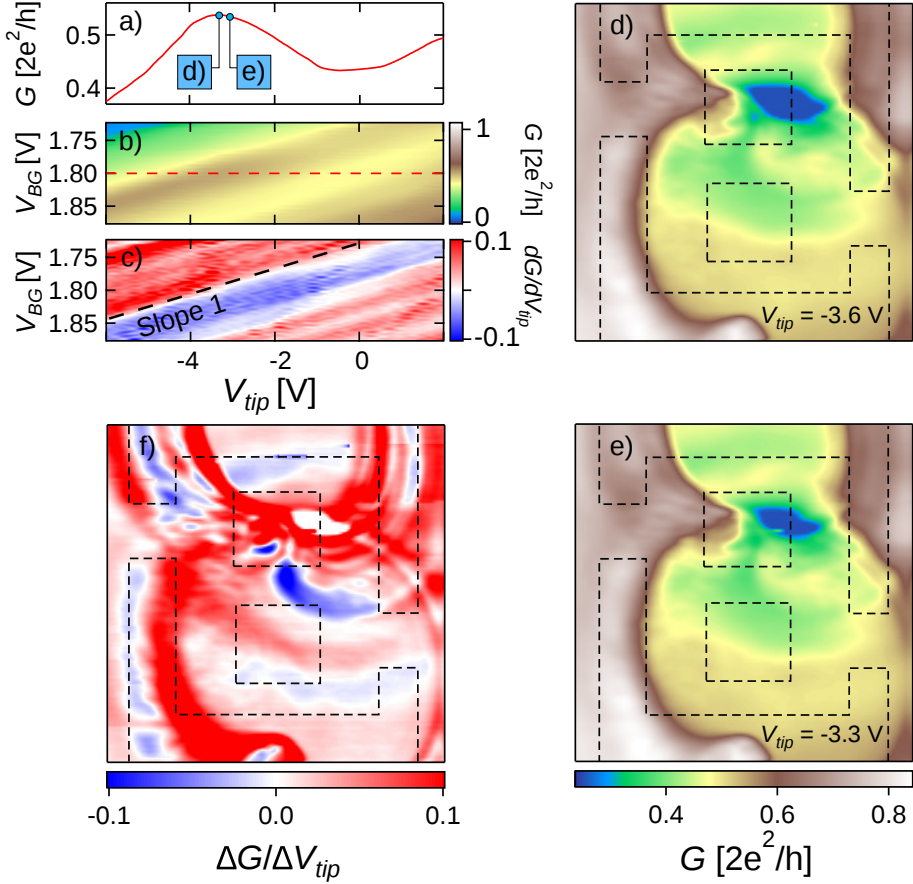


Figure 5.4: a) Conductance measurement extracted from b) (red dashed line) b) Conductance measurement as V_{tip} and V_{BG} are swept with the tip located 50 nm above the middle of the central channel (see Fig. 5.1a). c) Derivative of G with respect to V_{tip} at constant V_{BG} corresponding to the G mapping in b). d) and e) Scanning gate microscopy conductance maps of the network realised for closeby V_{tip} for the tip scanned in a plane parallel to the surface and located 50 nm above it and with $V_{BG}=1.8$ V. f) Subtraction of the two SGM conductance mappings showed in d) and e).

1-mechanism, while small local potential variations due to the tip induce the contrast in G-maps. Consistently with Fig. 5.3c, concentric fringes are also observed in SGM mappings realised for closeby- V_{BG} mappings at constant V_{tip} .

At this point, we revealed a first mechanism, CB, explaining the anomalous behaviour. Interestingly, we can strongly attenuate CB oscillations by increasing the global electron density. As shown on Fig. 5.3f, slope 1 - disorder related features vanish at higher density, which means that disorder-induced localized states do not rule conductance in the network any more. Let us recall that, in a disordered potential, potential wells can be present (see the one encircled in the disorder map presented in Fig. 2.4d). In such wells, electrons can be localized and this is what is meant by disorder-induced localized states.

Consistently, no concentric fringes can be observed for closeby- V_{tip} SGM images realised at higher density (see Fig. 5.6).

We now turn to slope 2 that may originate from a qualitatively different mechanism. At this point one has to recall what qualitatively changes when rising the 2DES density. In addition to a stronger tip potential screening, the amplitude of coherent effects rises. The latter is confirmed by magnetoconductance data taken at different densities (from $V_{BG} = 2$ to 4V) with no tip in the network vicinity. One of these curves is presented in Fig. 5.8a. Sweeping a magnetic field orthogonal to the 2DES clearly reveals G -oscillations with a period of 7 mT (see inset of Fig. 5.8a). This B-period corresponds to the AB period for electrons circulating around a single antidot present in the network (see Fig. 5.1a) [164, 171]. Indeed, the 7mT-oscillations amplitude increases strongly and then saturates as V_{BG} goes from 2 to 4 V, with a threshold near 2.2 V (see Fig.5.7 where a detailed analysis of the AB oscillations is presented) which is also the point where slope 1 leaves the floor to slope 2.

The picture is now even clearer as one expects AB effects to be much attenuated or absent in the presence of the potential barriers needed for CB. The transition to the AB regime occurs in the range $2.2 < V_{BG} < 3.2$ V, beyond which the amplitude saturates as the CB potential barriers are no longer significant. The presence of AB and even Altshuler-Aronov-Spivak (AAS) oscillations [172] (see the presence of a weak peak at 284 T^{-1} in Fig. 5.7e corresponding to twice the area of a single antidot) implies that a coherent mechanism could be at play in the case of slope 2 anomalies. In addition, from the analysis of the autocorrelation function of magnetoconductance traces [173, 174], we extracted a coherence length $l_\phi \sim 1 \mu\text{m}$ (at $T=4.2\text{K}$) comparable to the size of the network in the V_{BG} range where slope 2 related features are observable.

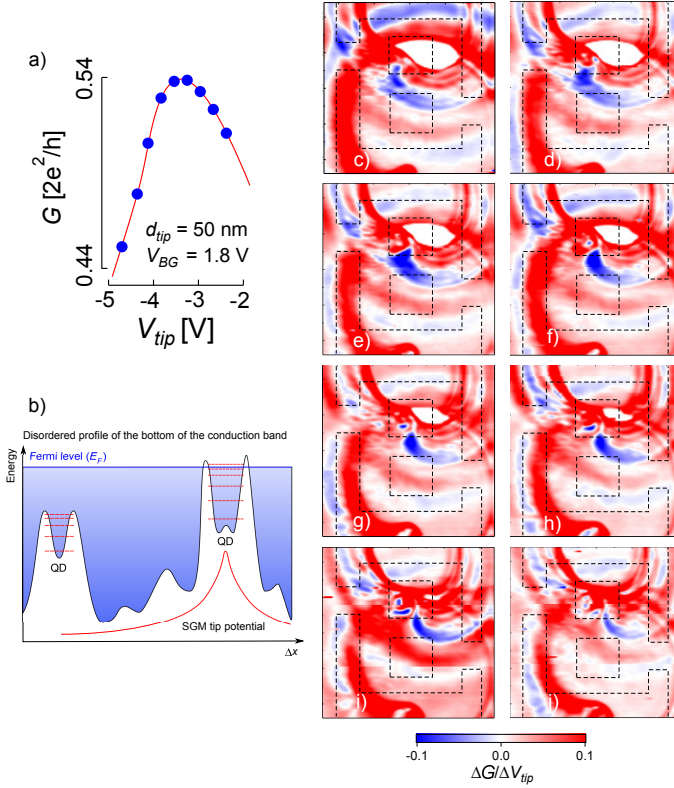


Figure 5.5: (a) Conductance vs V_{tip} , measured at $V_{BG} = 1.8$ V and $d_{tip} = 50$ nm, with the tip positioned above the middle of the central arm. The blue dots indicate the tip voltages where SGM mappings were realized. (b) Schematic picture illustrating the effect of the SGM tip potential (red continuous line) on the bottom of the conduction band (black line). The tip potential raises disorder-induced fluctuations up to the Fermi level. This creates a confined quantum dot structure, tunnel-coupled to the leads, with discrete energy levels brought in resonance with the Fermi energy for specific tip position and voltage. (c-j) Differential SGM mappings, obtained by subtracting conductance maps measured at two consecutive V_{tip} polarizations indicated by the blue dots in (a). Shrinking concentric fringes are observed, directly related to the tip-controlled tuning of energy levels inside quantum dots located within the device. The sequence of red-white-blue fringes crossing the central arm as V_{tip} is varied in (c-j) allows to deduce that the conductance maximum in (a) is directly related to the Coulomb blockade mechanism depicted in (b).

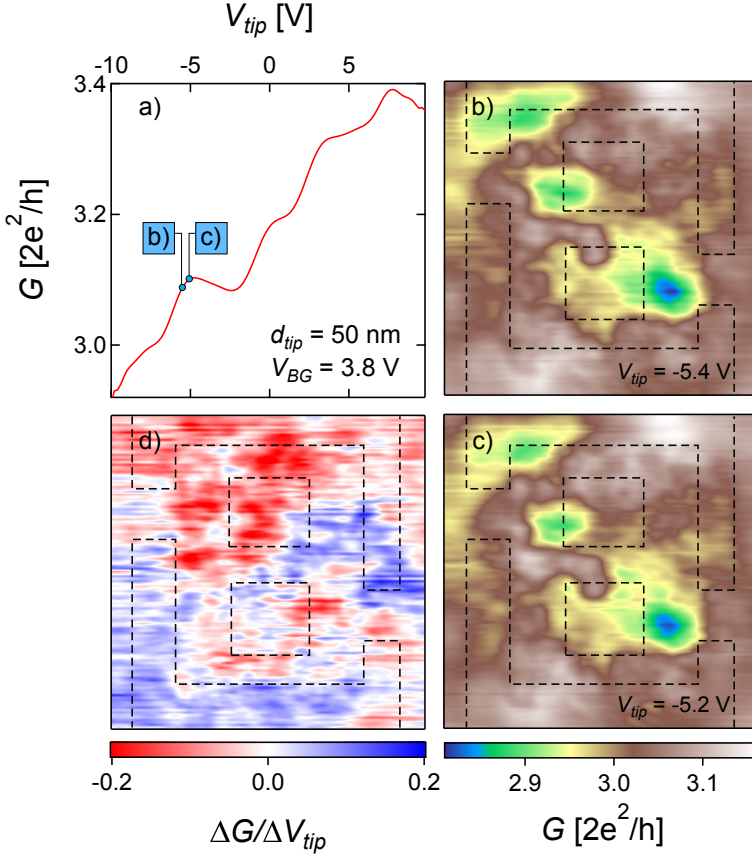


Figure 5.6: (a) Conductance vs V_{tip} with the tip located 50 nm above the middle of the central channel and $V_{BG}=3.8$ V. (b) and (c) SGM conductance maps of the device at $V_{tip}=-5.4$ and -5.2 V, with $d_{tip}=50$ nm and $V_{BG}=3.8$ V. (d) Subtraction of the two SGM SGM maps shown in (b) and (c).

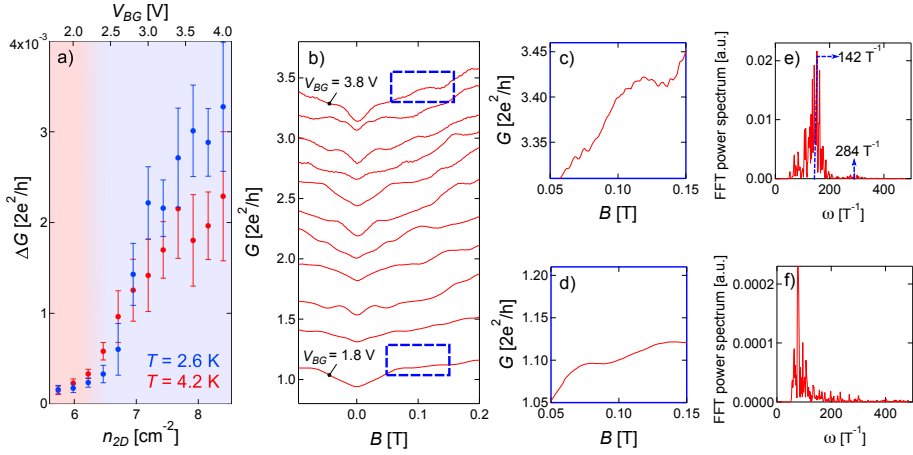


Figure 5.7: (a) RMS amplitude of the AB oscillations for different values of V_{BG} - *i.e.* different n_{2D} - at $T = 2.6$ K and $T = 4.2$ K. (b) G vs B measured for different values of V_{BG} between 1.8 and 3.8 V. (c), resp. (d), gives an enlarged view of a small range of magneto-conductance curves obtained at $V_{BG}=3.8$ V, resp. 1.8 V [blue dashed boxes shown in (b)]. (e), resp. (f), FFT of the magnetoconductance curves displayed in (c), resp. (d). Note that magnetoconductance curves were high - pass filtered. The FFT presented in (e) exhibits maxima at 142 and 284 T^{-1} , corresponding to 7 mT and 3.5 mT AB periods. The 142 T^{-1} maximum is consistent with orbits enclosing a single antidot in the device. The peak at 284 T^{-1} could correspond either to Altshuler-Aronov-Spivak (AAS) oscillations, or to AB oscillations associated with orbits enclosing both antidots. We favour the AAS hypothesis since the 284 T^{-1} peak disappears when time-reversal symmetry is lifted as B grows. None of the above-mentioned maxima could be identified in (f).

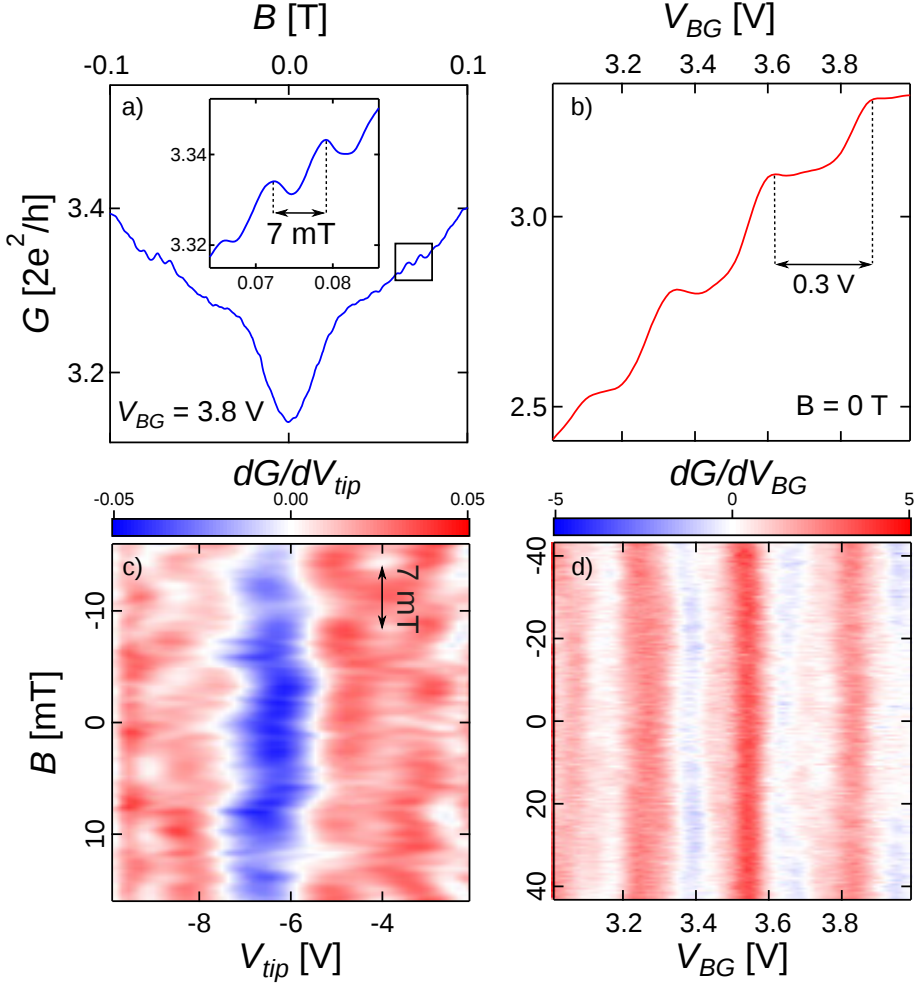


Figure 5.8: a) and b) Network's conductance as (a) the magnetic field (B) and (b) the back-gate potential (V_{BG}) are swept in absence of nearby AFM tip. The inset of Fig. 5.8a highlights the AB oscillations whose period are 7 mT. c) Derivative of G with respect to V_{tip} vs B and V_{tip} with a tip polarized above the middle of the central branch for $d_{tip} = 50$ nm with $V_{BG} = 3.8$ V. d) Derivative of G with respect to V_{BG} vs B and V_{BG} with no tip in the network's vicinity.

In this framework, one might suggest that the tip potential perturbation could modulate the interferences in the network through the electrostatic AB effect. However, the anomalous conductance variations are one order of magnitude larger in amplitude than the observed magnetic AB effect. Indeed, in the electron density range between 7 and $8.4 \times 10^{11} \text{ cm}^{-2}$ where AB effects are fully developed (*i.e.* V_{BG} between 3 and 4 V), the RMS amplitude of the anomalous effect is on average $\sim 1.5 \cdot 10^{-2} G_0$ while the magnetic AB amplitude is $< 2 \times 10^{-3} G_0$. Note that measurements on quantum rings realised in similar heterostructures revealed that the electrostatic and magnetic AB effect have equal amplitudes [175]. This is a strong indication that the anomalous phenomenon is not compatible with a tip-induced electrostatic AB effect associated with electron paths encircling an antidot in our sample.

However, interferences involving electron paths encircling smaller surfaces could lead to larger amplitude AB-like oscillations, which could be an alternative hypothesis explaining the origin of the anomalous phenomenon. Such oscillations should be visible in the magnetoconductance of the device, and their B -periodicity would allow to infer the area encompassed by interfering electron trajectories. To observe how the anomalous phenomenon evolves with the magnetic field, the derivative of G with respect to V_{tip} , measured with the tip over the middle point of the central branch of the network, is mapped as a function of B in Fig. 5.8c - note that the central branch is not depleted by the tip, over the full V_{tip} range in Fig. 5.8c (more details about tip-induced depletion are provided in chapter 4). Strikingly, the anomalous blue domain observed between $V_{tip} \sim -8$ and -6 V persists over the full B -range, and its average position is only weakly modulated by B with an AB periodicity associated with trajectories around antidots (*i.e.* 7mT). If an AB effect corresponding to a surface S trajectory was at the origin of the phenomenon, the sign of $\frac{dG}{dV_{tip}}|_B$ should be reversed at constant V_{tip} with a periodicity corresponding to ϕ_0/S (where ϕ_0 the quantum of flux). The absence of sign reversal in the B -range considered in Fig. 5.8c means that S is much smaller than $\phi_0/30 \text{ mT} = 0.068 \mu\text{m}^2$, which corresponds to a circle with a radius smaller than 148 nm. This radius is smaller than the antidot lateral size, but could fit into the central branch of the network. One could then imagine that the tip potential locally raises the potential in the central branch of the network, so as to create a small-area antidot which could generate AB-like oscillations with a large B -period. This hypothesis can be simply checked by putting the tip further away from the device. The anomalous behavior is now observed when varying V_{BG} , as shown

in Fig. 5.8b. The tip-induced antidot hypothesis is therefore ruled out. Note that the observation of the anomalous behavior for positive V_{tip} was anyway incompatible with this hypothesis. At that point, applying a magnetic field reveals that the anomalous phenomenon in $\frac{dG}{dV_{BG}}$ vs V_{BG} also remains essentially unaffected, up to 40 mT, as shown in Fig. 5.8d (although a small-amplitude 7 mT modulation also decorates the data). Since no sign reversal is observed for the anomalies over an even larger B-range than in Fig. 5.8c, Fig. 5.8d naturally leads to consider the possibility of coherent interferences between electron paths encompassing a vanishing surface and intrinsic to the 2DEG network, *i.e.* not induced by the tip.

The latter idea corresponds indeed to the picture of standing electron wave patterns located within a 1D cavity inside the network. The tip-induced perturbation area is critical in this framework as it can tune the local electron density and hence change the interference pattern. A simple way to tune this parameter is to vary the tip-surface distance d_{tip} as shown on Figs. 5.9a and b, obtained with the tip remaining above the middle of the central branch. Oscillations of G and $\frac{dG}{dd_{tip}}|_{V_{tip}}$ are visible as a function of both d_{tip} and V_{tip} in Figs. 5.9a and b, forming a fan-like structure. The observed convergence of the fringes on Fig. 5.9b at the level of the 2DES ($d_{tip} \sim -45$ nm) indicates that they originate from a phenomenon triggered by the tip perturbation, occurring in the central branch right under the tip (*cfr.* Fig. 5.1c), as opposed to the CB-related fringes, associated with the effect of the tail of the tip perturbation (*i.e.* a long range effect). Note also the clear qualitative differences between Figs. 5.5 and 5.6 which illustrate the very different effect of the tip in the CB and AB regimes.

The difference of behavior between both types of fringes with respect to changes of d_{tip} is further evidenced in $\frac{dG}{dV_{tip}}$ vs (V_{tip}, V_{BG}) maps (such data were already shown in Fig.3c where $d_{tip} = 50$ nm). Such a map is presented in Fig. 5.9c with $d_{tip} = 140$ nm where both slope 1 and slope 2 - identified on Fig. 5.3c - are reported for comparison purposes. One can directly observe that the slope of fringes emerging at low V_{BG} (slope 1 fringes) does not vary with d_{tip} , while slope 2 decreases as d_{tip} increases. A weaker slope means a larger V_{tip} period at constant V_{BG} , which is consistent with Fig. 5.9b. The increasing period of the oscillations of $\frac{dG}{dV_{tip}}|_{d_{tip}}$ vs V_{tip} when the tip goes away from the surface is compatible with the hypothesis of a local tip-induced variation of the interfering electron wavelength modifying resonant patterns in a 1D cavity [176],[177]. This is illustrated in Fig. 5.10: in the area where the density is modified by the tip-induced perturbation, the Fermi wavelength changes according to $\sqrt{\frac{2\pi}{n_{2D}}}$. In

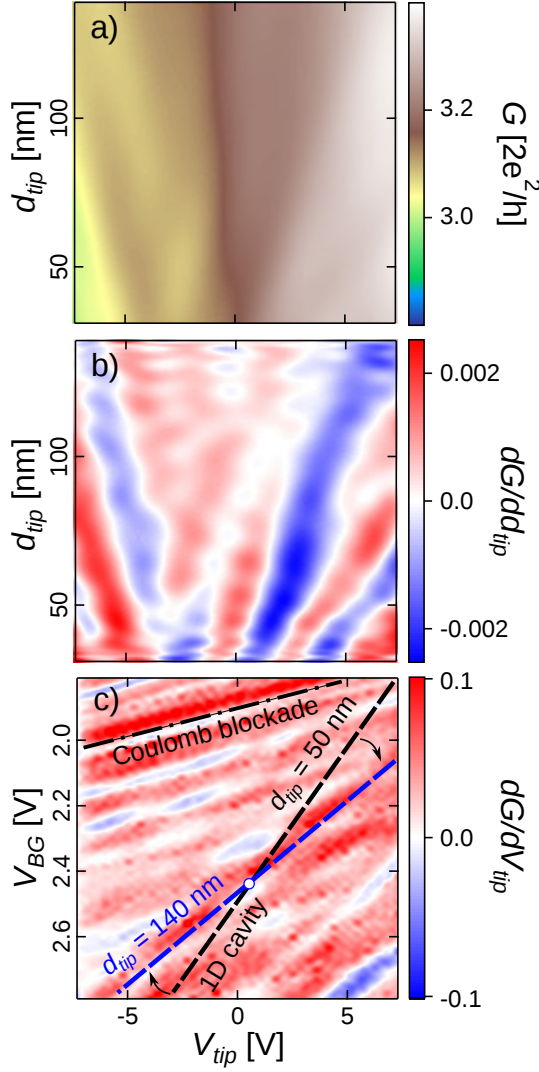


Figure 5.9: a) Conductance measured for different values of V_{tip} and d_{tip} for the tip centered above the middle of the central branch (see Fig. 5.1a) $V_{BG} = 3.8$ V. b) Derivative of G with respect to d_{tip} corresponding to the mapping a). The vertical axes of a) and b) are matched. c) Derivative of G with respect to V_{tip} at constant V_{BG} associated to the G mappings similar to Fig. 5.3b but realised for $d_{tip}=140$ nm. The slope labelled 'Coulomb blockade' is identical to the slope 1 presented in Figs. 5.3c and 5.4c. We also reported the slope 2 identified on Fig. 5.3c obtained with $d_{tip}=50$ nm (black dashed line) to compare with the slope observed with $d_{tip} = 140$ nm (blue dashed line). The horizontal axes of a) and c) are matched.

turns this affects the cavity resonance conditions, defined by:

$$\Delta\phi = \int ds k_F(s) = \int ds \sqrt{2\pi n_{2D}(s)} = N2\pi \quad (5.1)$$

where $k(s)$ is the local Fermi wavevector, s the spatial variable along the cavity and N the resonance order.

In this framework, an oscillation of $\frac{dG}{dV_{tip}}|_{d_{tip}}$ vs V_{tip} corresponds to a change of the resonance order N of the cavity. The increasing period of the oscillations observed when withdrawing the tip (Fig. 5.9b) is then explained by the diminishing lever arm of the tip over the local change in the Fermi wavelength. Applying a local potential variation leads to the following local equilibrium charge density :

$$n_{2D}(x, y) = \frac{m^*}{\pi\hbar^2} (E_F(V_{BG}) - e\phi_p(x, y; d_{tip}, V_{tip})). \quad (5.2)$$

To verify our hypothesis, one needs to identify the *locii* in d_{tip}, V_{tip} maps where $\Delta\phi$ is constant. This is equivalent to identify the locii $L(d_{tip}, V_{tip})$ where the integral S of the induced potential profile (or equivalently its square-root) is conserved. To do so we computed S in d_{tip}, V_{tip} ranges comparable to those presented in Fig. 5.9a. The iso-integral curves correspond in this framework to iso-phase curves and are presented in Fig. 5.11. The slopes observed in Fig. 5.9b are in rather good agreement with the iso-integral lines estimates with the model developed in section 4.4.1. "Slope 2" features in Fig. 5.9c and Fig. 5.11 advocate for a linear dependence of maximal tip induced potential variations at least for $|V_{tip}| \leq 7$ V. Larger values of V_{tip} may induce deviations from the intrinsically linear Thomas-Fermi model (see e.g. Fig. 4.7).

Let us bring another argument in favor of the resonance assumption. The V_{BG} -period associated to these oscillations is roughly equal to 0.3 V, as shown on Fig. 5.8b. Considering the resonance hypothesis, each maximum is assumed to correspond to successive resonance orders $N, N+1, N+2...$ At resonance, the following equation is relevant: $2L_c = N\lambda_F(V_{BG})$ where L_c is the cavity length and $\lambda_F = \sqrt{\frac{2\pi}{n_{2D}}}$ is the Fermi wavelength. One can therefore write:

$$\frac{N}{N+1} = \frac{\lambda_F^{N+1}}{\lambda_F^N} = \sqrt{\frac{n_{2D}^N}{n_{2D}^{N+1}}} \quad (5.3)$$

Since we measured n_{2D} as a function of V_{BG} (from SdH oscillations), we can determine N for the different resonances. For the maxima identified in

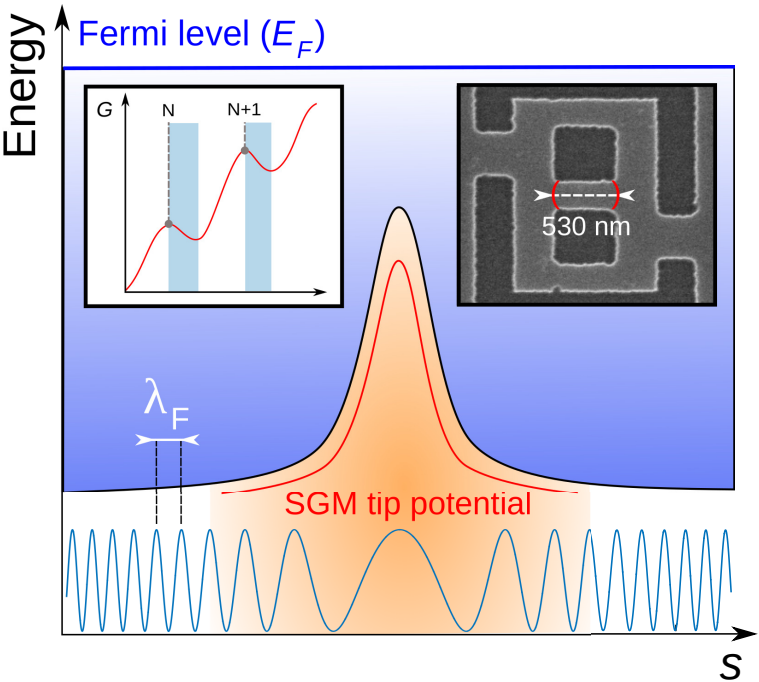


Figure 5.10: Illustration of how the SGM tip potential modifies locally the value of the bottom of the conduction band as the spatial variable s crosses the 1D cavity with the tip located above its middle. As the conduction band varies when the tip is polarised, it is possible to tune resonance conditions by locally modifying the electron wavelength. Insets: illustration of the G typical signature associated with successive resonance orders N , $N+1$ etc; estimated cavity length (530 nm) reported on a network scanning electron micrograph.

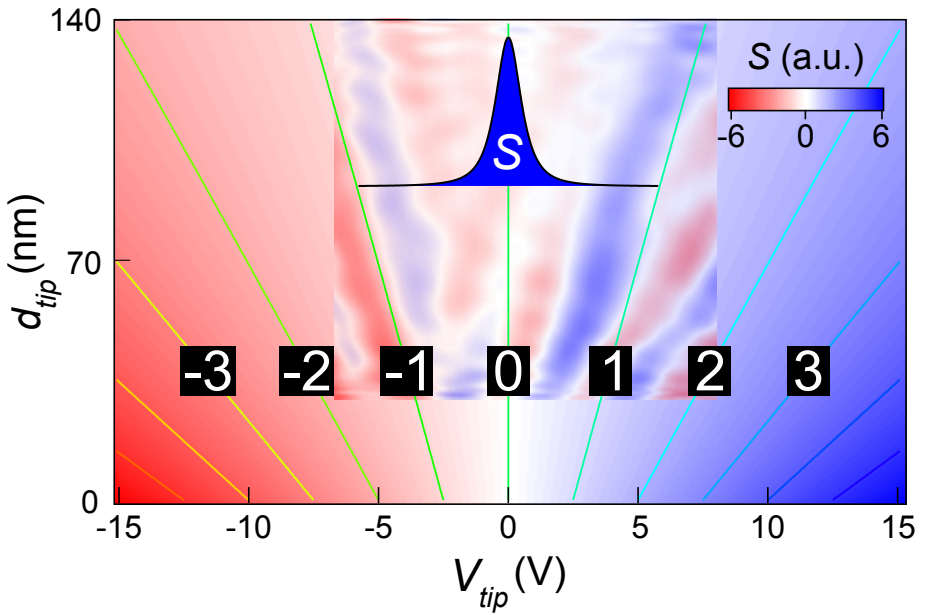


Figure 5.11: Integral of the tip-induced potential ϕ_p (S) mapped as a function of V_{tip} and d_{tip} . Contour lines are superimposed to the map to illustrate the linear relation needed to maintain a constant value of S . The map presented in Fig. 5.9b is superimposed to the figure to illustrate the good agreement of both fan diagrams.

Fig 5b at $V_{BG}=(3.1, 3.44, 3.8)[V]$, we have $n_{2D}=(7.34, 7.74, 8.15)[10^{15} \text{ m}^{-2}]$, hence $\lambda_F=(29.24, 28.48, 27.75)[\text{nm}]$. The corresponding values of N are 36, 37, 38, which leads to a cavity length L_c of $\sim 530 \text{ nm}$. Note that $2L_c$ is smaller than the elastic mean free path $l_\mu \sim 1.5 \mu\text{m}$ and comparable to the phase coherence length $l_\phi \sim 1 \mu\text{m}$ - for $V_{BG} = 3.8 \text{ V}$. L_c is larger than the central channel width so the 1D cavity is naturally located along the central branch, as schematically represented in Fig. 5.10 (white dotted line in the right inset). The edges of the cavity could correspond to the two open ends of the central branch: sharp changes of potential are expected at these locations, associated with the widening of the 1D channel, which could increase the reflection of electron wave functions into the 1D channel. However, further work is needed to confirm this hypothesis. In addition, the formula used for λ_F is valid for an infinite 2DES. In this case, we have a 1D central branch meaning that each transmission mode is characterized by a longitudinal wavevector k_n (associated to the n th mode) such that

$$E_F = \frac{\hbar^2}{2m^*} \left(\left(\frac{n\pi}{W} \right)^2 + k_n^2 \right), \quad (5.4)$$

where W is the width of the central branch.

The difference between λ_F and λ_n tends to be smaller and smaller as n tends to 1. No experimental data allowed us to precisely identify which mode leads to the aforementioned resonant feature. Also several modes could resonate, leading to a smearing of the conductance variation associated to the Fabry-Pérot resonance.

Now that we identified a clear d_{tip} -dependency of the resonance, one can naturally wonder why slope 2 remains roughly unchanged in (V_{tip}, V_{BG}) mappings as d_{tip} goes from 50 to 100 nm (see Figs. 5.3c and f). To clarify this observation we realised a set of measurements in the same (V_{tip}, V_{BG}) conditions as those presented in Fig. 5.3b but for several values of d_{tip} between 50 and 140 nm. The interest of such V_{tip}, V_{BG} maps is that the behavior of both slope 1 and slope 2 can be observed as d_{tip} changes. Qualitatively we would expect both slopes to decrease when rising d_{tip} - indeed no tip-induced conductance variation would be observed for a tip located at infinity. We are going to discuss both slopes separately. We observed that slope 1 remains unchanged no matter the value of d_{tip} between 50 and 140 nm (we can compare Figs. 5.3c and 5.9c). This is not surprising since the QDs leading to CB-related variations are located roughly at the center of the concentric fringes observed in Fig. 5.4f - *i.e.* a

few hundreds of nanometers away from the center of the central channel. So, the charging events are almost insensitive to a change of 90 nm in d_{tip} and slope 1 is the same in Fig. 5.9c. On the contrary, as presented in Fig. 5.9c, slope 2 decreases when d_{tip} goes from 50 to 140 nm. A smaller slope means a higher V_{tip} period at constant V_{BG} . This observation is consistent with Fig. 5.9b, where the iso-order resonance domains are going away from each other as d_{tip} increases. Also, slope 2 rotates around a particular point identified in Fig. 5.9c. Qualitatively, when the tip is brought away, one should apply a stronger tip potential in order to reach an equivalent resonant order. The domains located along slope 2 associated to the same resonance order should be rotating around a point where the tip is neutral with respect to the resonant pattern. It is possible to identify this particular point by checking where two equivalent domains for different d_{tip} are crossing. As expected, this point is located near $V_{tip} = 0V$.

At the end of this discussion, we can now come back to the interpretation of the temperature dependence shown in Fig. 5.2. One question indeed naturally arises if we consider UCFs on one side, and the Fabry-Perot resonance mechanism that we identified in the central branch on the other side : since both phenomena stem from electron interferences, why do they exhibit different temperature dependences ? To answer the question, another time scale should be considered, in addition to the phase coherence time : the electron dwell time [178],[179]. The electron dwell time in the central branch constituting a 1D cavity τ_d^{1D} is expected to be small compared to the dwell time in the whole device $\tau_d^{network}$. This has consequences on the temperature dependence of coherent oscillations. Indeed, as temperature decreases, the phase coherence time increases and a crossover is expected when it becomes larger than the dwell time : below this crossover temperature - typically in the range 1-5 K for cavities similar to the ones considered here [174] - the amplitude of coherent oscillations saturates. One can then understand the difference shown in Figs. 5.2a-b in the temperature dependence of the anomalous phenomenon associated to the 1D cavity characterized by a relatively small τ_d^{1D} , and of UCFs, related to interferences between semiclassical paths exploring the full device area (much larger than the 1D cavity). Figs. 5.2a-b show that UCFs dominates at 50 mK, but undergo a fast decay when temperature rises so that the anomalous conductance phenomenon becomes visible at 4.2K. One can therefore infer that the amplitude of the latter phenomenon saturates below 4.2K, because $\tau_\phi > \tau_d^{1D}$, while the UCFs amplitude increases below 4K, because $\tau_\phi < \tau_d^{network}$.

5.3 Conclusion

We identified two mechanisms at the origin of the paradoxical increase of conductance observed in a mesoscopic network while gradually depleting one channel : charging CB events associated to disorder-induced localized states and resonance mechanism due to the presence of a 1D cavity in the central channel. The first phenomenon has already been extensively studied in previous works, and in particular in Ref. 130 in the context of open nanodevices ($G > 2e^2/h$). Regarding the second mechanism, the electron wave resonance raises several questions and prospects. The formation of a 1D cavity is already puzzling: what causes electron wave reflections? Unlike resonant tunneling diodes, we revealed resonant state signatures in an open coherent and ballistic system where no tunneling mechanism is expected. Further work should allow to discriminate between different causes of reflection inside the structure (*e.g.* the abrupt variation of the electrostatic confining potential at the entrance of the 1D channel). By comparison with other channels in the network, only the central one could lead to a symmetric resonant cavity. In the top and bottom channels, one of their sides gradually curves to form the right angle that connects them to the vertical channels, while the curvature of the etched sides is very sharp at the opening of the central channel. Is this the only reason why a resonance is only present in the central branch? We also stress out that the two mechanisms described in the current work are intrinsically different from the one evidenced in the simulations results from Ref. 13, where the anomalous behavior was observed on the verge of full depletion of the central channel.



6. Two-dimensional Rutherford-like scattering in ballistic nanodevices

6.1 Introduction

Controlling collisions and scattering has always played an essential role in physics. Thanks to model experiments ranging from collisions of alpha particles with gold foils, conducted more than a century ago [180, 181], to high energy collisions between hadrons at the LHC [182], a wealth of intimate information were revealed about the nature of atoms and elementary particles as well as their interactions. In this framework, the most fundamental description of the interaction of a beam of particles and a scatterer is the famous Rutherford formula, describing the differential cross section dependence on the scattering angle, energy of incident beam, and potential shape of the scatterer [143]. Collisions are also ubiquitous in solid state physics, in particular when considering charge transport. Charge carriers indeed scatter on a large variety of "defects": lattice vacancies, phonons, potential of remote ionized impurities, etc. Due to this complexity, it is almost impossible to reach the same degree of control in charge transport scattering experiments as in the case of collisions involving beams of elementary charged particles propagating in vacuum.

However, in the ballistic regime of charge transport, the bulk carrier mean free path becomes larger than the device size, and transport properties can be tailored by tuning the device geometry [29, 183]. This is of course achieved most favorably in nanodevices, which are probably the most adequate system to

attempt to perform "ideal" scattering experiments with electrons in solids and their associated quasiparticles [184]. Nevertheless, even in the ballistic regime, a full treatment of scattering in solid-state devices requires to take into account complex many-body interactions with the Fermi sea [185, 186]. However, through all this chapter, no electron-electron interactions are taken into account and a single electron picture is sufficient to reproduce the simulation data that we will present.

The archetypal ballistic device is the so-called quantum point contact (QPC). Thanks to a metallic split gate deposited on top of a semiconductor heterostructure hosting a high mobility two-dimensional electron gas (2DEG), one can create a constriction whose width can be varied at will with gate voltage. The smooth resulting potential ensures adiabaticity, which leads to a quantized conductance of the QPC [187, 188]. This canonical realization of ballistic transport allowed to go one step further, in particular when combining transport measurements with a local electrostatic perturbation by a scanning probe. This method lead to explore deviations from this perfect picture of QPCs, such as the observation of branched electron flow in the leads or rich many-body physics [81–83, 86, 189]. In other studies, geometric scatterers with an asymmetric shape were designed to act as mirrors redirecting electrons towards a particular lead through specular reflection [190], leading to a rectifying behavior similar to diode bridges. Such devices could yield applications at high frequency, given the short electron transit time in the ballistic regime [191, 192]. In addition, the magnetic field is a particularly useful knob to focus electrons at desired locations through the so-called "magnetic focusing" effect [89, 90]. In a surprising way, up to our knowledge, there are much less examples where fine tuning of the electrostatic potential is used for similar lensing purposes [95].

Here, we study the geometry presented in Fig. 6.1 where specular electron reflection on the hard-wall facing the entrance of a quantum ring is either enhanced or reduced by tailoring the local electrostatic potential in the vicinity of the wall. The idea is that a Rutherford-like scattering effect - induced by an attractive/repulsive potential - should deflect electron trajectories and hence ease or unease electron injection in the QR arms. The injection of electrons has been shown to be critical in mesoscopic devices [183, 193] and our results provide a direct confirmation that even small changes in the electrostatic potential at a specific location in the device have strong impacts on ballistic charge transmission, and hence on the device conductance. Experiments fully reproduce the simulated behavior by applying positive or negative potentials on

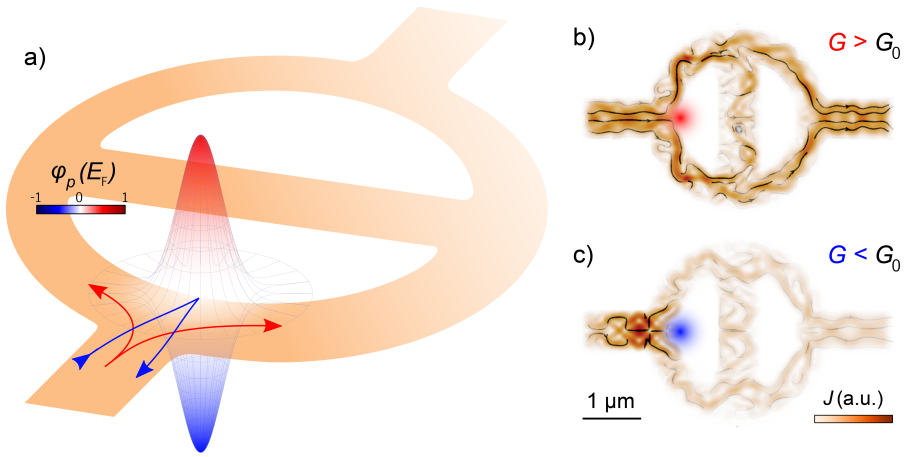


Figure 6.1: a) Illustration of the ring-like geometry (orange, not on scale), with the superimposed potentials used to tailor the potential landscape next to the edge of the ring antidot. b) and c) Tight binding simulations of the current density modulus (J) associated to the first electronic mode and iso-current density lines (black with arrows) for depletion (red potential) or accumulation (blue potential), respectively. G_0 is the total conductance without any perturbation potential.

a scanning metallic tip positioned over the hard-wall. Counter-intuitively, the highest conductance is observed for a depleting tip potential, and vice versa.

6.2 Results and discussion

Quantum transport simulation results were obtained using the KWANT package [25] for the ring-like geometry depicted in Fig. 6.1, where device boundaries are defined by infinitely sharp hard-walls (for details about the simulations, see appendix C). We focus here on the two T-shaped junctions located next to its leads, as this is where ballistic trajectories will be tuned. Note that the central branch connecting the two circular arms plays no role in this work.

The colored regions in Fig. 6.1a correspond to either raised (red) or lowered (blue) potential with respect to the otherwise flat background potential (disorder will be introduced later in the paper). This color convention will be followed all along the paper: red meaning depleting perturbation (raised potential) and blue meaning accumulating perturbation (lowered potential). Figs. 6.1b and c, corresponding to the simulated current density distribution (associated to the first electronic mode) in the potential landscapes of Fig. 6.1a, visually illustrate the impact of reversing the added potential experienced by electrons impinging on the T-junction. In Fig. 6.1b, the current injected through the left contact is favorably redirected towards the lateral branches of the device. In contrast, Fig. 6.1c reveals that current lines are focused on the hard-wall, which enhances reflection back to the entrance lead.

At this point it seems that current redirection might yield a strong signature in the device conductance G which may look counter-intuitive at first sight : simulations indeed predict that a repulsive perturbation should increase G while an attractive potential should degrade it. Furthermore, one can wonder how sensitive is this peculiar focusing/defocusing behavior with respect to the amplitude, spatial extension and location of the introduced potential perturbation presented in Fig. 6.1, as well as to the disorder in the background potential. The effect of all these parameters will be simulated in detail later in the paper where transmission through the device - converted in conductance - will be computed. In addition, it is tempting to test these predictions by measuring the conductance of a real-world device.

We thus carved out a ring-like structure from an InGaAs/InAlAs heterostructure hosting a 2DEG. The device geometry shown in Fig. 6.2a is lithographically very comparable to the one simulated above. The heterostructure is the one

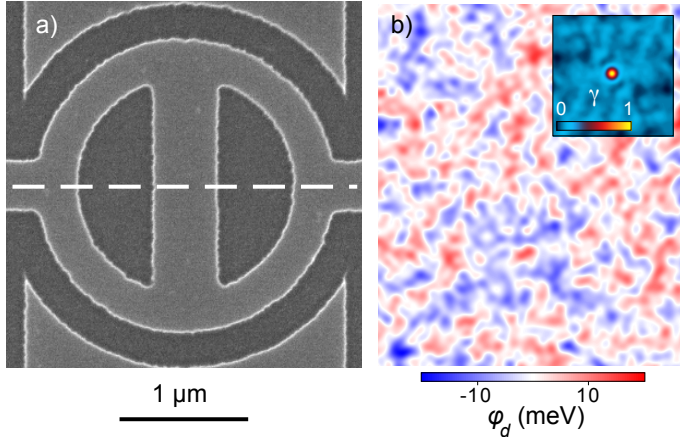


Figure 6.2: a) Scanning electron micrograph of the fabricated sample in an InGaAs/InAlAs heterostructure. b) Computed real-space disorder potential (ϕ_d) at the level of the 2DEG that will be used in the forthcoming simulations. The disorder standard deviation (S_d) is 4.78 meV, calculated taking into account a distribution of Si ionized dopants located 20 nm above the 2DEG (*i.e.* thickness of the InAlAs spacer). The inset to b) shows a map of the autocorrelation as the correlation lag becomes a vector in the $x-y$ plane.

presented in chapter 2 (see Fig. 2.2a). The 2DEG density and mobility can be tuned thanks to an applied electrostatic back-gate potential (V_{BG}). The following data were measured at the maximal accessible charge carrier density ($\sim 10^{16} \text{ m}^{-2}$) and mobility ($\sim 10 \text{ m}^2/\text{Vs}$) corresponding to $V_{BG} = 4 \text{ V}$. The Fermi energy is thus $E_F = 55 \text{ meV}$ and the Fermi wavelength is $\lambda_F = 25 \text{ nm}$. The 4-contacts conductance measurements were performed at a temperature $T = 40 \text{ mK}$ using a standard lock-in technique with a bias that remained comparable to $\frac{k_B T}{e}$. The physical characteristics of the host heterostructure allowed the modeling of a fixed disorder potential represented in Fig. 6.2b that will be used in the forthcoming simulations.

Experimentally, a convenient way to generate the kind of perturbation potential used in the simulations presented above is by approaching an electrically biased nanoscale tip (V_{tip}) at a distance d_{tip} above the patterned quantum ring (as illustrated in Fig. 6.3a). The tip can then be scanned along the transport direction, *i.e.* along the dashed line in Fig. 6.2b. In order to achieve a large effect, we brought the tip to a distance $d_{tip} = 60 \text{ nm}$ above the sample surface,

and biased the tip with large positive and negative voltages up to $|V_{tip}| = 14$ V.

The presence of the biased conductive AFM tip is numerically modeled using a Lorentzian-shaped perturbation potential $\phi_p(x, y)$ - illustrated in Fig. 6.1a - parametrized by the position of its center (x_{tip}, y_{tip}) , height ϕ_p^{max} , and width R_p which is half the potential FWHM. The superposition of $\phi_p(x, y)$ on the modeled disordered potential ϕ_d , together with the hard-wall boundaries that mark the edges of the nanodevice, define the potential landscape used in the simulations. The disorder potential ϕ_d models the influence of randomly located singly ionised silicon dopants located in a plane 20 nm away from the InGaAs 2DES. Screening of the dopant potential by the 2DES is taken into account via the Thomas-Fermi model (where we consider Lindhard's dielectric function). The value of disorder is estimated in the plane where the wavefunction for the ground state along the confinement direction (z axis) takes its average value. This wavefunction is solved within the Fang-Howard variational approach (which was found to differ only marginally from a 3D self-consistent Poisson-Schrödinger simulation).

In Fig. 6.3b, the conductance is computed as ϕ_p moves along the axis joining the entrance and the exit contacts (dashed line in Figs. 6.2a and 6.3a). Remarkably, when the tip position stands nearby the location of the hard-wall (vertical dashed lines), the conductance significantly deviates from that in the absence of perturbation ($\sim 11 \times \frac{2e^2}{h}$), e.g. when the tip stands at the center of the device. Beyond fluctuations originating from the presence of the random disorder, the effect is symmetric as positioning the tip near both T-junctions gives the same result. In other words, the Lorentzian potential has a similar effect on conductance when it modifies either the entry or the exit conditions. More importantly, this behavior is somewhat counter-intuitive: while a repulsive potential close to both T-junctions actually helps electrons crossing the overall structure (enhanced conductance), an attractive perturbation reduces their ability to pass through the device.

Looking further in the simulation results, we observe that reversing the sign of ϕ_p essentially reverses the change in conductance. Surprisingly, the back-scattering to the leads, due to current focusing on the hard-wall potential of the antidot (described in Figs. 6.1a and c), is similar in amplitude to the enhanced transmission due to defocusing (Figs. 6.1a and b). On the other hand, we observe that the symmetry naturally breaks when the tip locates above the leads. In that case, while depleting the lead strongly reduces the conductance, accumulating electrons has naturally a much weaker effect. Finally, when

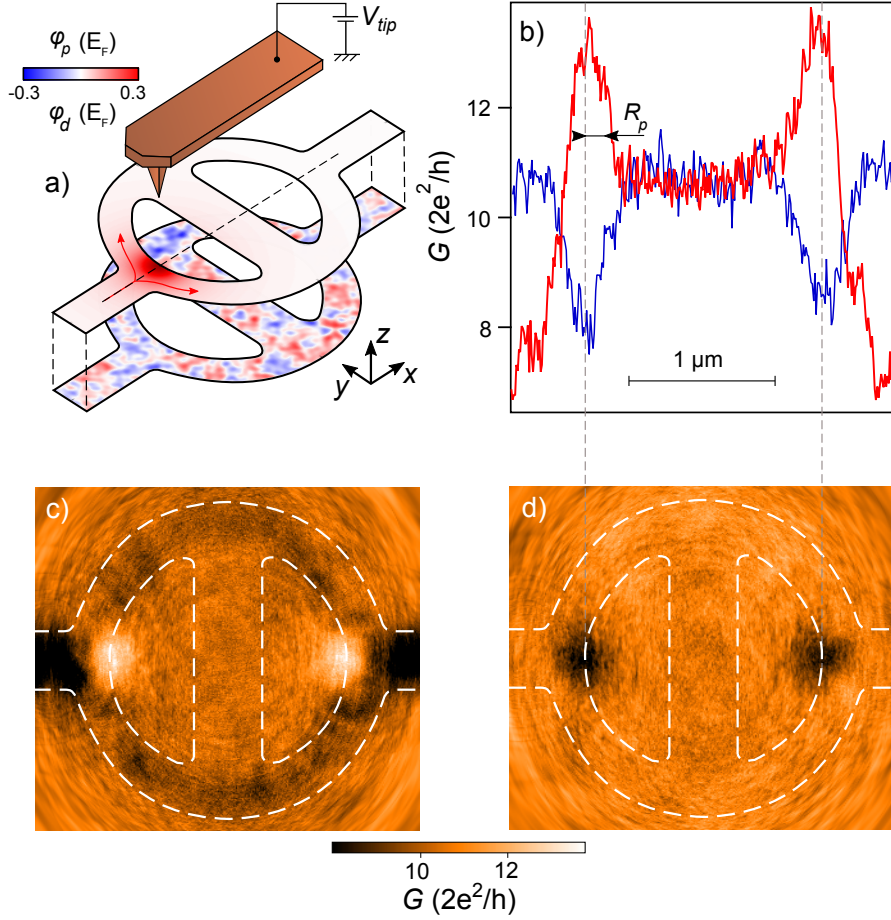


Figure 6.3: a) Illustration of the potential used for the simulations. It is composed of a disorder potential ϕ_d and a Lorentzian-shaped perturbation potential ϕ_p caused by a biased conductive AFM tip located above the 2DEG. b) Simulated conductance profiles as ϕ_p is swept along the dashed line in a). Simulation parameters are as follows (same conditions as in Fig. 6.1) : the red profile corresponds to $\phi_p^{\text{max}} = 0.9 E_F$ (depletion) and $R_p = 150 \text{ nm}$; the blue profile corresponds to a reversed perturbation potential (accumulation; $\phi_p^{\text{max}} = -0.9 E_F$). These profiles are extracted from the conductance mapping obtained when ϕ_p is swept in the (x, y) plane. They are presented in c) ($\phi_p > 0$) and d) ($\phi_p < 0$). The vertical dashed lines correspond to the locations of the hard-walls along the scanned line in a).

moving the perturbation from the T-junction area towards the center of the device, the effect on G naturally vanishes over a distance corresponding roughly to R_p (Fig. 6.3b).

Beside moving the tip along the device axis, one can also wonder how sensitive G -variations are to the perturbation position in the (x, y) plane. This aspect is examined in the G maps plotted in Figs. 6.3c and d obtained for locally raised or lowered moving potentials, respectively. The main contrast is observed over the T-junctions as well as over the device leads for a depleting potential. In both x and y directions, this contrast fades away over distances comparable to R_p . When positioning the perturbation potential over the device arms and their vicinities, the G map is decorated with short characteristic length scale fluctuations which are similar to those reported in previous works [96, 103, 194]. This weaker amplitude contrast was attributed to the perturbation of resonant states in the local density of states (LDOS) by the moving potential [96, 194], as well as to the electrostatic Aharonov-Bohm effect [103]. Note that here the mapping conditions are not suitable for imaging the LDOS because the moving potential is in the strong perturbation and not in the linear regime discussed in Ref. 194. In this framework we are not using the scanning gate with microscopy purposes in mind.

It is now time to compare these predictions with experimental results on the sample described above. Figure 6.4 summarizes the data in a way to ease the comparison with the simulations. We first scanned the biased tip along a line linking the device leads for two opposite sign polarities. Figure 6.4a shows, like simulations in Fig. 6.3b, that a depleting (red) potential, near the border of the inner quantum dot, eases electron injection, while an accumulating potential (blue) located at the same place tends to reduce electron transmission through the device.

For a strongly depleting potential ($V_{tip} = -14$ V - red curve in Fig. 6.4a), corresponding roughly to $\phi_p^{max} \sim 0.4 * E_F$ (see chapter 4 and more peculiarly Fig.4.8), G exhibits local maxima when the tip is located above the limit of the etched area in front of the entrance and exit leads (dashed lines in Fig. 6.4a). As expected, the conductance is reduced all the more as the tip decreases the 2DEG density over the leads. But, counter-intuitively, a strongly accumulating potential ($V_{tip} = 8$ V) brings G to a minimum. Moreover, the effect is essentially symmetric when the tip moves from one T-branch to the other. The qualitative match with the curves presented in Fig. 6.3b (obtained

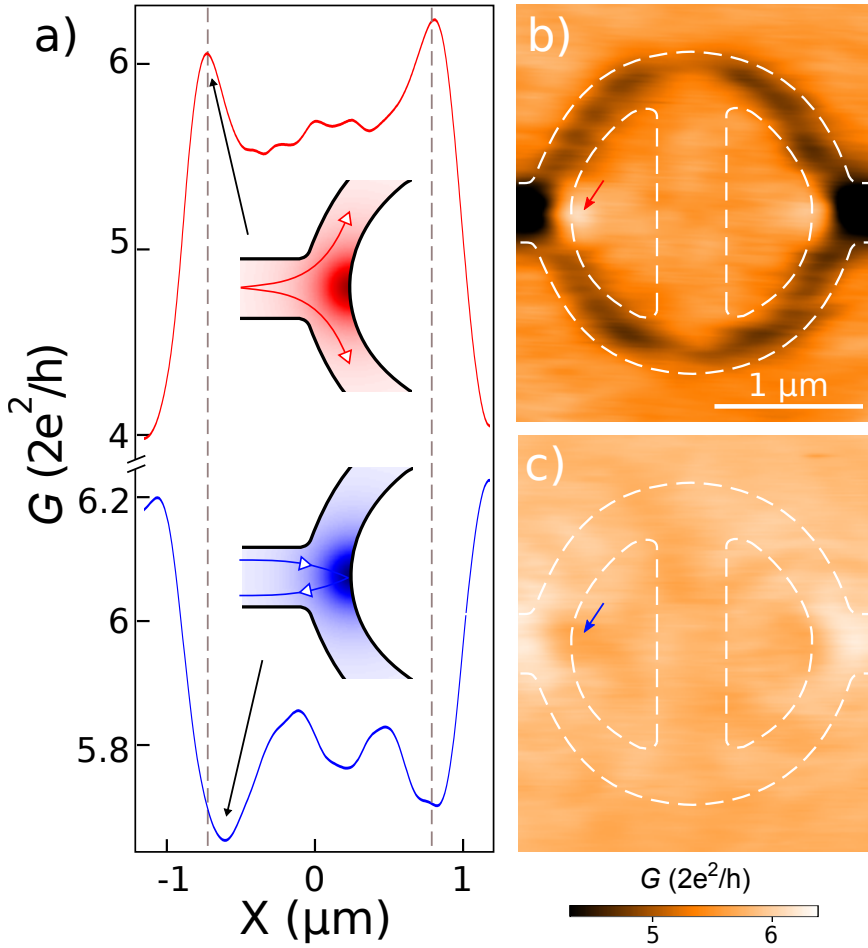


Figure 6.4: a) Experimental conductance profile as a voltage biased tip is scanned along the dashed line presented in Fig. 6.2a. The tip is scanned at a distance of 60 nm from the sample surface with $V_{tip} = -14$ V (red curve) or $+8$ V (blue curve). A qualitative scenario is also illustrated for the peculiar electron forth-scattering (red) and back-scattering (blue). b) Conductance map as the polarised tip ($V_{tip} = -14$ V - depletion) is scanned in a plane at the same constant distance from the sample surface. c) Same map as the one presented in b) but with $V_{tip} = +8$ V (accumulation).

for $\phi_p^{max} = \pm 0.9 * E_F$) is very good ¹, and the experimental conductance maps presented in Figs. 6.4c and d compare well with the simulations presented in Figs. 6.3c and d. We observe a remarkable coincidence of simulated and experimental positions and lateral extensions of the peaks and dips located around the hard-walls in the T-junctions. Note also that, in addition to the main dips and peaks discussed above, some fluctuations are visible in the experimental data in Fig. 6.4a, in particular when moving the tip towards the center of the device. Such fluctuations are reproducible, and associated with Universal Conductance Fluctuations (UCFs), stemming from tip-induced changes in the potential causing phase and momentum changes for charge carriers at the local scale, thereby affecting charge carrier interferences. The amplitude of similar fluctuations is larger in the simulations results shown in Fig. 6.3b, as temperature in the simulations is 0 K, so that thermal smearing and decoherence are not taken into account.

Resonant features along the ring circumference are also observed in all cases, but the smallest ones that are visible in simulated G maps in Figs. 6.3c and d, in particular those with concentric shape observed mostly outside the device area, are absent in the experimental data. This is most probably related to thermal averaging, which is not taken into account in the simulations.

At this stage, we can conclude that the experiments confirm, at least qualitatively, that a focusing/defocusing can be induced by a Lorentzian perturbation combined to a hard-wall potential in a ballistic device. While defocusing (Fig. 6.4a red) is clearly reminiscent of the Rutherford scattering - here in 2D -, focusing on the hard-wall induces a peculiar back-scattering mechanism as the lensing is combined with the specular reflection illustrated in Fig. 6.4a (blue).

At first sight, the weaker absolute value of the voltage applied on the tip in accumulation (blue in Fig. 6.4a) could explain why the effect on the conductance is weaker than in depletion (red in Fig. 6.4a). However, we need to dig deeper in the simulations to test the quantitative correspondance between experiments and predictions.

¹However, a significant increase of conductance is observed in the experimental data when the accumulating tip stands above the entrance leads, which contrast with the behaviour observed in the simulated conductance (Fig. 6.3b), where no increase is found in the lead region. This can be explained by the different geometries in both cases : the simulated device has semi-infinite leads, without edge roughness, while the actual device has 400 nm-long leads, with some edge roughness due to chemical etching (on the scale 10-20 nm). An accumulating tip can therefore enhance charge carrier injection in the actual device, and can bring electrons away from the rough edges, and therefore cause the device conductance to rise.

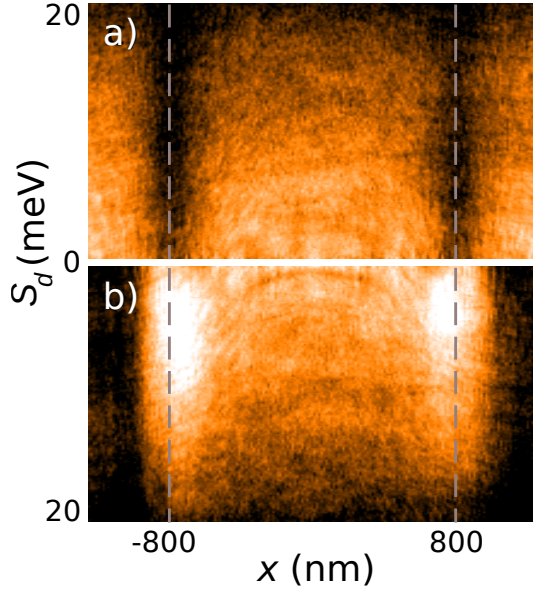


Figure 6.5: Simulated conductance profiles as the potential perturbation is swept along the black dashed line in Fig. 6.2a, as a function of S_d , with $R_p = 150$ nm and a) $\varphi_p^{max} = -0.9 E_F$ b) $\varphi_p^{max} = +0.9 E_F$

Figure 6.6 shows the evolution of the conductance when φ_p travels along the axis of the quantum ring, and when either φ_p^{max} or R_p is varied, the other parameters remaining constant (a similar map with a varying disorder amplitude S_d is shown in Fig. 6.5 - the effect robustness with respect to S_d is also confirmed).

We obviously focus our attention on the two regions near the edge of the inner QR, i.e. $x \sim \pm 800$ nm (dashed lines in Figs. 6.6a and b). We first observe no obvious threshold when $|\varphi_p^{max}|$ increases (Fig. 6.6a). G undergoes a smooth evolution at least up to $2 * E_F$. However, on the depletion side ($\varphi_p > 0$), the positions of the local G maxima are gradually shifting towards the center of the device as φ_p^{max} is made more positive. This reflects the fact that roughly identical potential perturbation conditions are found in the T-junctions both for a weakly perturbing potential ($\varphi_p^{max} < E_F$) centered close to the hard-wall, and a strongly perturbing potential ($\varphi_p^{max} > E_F$) centered further away from the hard-wall. On the accumulation side ($\varphi_p < 0$), the position of the dips' centers remains essentially unaffected : charge accumulation in the T-junctions does

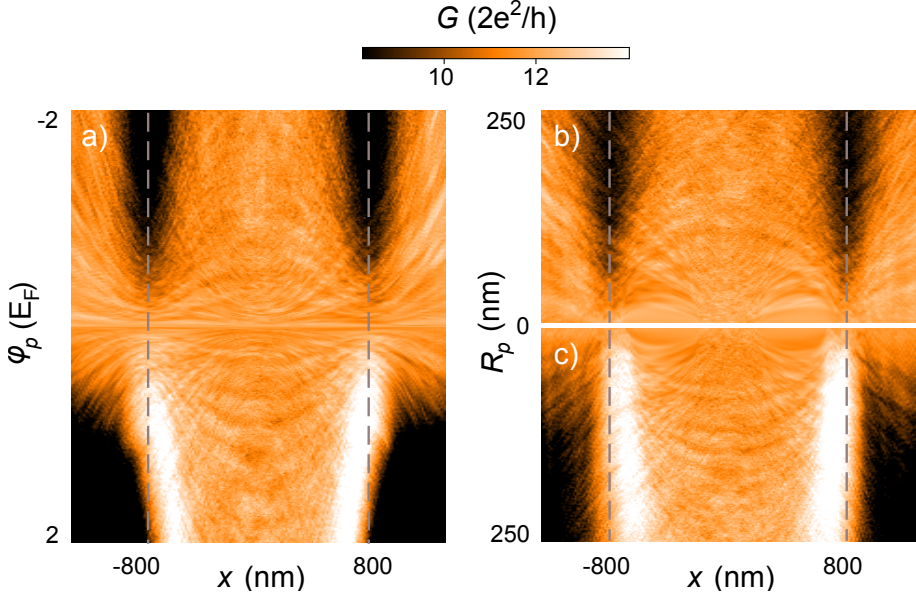


Figure 6.6: Simulated conductance profiles as the potential perturbation is swept along the black dashed line in Fig. 6.3a for several values of : (a) ϕ_p^{max} with $R_p=150$ nm and disorder strength $S_d=4.78$ meV, (b) R_p with $\phi_p^{max}=-0.9 E_F$ and $S_d=4.78$ meV, (c) R_p with $\phi_p^{max}=0.9 E_F$ and $S_d=4.78$ meV. The vertical dashed lines correspond to the locations of the hard-walls along the scanned line.

not modify its geometry.

Varying R_p has an interesting effect on the conductance peaks and dips. Beyond a few tens of nm, and up to 200 nm where the arms themselves start to be narrowed, varying R_p has essentially no effect on the amplitude of conductance extrema, either for negative (Fig. 6.6b) or positive (Fig. 6.6c) perturbation potentials. Indeed, the amplitude of conductance peaks and dips saturates for $R_p \geq \lambda_F = 25$ nm, i.e. in the classical regime. In such situation we expect that the classical limit is valid for $\lambda_F \leq R_p$ (see Fig. 6.7).

On the other hand, the evolution of the width of conductance extrema (Figs. 6.6b and c) is smoother and gives us the possibility to determine the value of R_p^{exp} that characterizes our experimental configuration. Based on the FWHM of the strongest (red) conductance peaks in Fig. 6.4a, we obtain that $R_p^{exp} \sim 135$ nm. This value is well in the range investigated in the simulations and indeed

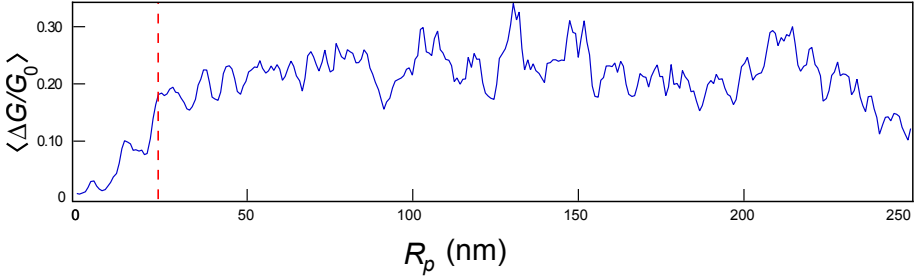


Figure 6.7: Average of two profiles of $\Delta G/G_0$, measured along the two vertical dashed lines in Figs. 6.6b and c, as R_p is varied. The red vertical dashed line corresponds to the Fermi wavelength.

consistent with data discussed in chapter 4.

Finally, our results show that increasing the disorder dampens the effect but no qualitative change is observed even when multiplying the initial disorder (Fig. 6.2b) by a factor of four (see Fig. 6.5). This robustness is a clear signature that distinguishes the present effect from universal conductance fluctuations [163], even sensitive to a change of potential amplitude on a single tight-binding site.

To go beyond the good qualitative correspondance between Figs. 6.3 and 6.4, we now need to question the experimental data more quantitatively. This is the purpose of Fig. 6.8 that addresses the effect of the density, or Fermi energy, and finally provides a quantitative comparison between experiments and simulations.

The simulated variation of the conductance with E_F , while keeping constant the absolute value of the ratio $\frac{\varphi_p^{max}}{E_F} = 0.9$, is presented in Figs. 6.8a (for depleting tip-induced potential) and 6b (for accumulating tip-induced potential). Since G increases with E_F , it makes sense to examine the relative change of conductance $\Delta G/G_0 = (G - G_0)/G_0$, where G_0 is the conductance of the device when the tip is above the device center ($x = 0$). It is immediately apparent that the simulated $\Delta G/G_0$ is insensitive to E_F provided that the ratio φ_p^{max}/E_F is kept constant at 0.9. In other words, the efficiencies of both focusing and defocusing are not sensitive to E_F alone, but, as Fig. 6.8c reveals clearly, to the ratio $\frac{\varphi_p^{max}}{E_F}$ (see Fig. 6.9).

More precisely, Fig. 6.8c shows a linear dependence of $\Delta G/G_0$ as a function of $\frac{\varphi_p^{max}}{E_F}$ up to $\frac{\varphi_p^{max}}{E_F} \simeq 1$ in the depletion regime. Noteworthy, the linear behaviour

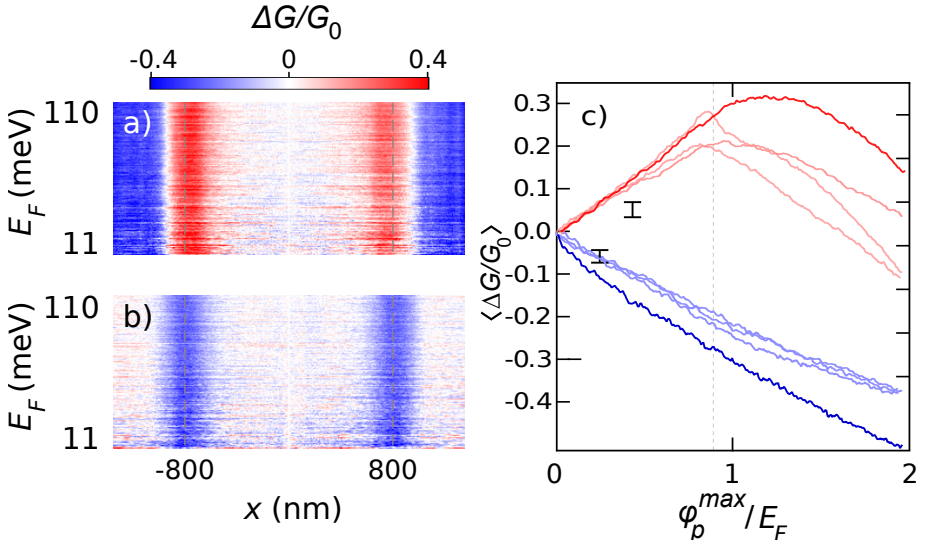


Figure 6.8: a) and b) Relative variation of conductance ($\Delta G/G_0$) with respect to that at $x = 0$ for $\frac{|\phi_p^{max}|}{E_F}$ fixed at 0.9 for all E_F in the presented range. a) is a map realised for a depleting tip-induced potential and b) for an accumulating tip. c) Relative conductance variation - averaged over the range $11 \text{ meV} < E_F < 110 \text{ meV}$ - at $x = -800 \text{ nm}$ (red/blue for depletion/accumulation). These curves are obtained for different disorder configurations (keeping the same disorder standard deviation). The dark red/blue curves correspond to the disorder configuration that was used to generate all the simulated data presented above and the light red/blue curves correspond to different disorder configurations. The two experimental data points are indicated in c) in the form of two vertical bars.

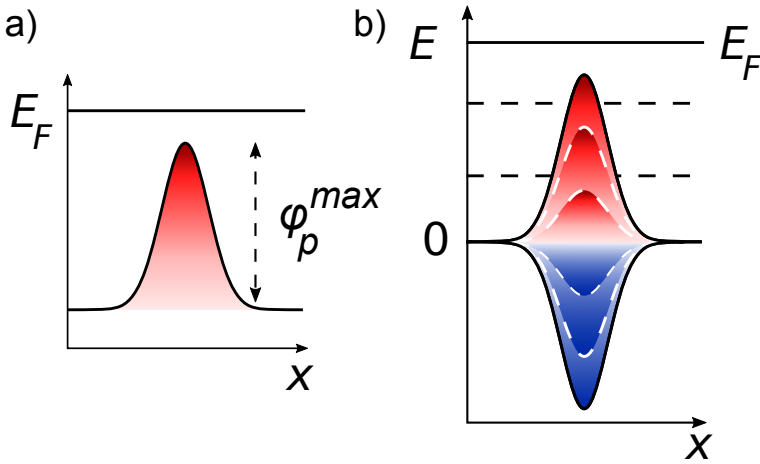


Figure 6.9: a) In a 2DES, this ratio is between the tip-induced potential ϕ_p and the Fermi energy E_F . b) For a given $\frac{|\phi_p|}{E_F}$ ratio, the relative increase of conductance is constant as computed in Fig.6.8c.

is only weakly sensitive to disorder in this regime. For $\frac{\phi_p^{max}}{E_F} \geq 1$, defocusing by a depleting tip becomes less efficient as the arms' width start to shrink, and the overall behaviour is much more sensitive to the precise disorder configuration. No such deviation from linearity is observed in the case of accumulation (blue lines in Fig. 6.8c). The counter-intuitive entry/exit symmetry persists in all the range investigated and the linearity with respect to $\frac{\phi_p^{max}}{E_F}$ is preserved.

How can we understand this linear dependence, at least in the depletion regime? The defocusing of ballistic electrons facing a Lorentzian-shape repulsive potential is clearly reminiscent of the Rutherford scattering. The original Rutherford formalism provides an expression for the differential cross-section in three dimensions (3D) for a scattering potential $\frac{C}{r}$ - where C is the amplitude and r the distance from the scattering center - as a function of the energy of incident particles E and of the scattering angle θ . Since the arms of the quantum ring capture electrons in a finite angle range from the leads, one can consider the differential cross-section at a given angle as related to the conductance of our ballistic device. Coincidentally, in the 3D case, the Rutherford formula is independent of whether you treat particles classically or quantumly [143]. In 2D, this elegant result is no longer valid in general. In the 2D quantum regime, one has to find an analytical expression of the differential cross-section ($\frac{d\lambda}{d\theta}$)

by solving the 2D version of the Lippmann-Schwinger equation [144] with a Lorentzian-shaped potential distribution. This has been done numerically in chapter 4 but no analytical formula prescribe a linear dependency (which is characteristic of the specific transport regime detailed in this chapter).

In the 2D classical regime however, an equivalent formula was derived [143, 195]. For the same $\frac{C}{r}$ potential :

$$\frac{d\lambda}{d\theta} = \frac{|C|}{4E\sin^2(\theta/2)} \quad (6.1)$$

One readily finds from Equ. (6.1) that, for a given angle θ , the scattering amplitude is fully determined by the ratio between the amplitude C of the perturbative potential and the energy of the particles. In the case of our device, this ratio would correspond to $\frac{\phi_p^{max}}{E_F}$. The linear response of the simulated $\Delta G/G_0$ with respect to changes in $\frac{\phi_p^{max}}{E_F}$ revealed in Fig. 6.8c is thus reminiscent of the 2D Rutherford scattering in the classical regime (note that Eq. (6.1) is also valid in the accumulation regime, but in our QR geometry, specular reflection on the hard-wall must also be taken into account). Beyond Eq. (6.1) that is probably not strictly applicable to our Lorentzian-shape potential, the Rutherford analogy helps visualizing the observed ballistic defocusing.

We finally turn to what is probably the most important information presented in Fig. 6.8c : the quantitative comparison between experiments and simulations. To reach that point, we first need to evaluate the amplitude of the perturbation potential induced by the tip. A direct view of the shape of the tip-induced potential experienced by electrons inside the device is obtained by mapping the conductance of a narrow channel in a similar device (whose width is comparable to the leads of the device) close to pinch-off as a function of the electron density, with the tip scanning along a line perpendicular to the channel axis (see Fig. 4.8). Following this procedure, we determined that $\phi_p^{max} = 3.9$ meV for $V_{tip} = -4$ V and $d_{tip} = 80$ nm, and scaled this value taking into account the parameters used in Fig. 6.3a. Knowing the values of ϕ_p^{max} for both the depletion and accumulation potentials data in Fig. 6.4a, we were able to plot the experimental $\Delta G/G_0$ vs $\frac{\phi_p^{max}}{E_F}$ in Fig. 6.8c, for two different values of ϕ_p^{max}/E_F . The good agreement between experiments and simulations reveals the global consistency of our study and that it is indeed possible to strongly enhance or reduce the injection of ballistic electrons in a ballistic device by tuning the shape of the potential faced by electrons. It also means that the simple tight-binding model used here captures the essential physics of the phenomena. In the

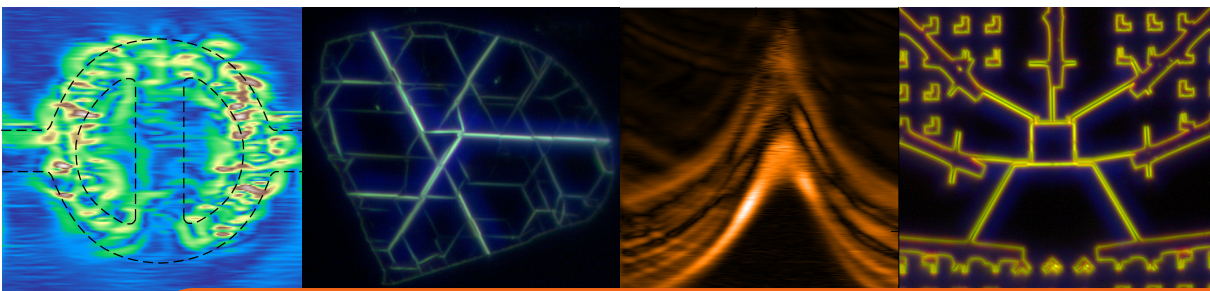
experiment, a conductance change of up to $\sim 10\%$ relative to the unperturbed device conductance is observed, which is relatively important, compared to e.g. coherent effects at this temperature (40 mK). The phenomenon seems also particularly robust with respect to disorder. This may seem surprising at first sight if its origin is a "ballistic redirecting effect" induced by the tip potential. However, high contrast magnetic focusing effects were observed in semiconductor heterostructures with comparable or lower mobilities [39]. This common robustness in both cases further reinforces the idea that ballistic focusing is at the heart of the observed phenomenon.

6.3 Conclusion

In conclusion, we have evidenced surprising ballistic electron focusing and defocusing behaviors governed by a local electrostatic potential. The phenomenology is similar to the 2D Rutherford scattering assuming classical electron dynamics. The applicability of this relatively simple classical formalism in the case of a 2DEG-based device was not expected. Indeed, the scattering amplitude for the interaction between charged particles and a sharp electrostatic potential should in principle be governed by complex interactions related to the presence of the many-particle background of the Fermi sea [186]. Another unexpected result of this work resides in the unexpected symmetry revealed between the amplitude of the Rutherford defocusing effect (when a depleting potential is applied) and reflective focusing, as experienced by electrons scattered by an accumulating potential in front of a hard wall. All these puzzling fundamental questions will require additional scrutiny and will probably foster further experimental and theoretical work.

In a broader context, our observations help in the understanding of charge carrier injection in ballistic devices, as it shows that fine tuning of the potential in the vicinity of the entrance and exit leads can have huge effects on transmission through the whole device. In turn, this work provides useful tools in the perspective of building 'electron optics' devices, where a local modulation of the electrostatic potential inside a device redirects the electron flow in a similar way as an optical lens curves light rays [154]. In this framework, scanning gate microscopy can play an important role, as pointed out in various theoretical proposals where scattering is investigated by tuning the electrostatic potential at the local scale using a charged metallic tip [186, 196, 197]. Although the description of scattering in two spatial dimensions was considered as a

curiosity up to the early eighties [195], nowadays high mobility two-dimensional charge systems gives this fundamental question a complete relevance and the possibility of testing this description, even with relativistic Dirac particles [101, 102, 151, 198, 199], also opens new directions of research.



7. Conclusion and perspectives

To conclude, we would like to recall the initial motivation of this thesis : based on the work presented in Ref. 13, we started to transport phenomena present in nanoscale networks when modifying the electrostatic potential landscape at the local scale, and to unveil the underlying physics thanks to a combination of experiments and simulations. Doing so, we realised a first experiment on the network presented in chapter 5. The goal was to pinpoint experimental conditions where the electrical conductance rises when closing a transport channel [13], which would have been analogous to the Braess paradox - presented in chapter 1. Experimentally, we identified different experimental conditions yielding conductance variations that could be ascribed to the Braess paradox but their physical origin was demonstrated to be either related to a Coulomb blockade mechanism or to a Fabry-Pérot resonance in a unidimensional cavity hosted in a transport branch so no true paradox was observed in the data presented in chapter 5. Even though no new physics was at the origin of the conductance variations we discovered, this work allowed us clarify the mechanisms at play in such complex network devices.

In the second part of the thesis (see chapter 6), we studied an injection mechanism that we identified in another network geometry. Finely tuning the electrostatic potential in the vicinity of a hard-wall facing the injection lead resulted in enhancing or decreasing the overall conductance of the network, depending on whether we used a repulsive or an attractive perturbation poten-

tial. The peculiarity about this injection process was that a repulsive potential enhanced the value of the conductance and vice-versa. This injection mechanism oriented us towards the problem of ballistic scattering in two dimension and it turned out that the results presented in chapter 6 were analogous to the two-dimensional Rutherford scattering.

We have therefore been playing mainly with three types of phenomena (electron interference, electron ballistic scattering, and Coulomb blockade) in semiconductor-based heterostructures. The strength of scanning gate microscopy is in its capability to spatially isolate the origin of each macroscopically observed phenomenon. In this sense, the experiments on the tunability of 1D resonances inside "Braess networks" represent an interesting application of the SGM technique to disentangle a complex problem. Note, again, that the "local tuning" of the electrostatic landscape by the SGM tip was the critical lever arm in this work, and not the microscopy.

At the end of this work, it is necessary to take some distance from the results and examine which kind of future research directions could emerge from this work.

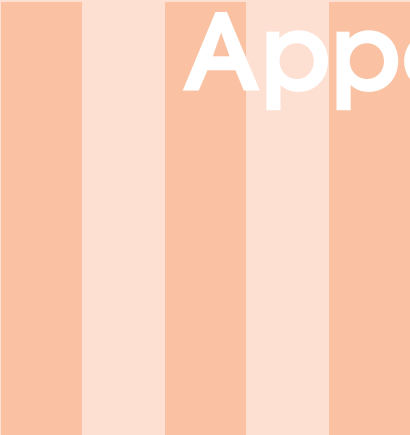
First, keeping the same type of 2D electron system, one could optimize the geometry to maximize the scattering effect. On one side this could lead to a more in-depth verification of the 2D Rutherford model, and on the other side, this could also yield quantitative data on the influence of the injection geometry on device characteristics, which could be valuable in the framework of device engineering (transistors, ballistic devices, etc.).

Second, the scattering center created by the tip can also be modified, either to examine and fully characterize different types of scattering (for example by a magnetic tip, or by a different shape of scattering potential).

Third, one could also try to reproduce the results presented in chapter 6 but in 2D electron systems with different scattering rules. For example, in single layer graphene, the Berry phase of π prohibits backscattering, while for bilayer graphene, the Berry phase of 2π prohibits rectangular deflection. These peculiarities clearly distinguish single and bilayer graphene from the heterostructure system we studied.

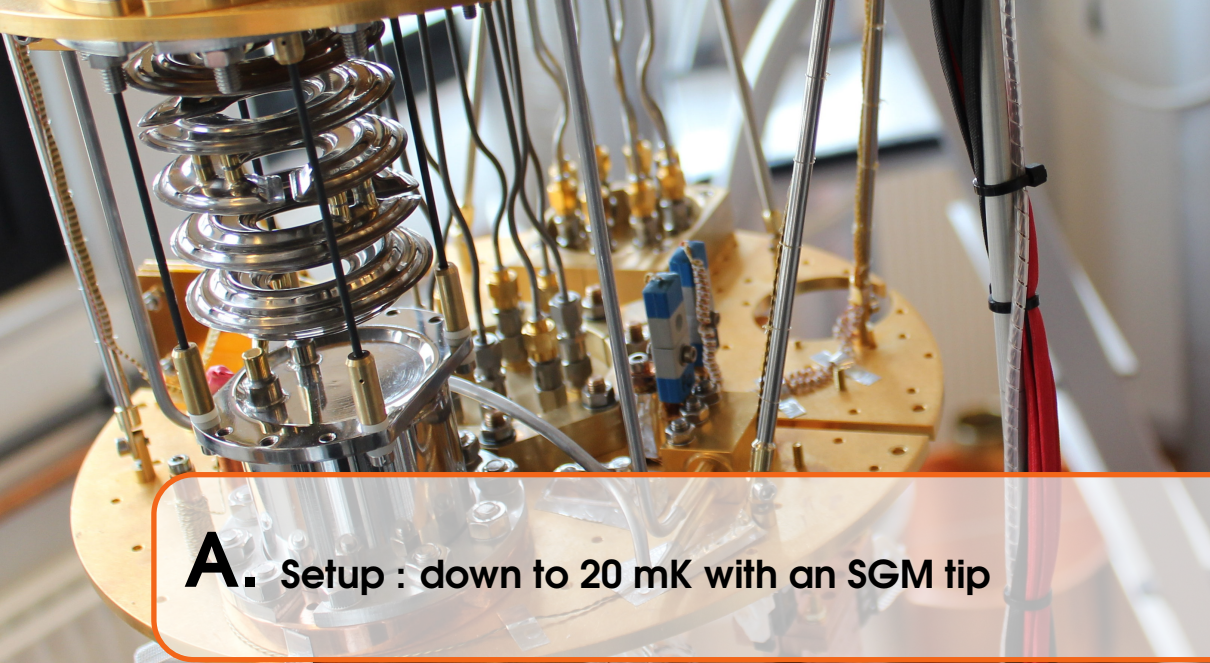
As a last perspective, we would like to point out that viscous electron flow has been reported in Refs. 200, 201 for encapsulated graphene measured between 100 and 200K. This regime of transport is characterized by a very short electron-electron interaction length, compared with the ballistic mean free path. In this case, a viscosity can be associated with the electron motion,

and hydrodynamic phenomena such as vorticity or the emergence of a gradient of electron velocity close to device edges should be observed. It would be interesting to tune of electron whirlpools present in the vicinity of the T-junction and see the deviations form the results presented in chapter 6. A systematic exploration of the effect of temperature would then allow to fully characterize transitions between regimes governed by Dirac, Rutherford and Navier-Stokes equations.



Appendices

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A. Setup : down to 20 mK with an SGM tip

This appendix aims to provide an overview of the general architecture of the experimental setup we used to perform a scanning gate experiment. A detailed description is presented in the following manuscripts [202, 203]. An exhaustive presentation of the cryogenic system and the electrical connections realised between the sample and the sources can be found in [203].

The system we worked on is referred in our group as the *fridge*. This generic name refers to different modules (cryogenic, low-noise measurements, precise positioning, gases management and data acquisition). The 'kernel' of the fridge is a He^3/He^4 dilution refrigerator (Kelvinox 300[®] - Oxford instruments) that was modified in-house in order to perform SGM measurements (see Fig. A.1a). The generic abilities of such system are : scanning gate microscopy measurements under high magnetic field (17 T) down to the bottom temperature of 20 mK.

This section presents lots of products named by their specific name. There is no advertising goal but rather to guide the reader towards equipments that are eventually presenting (in their manual) their *generic* capabilities. Also we found important to communicate about the technical environment of this thesis so that the reader could identify possible technical limitations in our specific implementation of the measurement setup.

A.1 Tip positioning control

An appendix (see Fig.A.1b) to the dilution refrigerator mixing chamber was designed to host motors and scanners to control 'roughly' and finely, respectively, the AFM tip position (see A.1c). Fig.A.1c, presents a stacking consisting a 3 attocube[®] motors (two ANPx101 for the x,y horizontal linear motions and one ANPz101 for the vertical linear motions - up to 5 mm). The same idea is followed for the scanners (ANSx100 and ANSz100) where controlled motions can be obtained in a $50 \times 50 \mu\text{m}^2$ window and motion up to $50\mu\text{m}$ along the z axis. The motors are controlled with an attocube voltage source generating ramp waveform signal up to 70V. This allows motor motion based on the stick/slip working principle [204]. The scanners are controlled through a Nanonis[®] module. Nanonis provides a convenient interface to control low noise, high voltage sources used for the scanners. At low temperature, voltages up to 40V were applied to the scanners. A scan control panel allows to record different channels (lock-in voltage, currents etc) in order to map them in real space. Storing all the data in a .sxm file. The scanner along the z-axis has the peculiarity to be coupled to an AFM-type feed-back loop (the 'z controller').

Indeed, an home made PCB board (see Fig.A.2a) is mounted on the motor/scanner stacking (shown in Fig.A.1c). This board is designed in order to embed the AFM tip/cantilever (see Fig.A.1e) itself mounted on a tuning fork (see Fig.A.1d). The AFM tip/cantilever is provided by μ -masch[®] and has the following characteristics. The system illustrated in Fig.A.1e is made out of silicon and both the cantilever and the tip are fully coated by two consecutive continuous films of titanium (1st layer of 15nm) and platinum (2nd layer of 10nm). The tip has a radius of curvature less than 30nm, a tip height between 20 and 25 μm , a full tip cone angle less than 30° , a typical resonance frequency and a force constant of about 75 kHz and 3.5 N/m, respectively. The quartz tuning fork (X32K768L104 - Crystal by AEL[®] - see Fig.A.1d) on which the tip is glued with the help of μ -manipulators (with EPO-TEK H31D by epoxy technology[®])(see Fig.A.2b), is glued on the board and its pins are soldered to electrically conductive pads (see Fig.A.2a). This epoxy is conductive and needs to be annealed about one hour at 150° to become solid. The different scale bars can guide the reader towards the smallest scale - nanometers. We want to make a little digression about the gluing of the cantilever on the tuning fork. Although the cantilever/tuning fork relative motion is controlled with micromanipulators, the epoxy droplet is deposited 'by hand'. The droplet characteristic dimensions have to be of a comparable size with respect to the cantilever dimensions (see

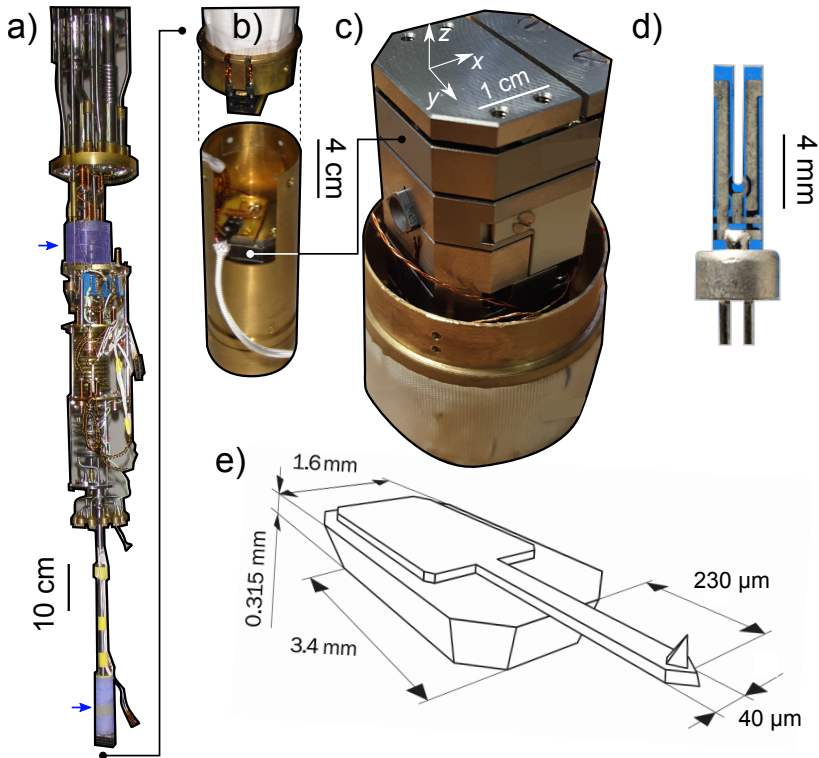


Figure A.1: This figure presents a comparison of the different scales at play in the measurement. a) Dilution refrigerator insert that we used for the measurements of this thesis, known as the "fridge". b) Lower part : Home-made atomic force microscope hosting the stack of motors/scanners with the tuning fork cantilever assembly on top. Upper part : mixing chamber of the dilution refrigerator with the sample anchored to the cold finger (the coldest place in the fridge - 40 mK). c) Piezoelectric motors/scanners stack used to control the tip location. d) Tuning fork typically used in the measurement upon which is pasted the AFM cantilever. e) Illustration of the AFM cantilever used in the SGM measurements. Only the cantilever (and not the main support) is pasted to the tuning fork.

Fig. A.1e). This 'human' connection with this rather small lengths makes you realize how actually small is the nanoscale. Hopefully, technological innovations allowing to control smaller and smaller objects keep getting developed, demonstrating the fascinating aspect of the infinitely (?) small. Let us now come back to the manipulations performed after the cantilever/tuning fork system is realized. When approaching the tip towards the surface, one needs to be able to stop the z-scanner once the tip apex "touches" the sample surface. To do this, as mentioned, the z-scanner is coupled to a feedback system preventing the tip from crashing. The tuning fork is indeed used as a force sensor. We start by applying an oscillatory voltage signal (with a precise frequency that can be swept using the nanonis oscillation panel) to the tuning fork (ΔV in Fig.A.2a). The amplitude of this signal at low temperature is typically few mV because the tuning fork is known to deform around 300 nm per volt (motions around 3 nm of amplitude for 10 mV). We then find the resonance frequency of the tip/tuning fork system. The free¹ tuning fork is expected to resonates around 2¹⁵ Hz but once tip is pasted, the tuning fork becomes unbalanced and this can shift the free resonant frequency. Moreover, a gold wire is glued in the same epoxy drop as the conductive cantilever (see the black arrows on Fig.A.2c and d). Also, the tuning fork cylindrical protective cap (grey cylinder on Fig.A.2a, is opened in order to glue the tip. Opening a tuning fork cap with a dremel tool and gluing a micrometric cantilever on this tuning fork are side features of SGM-related work that bring a lot of charm to this disciplines (with our original trial and errors method). As the tuning fork deforms under the sine stimulus, a current is generated with an amplitude (amplified by a current amplifier) and a phase difference with respect to the excitation frequency. At the resonance frequency, the current and voltage signals are phase-shifted by 90° and the current amplitude (hence deformation) is maximal - as presented with our toy model (see Fig.4.5c). The resonance frequency of a tuning fork is an extremely sensitive quantity allowing to collect informations on a wide variety of interactions but can also be a good indicator with respect to temperature and pressure. For example, once you stop pumping on a chamber, if you have shifts of the resonance frequency, the chamber is probably leaking. This is a valuable indirect indicator during an experiment.

As it is brought in the vicinity of the surface, surface interaction shifts the resonance frequency, that we can track using the Nanonis build-in feed-back loop on the resonance phase and amplitude. The "contact" criterion to stop the

¹in absence of surface in its vicinity

z-scanner is arbitrarily set when the frequency shift reaches a value of e.g. 300 mHz (we had a noise of typically 15 mHz rms). Once "contact" is reached we can measure a topographic image consisting in realising an *iso frequency-shift* real space mapping. A topography of the flat surface in the vicinity of the device allows to measure the two tilt angles of the surface around the x and y axis. Once this is done, we lift the tip by a few tens of nanometers to a home position before scanning in a 2D plane parallel to the surface. A topography is not necessary to locate the nanonetwork (as detailed in chapter 4 where "EFM"-like images are obtained to locate the device). Indeed as presented in Fig.4.5d, we can also map the frequency shift in the (x,y) plane as we scan at a constant height (no feed-back on the z positioner). This has the advantage of preserving the pristinity of the device because of the absence of contact between the tip and the device.

A.2 Electrical measurements

As the tip is scanned in the vicinity of the nano-device, its electrical conductance (G) is measured using a standard low frequency *lock-in* [205] amplifier technique. Lock-in amplifiers are used to detect and measure very small AC signals, often in the presence of noise sources that can be orders of magnitudes larger. An output signal coming from the lock-in can be used to bias the device with a voltage AC bias of typically 40 μV and a current of a few tens of nano-ampere is measured. At twenty mK, the thermal energy corresponds to a broadening of the Fermi-Dirac statistics of 1.7 μeV . 40 μV is a good compromise between near-equilibrium conditions and a good signal to noise ratio. Regarding the frequency, the Drude model predicts a 2DES given by :

$$\sigma_{2DES}(\omega) = \frac{ne^2\tau}{m^*} \frac{1}{1 + i\omega\tau} \quad (A.1)$$

where τ is the transport scattering time discussed in equ.2.39. Typically, τ is of the order of few pico seconds meaning that sensitive effects could be obtained for Tera Hertz [206]. Anyway, the measurements are realised at low frequency to avoid frequency dependent capacitive/inductive effects associated to the coaxial lines and the different measurement equipments. The frequency is typically $f = 28.5$ Hz. All the electronic signals coming from the different equipments are read by voltmeters (Keithley 2000) and communicate their values to the central acquisition computer via a GPIB/ethernet controller. The

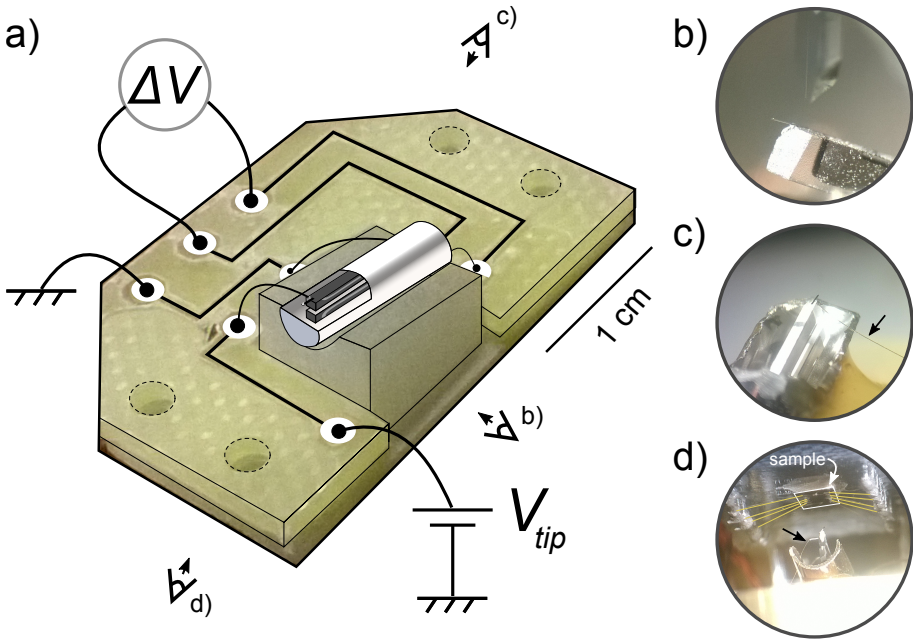
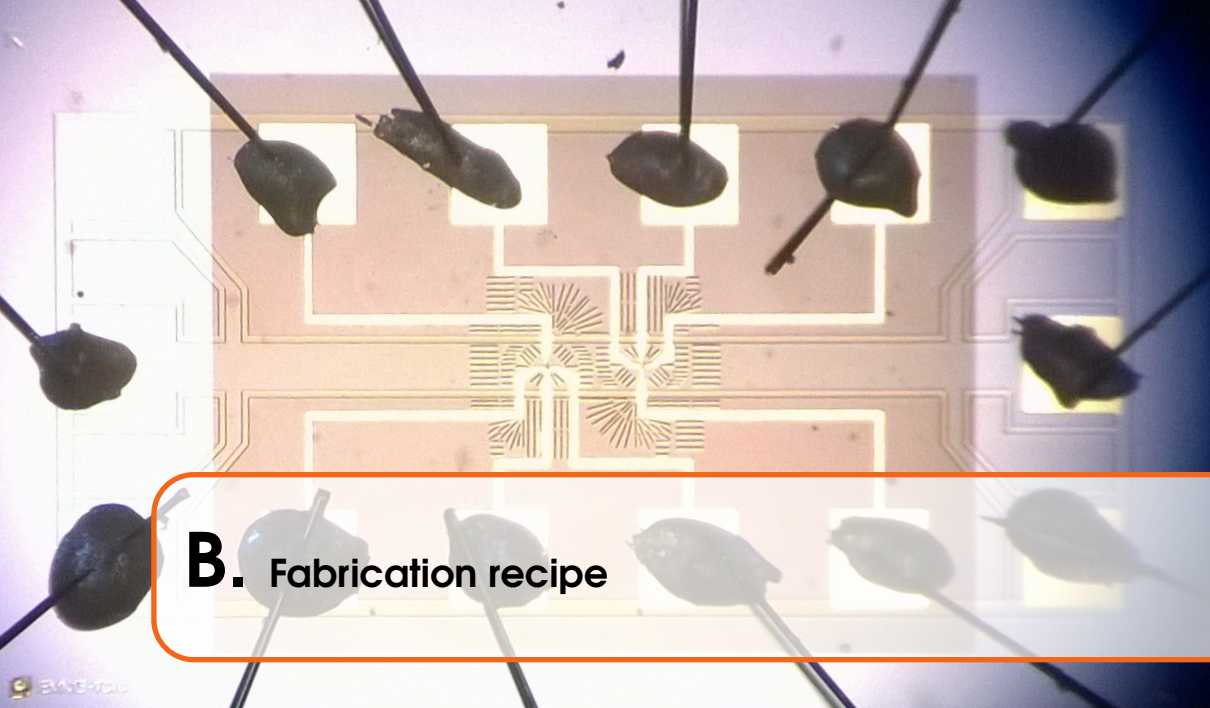


Figure A.2: a) SGM PCB board hosting the tuning fork where the tip cantilever is pasted. The black lines are conducting copper lines used to bias the tip (V_{tip}), ground the tuning fork cap and apply the oscillatory voltage (ΔV) responsible for the tuning fork periodic motion. Three different views are presented in b), c) and d). Each respective observation points are presented in a). The gold wire is showed with the black arrow in c). The tip approaching the sample is showed in d) where the tip/tuning fork is indicated with a black arrow.

different equipment used where : low noise lock-in amplifiers (Princeton applied research model 124a and a MFLI from Zurich instruments), low noise current amplifier (SRS model SR570). The different data where managed thanks to different Labview's virtual interfaces communicating via the virtual interface software architecture (VISA). SGM mappings were obtained via the 'scan control' module of Nanonis. Nevertheless, our reference format is '.txt' so we wrote a little python script (available upon request) converting the hexadecimal data of the selected channel of the .sxm file into a .txt file.

Cooling down the system

To go down to few milli-Kelvin with a scanning gate AFM tip, one follows a typical cascade cooling down with 3 key temperatures corresponding to 3 cryogenic fluids (77K for liquid nitrogen, 4.2K for liquid helium 4 and 40 mK for the He3/He4 mixture). Due to the erratic costs of helium 4 [207], the dilution refrigerator is coupled with a liquefier. Cryogenics manipulations to reach the bottom temperature are fully detailed for the setup we used in the following PhD thesis [202, 203].



B. Fabrication recipe

The fabrication of the nanoscale network presented in chapters 5 and 6 is fully detailed in [203, 208, 209]. Nevertheless, we will briefly detail here the main steps followed to realise the device and give some additional details about the InGaAs/InAlAs heterostructure (see Fig. B.1) we used.

B.1 Realizing a nanoscale network

The idea to design a network simply consists in removing the part of the heterostructure, so that electrons only flow in the remaining part. The process is realized in three main steps : (1) patterning the device (Electron beam lithography), (2) Metallization/lift-off (Ohmic contacts realization), (3) removing the locations where we do not want for electrons to flow by etching the sample. Although steps one and two are rather standard procedures, laterally etching a heterostructure is a process that has to be further discussed because the formation of these extra surfaces leads to the creation of surface states that can significantly modify the charge density profile near the etched edges.

The very first step of the process consists in cleaving the heterostructure wafer to a size of 5 by 6 mm. The initial substrate (before the molecular beam epitaxy growth) is obtained from a large n-doped single crystal of indium phosphide whose growth was performed using the Czochralski process with a crystallographic growth direction $\{100\}$. This discussion is followed by a

process sheet where all the fabrication parameters are reported.

We then aim to realize the metallic pads that serves as contact. The question is : how do we connect a 2DES given the fact that it is sandwiched in materials that are insulating at low temperature (semiconductor layers) ? Even the doped InGaAs CAP layer (see Fig. B.1). Indeed, initially, the top surface is a n-doped InGaAs layer (called the CAP layer) with $n_{3D} = 6 \times 10^{18} \text{ cm}^{-3}$, the electron charge density. At the surface, the abrupt termination leads to unsatisfied electron bonds, known as dangling bonds. Due to translational symmetry along the surface, these bonds can form two-dimensional bands with a remarkably high density of surface states. In such conditions, the Fermi level falls into this narrow band because even high dopant concentration would not be sufficient to put enough charges into the surface states in order to fill the band. The quasi-constant position of the Fermi energy (due to the narrowness of the band) is referred to as Fermi pinning. The surface states depends on many aspects. To quote W. Pauli, God made solids, but surfaces were the work of Devil. Obviously, different growth conditions lead to different surface crystallography. For example, the growth direction of our heterostructure is $\{100\}$ which leads to a specific surface crystallography and thus specific surface states. Also, the surface states can be influenced by surface reconstruction mechanisms, impurities adsorption... Depending on these aspects, the surface states can act as donors or acceptors. Charge depletion or accumulation profile can then occur in the vicinity of the surface and a bended potential profile will be present near the surface (with the bending characterised by Poisson equation). The charge depletion or accumulation is fixed by the surface charge neutrality condition. The potential varies on a characteristic length d given by [210]

$$d = \sqrt{\frac{2\epsilon_R\epsilon_0\phi_0}{n_{3D}}}, \quad (\text{B.1})$$

where ϕ_0 represents the total band bending (this difference is due to the space charges compensating the occupied surface states, curving the bands).

For example, for the InGaAs cap (as epitaxially grown), where the Fermi level is known to pin at mid gap, ϕ_0 is approximately equal to 0.3 eV. This gives us a depletion d of 8.45 nm. Hence, the 7 nm top cap layer is depleted by surface states and thus, no conduction is expected in parallel with the 2DES if the contacts are deposited at the surface.

To realize the contacts pads, we opted for a lift-off standard procedure. We spin-coated PMMA on the heterostructure surface. PMMA is in an anisole

solution (7% of the total solution mass). The thickness of the PMMA baked layer is to be compared with the contact pads thickness (that is 175 nm). This thickness depends on the percentage of PMMA in anisole and the spin-coating selected parameters.

Once baked, pads of 300 by 300 microns (see dark grey pads in Fig. B.1) are exposed. All lithographic patterns that we mention in this appendix are realised using a scanning electron micrograph (FEI/Philips XL30 FEG) coupling with Elphy Plus control software. This SEM has an ultimate resolution of 10 nm. When exposed to the electron beam, the long PMMA polymer chains (950K) are broken in smaller parts that can be dissolved in a MIBK solution. Once the pads location are emptied of PMMA, we can evaporate the contacts.

A so-called eutectic mixture of gold and germanium (this is an AuGe alloy with weight fractions 88) is evaporated onto the semiconductor surface. Then, a thermal annealing is realized. The purpose of the rapid thermal annealing (RTA) is to cause the diffusion of germanium in the heterostructure. It is believed that germanium replace gallium in the cap layer. The cap layer is then extra doped due to Ge. This doping profile aims to reduce the width of the Schottky barrier inevitably present near the contact/InGaAs interface. As the width gets thinner, electron tunneling through the barrier becomes prominent and the rectifying behavior of this metal/semiconductor contact leaves the floor to a Ohmic responds where the current flow is proportional to the voltage difference between each sides of the contact.

Although this procedure is well described *e.g.* in Ref. 211, our heterostructure is characterised by the presence of a highly n-doped InP backgate located 600 nm below the surface. It is then critical that the germanium diffusion profile does not reach the backgate (short-cuttet otherwise). After few iterations and a qualitative analysis based on the diffusion constant of Ge in the heterostructure, we established a suitable RTA temperature profile (see process sheet below).

After this metallization, a Hall bar was etched from the heterostructure. To do so, we spin-coated a PMMA layer on the surface, and patterned - using the SEM - the edges of the Hall bar (illustrated in light grey in Fig. B.1). The etching agents are detailed in the process sheet. The etching time is tailored so that we make sure to etch underneath the 2DES located approx. 45 nm beneath the surface.

To etch nano-networks the procedure is similar to the realization of the Hall bar but since the characteristic lengths are significantly smaller, the electron beam current is lower and the thickness of PMMA is thinner (see process sheet).

Also note that, in addition to the nanodevice, we also etched arrows in the vicinity of the device, pointing towards it. This has nothing to do with electronics, it concerns the SGM microscopy measurement. Indeed, the first step of the measurement (after electrically bonding the Ohmic contact pads) consists in the alignment of the AFM tip with respect to the nanostructure. This procedure consists in two steps. First, the tip is roughly aligned through with the help of a binocular (the tip has to be aligned above the arrow region - $450 \times 450 \mu\text{m}^2$). Second, the tip is approached to the surface and then scans the surface. Given the relatively high arrow density, we hopefully scanned (see scanners in Fig. A.1) an arrow and the quest for the device can begin ! At room temperature, this quest is not really time consuming since we can see through the binocular that remains roughly in the good region nearby the device. Once the device found, we withdraw the tip by a few microns and put the electromagnetic shields and seal (indium seal) the inner vacuum space.

B.2 Lateral etching

Let us come back to the etching of the nanodevice. The peculiar chemistry of this process step is described in the process sheet below. Nevertheless, before etching a nanonetwork, we will make few comments about the physics of the surfaces of semiconductors. Indeed as shown on the different lithographies (see Figs. 5.1a and 6.2a), we laterally etched complex structures not only involving a single crystallographic orientation. In addition, one can note a certain edge roughness. Characterising the surface states along the edges of our devices is extremely complex as well as the evolution of the conduction band in the vicinity of the edges with respect to the peculiar pinning of the Fermi energy at the etched surfaces. Indeed, as mentioned above, the vicinity of the edges could be depleted on a finite length d (or in accumulation) only due to the freshly made surfaces. This distance d , if known, would have to be compared with the smallest width of the channels in the network. In case d is larger than half the width of a channel, we would make a huge mistake when considering the average density in the network as the one measured in the Hall bar. We have no number for this length d but we have few heuristic arguments that we found quite convincing. First, in Ref. 208, comparable devices were realized and the author observed that single nanowires were no more conducting under a width of 50 nm. Second, as presented in Ref. 128 where similar devices were measured, we compared the magnetoconductance

traces between the longitudinal resistance of the Hall bar and the network. The comparison consists in checking if the longitudinal resistance peaks (in the quantum Hall regime of transport) for the same values of the magnetic field B for both the Hall bar and the device. If so, the domains in B of identical filling factors are the same and thus, the density can be considered as identical. Nevertheless, we have no experimental 'smoking gun' to estimate the length d . Our $\text{In}_{0.75}\text{Ga}_{0.25}\text{As}$ is closer to InAs than GaAs so to get insights about this length, we could look at the situation of Fermi level pinning in the case of InAs [212, 213]. We can mention that the complex problem of Fermi pinning to the surface of InGaAs (with a varying Ga concentration) was investigated in the following paper [214]. Also, further informations about InGaAs/InAlAs quantum wells can be found in [215].

B.3 Process sheet

Ohmic contacts

- Sample cutting : size 5×6 mm
- Standard cleaning
 - 3 min in acetone + 1 min ultrasonic bath
 - Acetone/IPA rinsing
 - N_2 dry
- Sample sticking on 2 inch Si wafer using PMMA 9% (10 min in stove at 140°C)
- Resist spin-coating : 7% PMMA (6000 rpm, 6000 rpm/s, 1min)
- Resist baking 15 min in the stove at 140°C
- E-beam lithography (dark grey pads in B.1)
 - Electron energy : 30 keV
 - Write field: $216 \mu\text{m}$
 - Sticking
 - dose: $300 \mu\text{C}/\text{cm}^2$
 - $I_{\text{beam}} \approx 2480$ pA
- Development : 90 s in

MIBK:IPA (1:3), 30 s rinsed in IPA, N_2 dry

- Metallization (deposition rate ≈ 0.1 nm/s)
 - 2.5 nm Ni
 - 40 nm Ge
 - 80 nm Au
 - 5 nm Ni
 - 50 nm Au
- Lift-off : few min in acetone the standard cleaning
- Rapid thermal annealing (RTA) : 10 min N_2 purge at 20°C , 30 s heating up to 285°C , 1 min at 285°C , cooling

Hall bar etching

- Standard cleaning
- Sample sticking on 2 inch Si wafer using PMMA 9% (10 min in stove at 140°C)
- Resist spin-coating : 4% PMMA

(6000 rpm, 6000 rpm/s, 1min)

- Resist baking 15 min in the stove at 140°C
- E-beam lithography (green pattern)
 - Electron energy : 30 keV
 - Write field: 216 μm
 - Stitching
 - dose: 300 $\mu\text{C}/\text{cm}^2$
 - $I_{\text{beam}} \approx 2480 \text{ pA}$
- Development : 90 s in MIBK:IPA (1:3), 30 s rinsed in IPA, N_2 dry
- Wet etching : $H_3PO_4:H_2O_2:H_2O$ (5:1:300) - 5 min, H_2O - 1 min
- Standard cleaning

Cap layer etching

- Succinic acid solution (pH=5): H_2O (30:4) - 1 min 30
- rinsed in H_2O - 1 min 30

Nano-network etching

- Standard cleaning
- Sample sticking on 2 inch Si wafer using PMMA 9% (10 min in stove at 140°C)
- Resist spin-coating : 2% PMMA (6000 rpm, 6000 rpm/s, 1min)
- Resist baking 15 min in the stove at 140°C
- E-beam lithography (red pattern)
 - Electron energy : 30 keV
 - Write field: 43.7 μm
 - dose: 250 $\mu\text{C}/\text{cm}^2$
 - $I_{\text{beam}} \approx 25 \text{ pA}$
- Development : 90 s in MIBK:IPA (1:3), 30 s rinsed in IPA, N_2 dry
- Wet etching : $H_3PO_4:H_2O_2:H_2O$ (5:1:300) - 2 min, H_2O - 1 min
- Standard cleaning

The contacts pads are then manually bonded with gold wires and silver epoxy (EPO-TEK H31D).

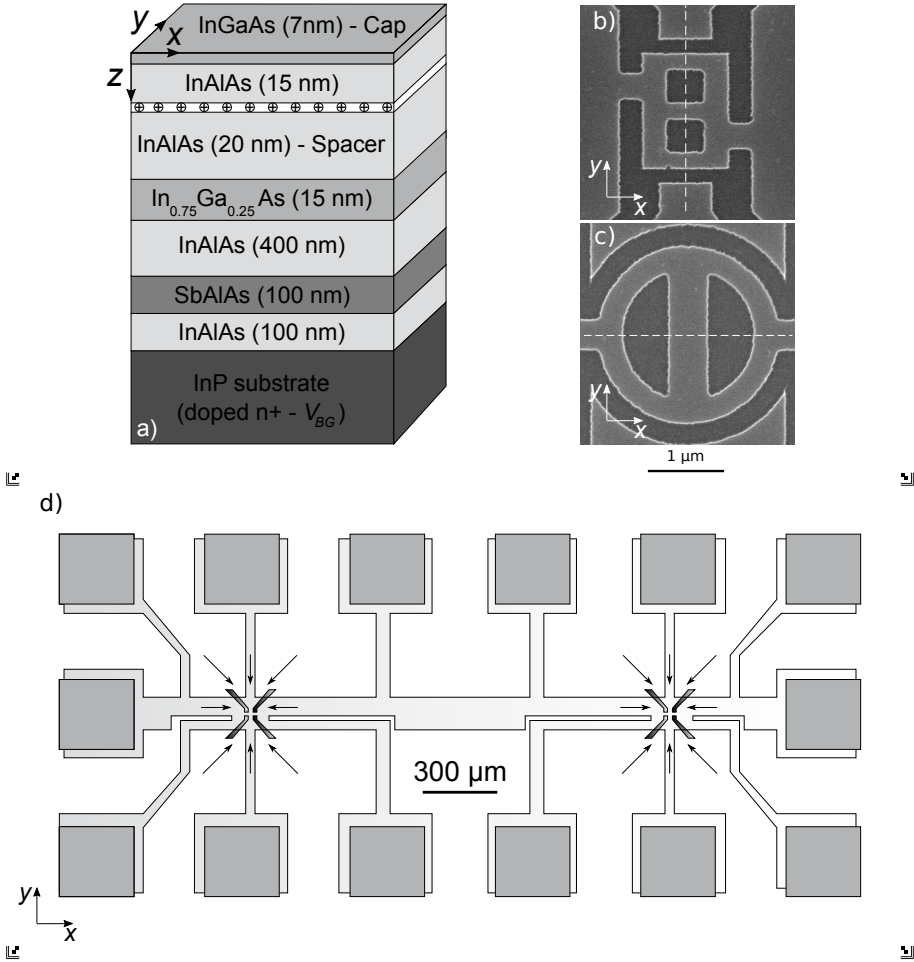
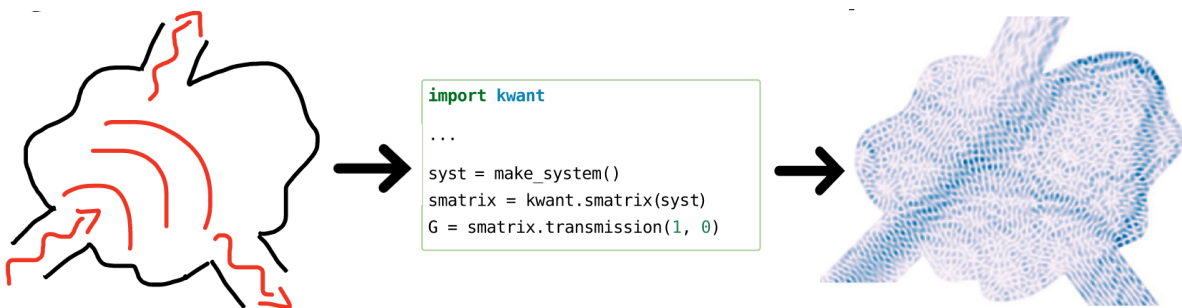


Figure B.1: a) Reminder of the heterostructure presented in Fig. 2.2a that we processed. b) d) Illustration of the Hall bar hosting the different samples measured. The nano-networks presented in b) and c) are centred on the converging point of the black arrows.



C. Quantum transport simulations : Kwant codes and parameters

As presented in chapter 3, the conductance of our nano-networks is simulated by solving the scattering problem with the S – *matrix* formalism. This problem is solved using Kwant. Kwant is an open source python package implemented to solve the scattering problem on tight-binding modelled systems [25, 51, 61] (see Fig.C.1).

C.1 Implementation details

The lattice parameter of the square lattice describing the device is a (see Fig.C.1) and is chosen to spatially resolve the electron propagative modes whose characteristic periods are at most given by λ_F , the Fermi wavelength. Our criterion to fix a is thus that $a \ll \lambda_F$. Typically, considering 2D electron densities close to $n_{2D} \sim 10^{16} \text{ (m}^{-2}\text{)}$, λ_F is close to 25 nm and after convergence test with a going from 10 to 1 nm we choose $a=2$ nm as our reference value for all the simulations presented in chapter 6.

The *first-neighbour* tight-binding parameters are chosen as follow : the hopping energy is set to one, the Fermi energy is scaled with respect to Eq. (3.22). The tip-induced potential ϕ_p is modelled as a Lorentzian distribution as presented in Eq. 4.20 and summed, for a given set of parameters $(V_{tip}, R_{tip}, x_{tip}, y_{tip})$, to the onsite energy of $4t$ (2D situation). No electron-electron interactions were considered in the simulations. As a matter of fact, no many body effect is

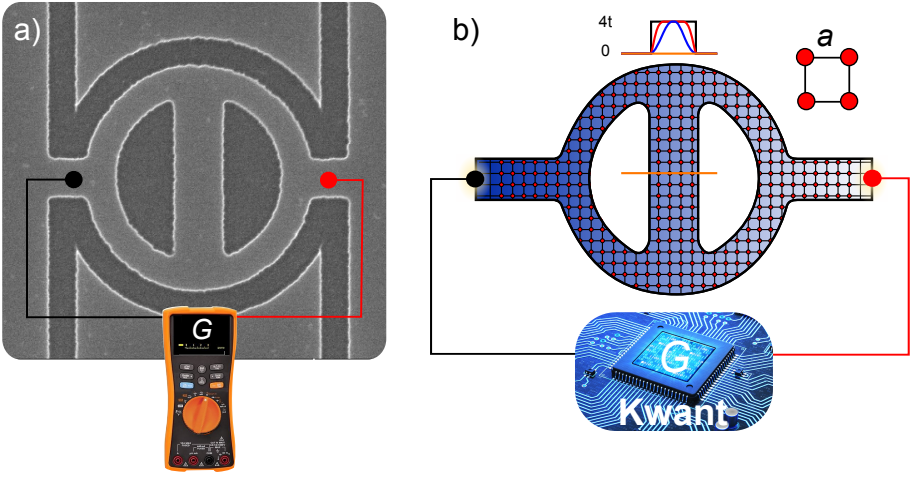


Figure C.1: a) Scanning electron micrograph of the network studied in 6 that is modelled in b) by a tight binding discrete lattice of lattice parameter a . Conductance (G) is measured on the one hand (a) and simulated on the other (b). On inset profiles shown in the upper part of (b) we can see, in black, a line profile of the onsite value (with no disorder considered) of the onsite value ($4t$). The red and blue profiles are illustrations of tentative profiles that should be considered to model more appropriately the onsite value through a channel (perpendicularly to the edges). The orange horizontal lines are matched.

required to reproduce qualitatively the results presented in chapter 6, the single electron picture is sufficient to understand the mechanism.

To perform the resolved mappings presented through chapter 6, we used the facilities offered by the CECI (Consortium des Équipements de Calcul Intensif) that provide researchers with access to powerful computing equipment. Since the value of the conductance computed for a given set of parameters is independent of the others, these computations were the ideal candidates for parallel computing. Our job¹ submission were managed by the open-source program SLURM (Simple Linux Utility for Resource Management). Even though Kwant is optimised to manage multiple jobs with the MPI (message passing interface) norm, we used *job arrays* for running collections of almost-identical jobs (called subjobs). To realise an M by N mapping (where M and N are the number of points along the horizontal and vertical axis respectively), we asked for N processors on a cluster to compute M data points. This was done thanks to the environment variable '*SLURM_ARRAY_TASK_ID*' that provides an integer label to a subjob. The different lines were then gathered together into a single file thanks to the following bash line : `awk 'FNR==1print ""'1' $(ls ABS_* | sort -n -t _ -k 2) > finalfile.dat`. A typical example of the code used to generate an SGM map is presented below.

C.2 Possibilities of improvement

As discussed in chapter 6, there is still a quantitative difference between the experimental and the simulation results (roughly a factor two between the simulated value of the conductance and the measured one). We think that a better match could be obtained if we modelled more accurately the potential landscape in the network. For example, if in addition to the dopant induced disorder potential, we take into account : the ternary alloy (InGaAs) scattering [216], interface roughness scattering and correctly simulated the unavoidable density variation present at the edges of the device due to Fermi pinning (see appendix B). Even without a precise estimate of the potential profile in the vicinity of the edges, improvements can be made. Indeed, in the work presented in chapter 6, the onsite values (disorder left apart) are considered as $4t$ inside the leads/device system and zero elsewhere. This discrete jump occurring over one lattice parameter a is really abrupt (see the black profile in the inset of Fig. C.1) and does not at all model correctly what is happening near the edges

¹a job is an execution of the code for a given set of parameters

(this abrupt jump models infinitely sharp potential hard walls, which is not what causes a chemical etching). One could try smoother onsite value profiles (see the red or blue profile in the inset of Fig. C.1). One should, *e.g.*, define a 2D map with these profiles, matching the nanodevice and that could multiply the onsite value ($4t$ + disorder) everywhere on the device. The lithographic lengths (shown in Figs. B.1b and c) should not correspond to infinite hard walls as it was done in chapter 6. The analytic expression of the electron density profile near the etched edges and the characteristic lengths over which it varies requires a deep analysis. Nevertheless, qualitatively, this procedure would reduce the value of the conductance with respect to the device we defined from the lithographic images. In addition, as we can see on the SEM image (see Fig. C.1a), there is some edge roughness (typically 20 nm). Taking into account this roughness would lead to another source of scattering, reducing the value of the conductance.

Also, higher order tight-binding simulations (second nearest neighbour etc.) would surely play a role to get a better conductance estimation.

C.3 A typical job

```

1  #!/usr/bin/env python3
2
3  # Importing general packages to manage arrays manipulation and
   basic math operation
4  from __future__ import division
5  import math
6  from math import atan2, pi, sqrt, cos, sin, acos
7  from cmath import exp
8  import numpy as np
9  from numpy import prod
10 from types import SimpleNamespace
11 import os
12
13 # Importing Kwant module
14 import kwant
15
16 # The present code is dedicated to the simulation of electron
   transport through a nano-network
17 # where the potential distribution is modified by potential
   disorder and a voltage bias SGM tip.
18
19 def main():
20
```

```

21  # Physical constants
22
23      h = 6.626069e-34      # Planck constant (Js)
24      e = 1.602e-19        # electron charge (C)
25      c = 299792458        # Light celerity (m/s)
26      kB = 1.3806503e-23   # Boltzmann constant (kg m2/Ks2)
27      m_e = 9.1e-31        # electron mass (kg)
28  m_eff = 0.036            # effective mass (-)
29
30  # General lattice parameters
31
32      a = 2 # Lattice parameter
33
34      lat = kwant.lattice.Monatomic([(a, 0), (0, a)],(0., 0.)
35  )
36
37      t = 1                # hopping parameter at B = 0
38      W = 300              # width of the ring (nm)
39      L = 950              # radius of the ring (nm)
40      l_lead = 950         # extra length for the leads
41  - in order to avoid side effects (nm)
42      W_central = 464      # width of the central
43  channel
44
45      # In order to round angles in the structure , we define
46  the radius Rc of small circle
47      # that are tangent to curves around the angular point
48  defining the inner and outer ring radius
49
50      Rc = 100             # aforementioned radius
51      Rout = L+W/2         # outer ring radius
52      Rin = L-W/2          # inner ring radius
53  W_lead = W*(407/300)    # width of the leads
54
55      # Location of the first circles (lead tangent)
56
57  y0 = Rc+W_lead/2
58      x0 = sqrt((Rout+Rc)**2-(y0)**2)
59      alpha = acos(y0/(Rout+Rc))
60
61      # Location of the second circles (central channel
62  tangent)
63
64      x1 = Rc+W_central/2
65      y1 = sqrt((Rin-Rc)**2-(x1)**2)
66      beta = acos(y1/(Rin-Rc))

```

```

61     # analytic definition of the nanodevice
62
63
64     def myshape(pos):
65         x, y = pos
66         return ((x**2 + y**2) > (Rin)**2 and (x**2 + y**2)
67 < (Rout)**2)\
68         or (abs(x)<W_central/2 and abs(y)<L) or ((abs(x)-L
69 >0) and (abs(x)-(L+l_lead)<0) and (abs(y) < W_lead/2))\
70         or ((x+x0)**2+(y-y0)**2 > Rc**2 and y>W_lead/2 and
71 x > -x0 and x<-x0+Rc*sin(alpha))\
72         or ((x+x0)**2+(y+y0)**2 > Rc**2 and y<-W_lead/2 and
73 x > -x0 and x<-x0+Rc*sin(alpha))\
74         or ((x-x0)**2+(y-y0)**2 > Rc**2 and y>W_lead/2 and
75 x < x0 and x>x0-Rc*sin(alpha))\
76         or ((x-x0)**2+(y+y0)**2 > Rc**2 and y<-W_lead/2 and
77 x < x0 and x>x0-Rc*sin(alpha))\
78         or ((x+x1)**2+(y-y1)**2 > Rc**2 and x<-W_central/2
79 and x>-x1-Rc*sin(beta) and y>y1 and (x**2 + y**2) < (Rin)
80 **2)\
81         or ((x-x1)**2+(y-y1)**2 > Rc**2 and x>W_central/2
82 and x<x1+Rc*sin(beta) and y>y1 and (x**2 + y**2) < (Rin)
83 **2)\
84         or ((x+x1)**2+(y+y1)**2 > Rc**2 and x<-W_central/2
85 and x>-x1-Rc*sin(beta) and y<-y1 and (x**2 + y**2) < (Rin)
86 **2)\
87         or ((x-x1)**2+(y+y1)**2 > Rc**2 and x>W_central/2
88 and x<x1+Rc*sin(beta) and y<-y1 and (x**2 + y**2) < (Rin)
89 **2)
90
91     # definition of the onsite energy of the tight-binding
92     model
93
94     # disorder flag - 0 : no disorder / 1 : disorder
95
96     disorder_flag=1
97     disorder = np.loadtxt('disorder_map.dat') # disorder
98     map in eV
99
100     def onsite(site, params):
101         x, y = site.pos
102
103     # Computing the actual Fermi energy in the 2DES
104
105     M = (m_eff*m_e/(pi*((h/(2*pi))**2)))*abs(e)

```

```

91         n2D = 1e16      # [m-2] value of the average
    measured 2D electron density
92         Ef = 1000*n2D/M # measured Fermi energy (meV)
93
94     # Definition of the 2DES potential landscape via the onsite
    energy value
95
96         if (disorder_flag):
97             return (1000*disorder[ int((Rout-y)/a) , int((L+
    l_lead-x)/a) ]/Ef)*energy + 4 * t + params.Vtip /(1+((x-
    params.xtip)**2+(y-params.ytip)**2)/params.Rtip**2)
98         else:
99             return 4 * t + params.Vtip /(1+((x-params.xtip)
    **2+(y-params.ytip)**2)/params.Rtip**2)
100
101     # definion of the hopping parameter taking into account
    the presence of a magnetic field
102     # orthogonal to the 2DES
103
104     def hopping(site_i , site_j ,params):
105         xi , yi = site_i.pos
106         xj , yj = site_j.pos
107         return -t*exp(-0.5j * params.phi * (xi - xj) *
    (yi + yj))
108
109     # build the 2DES (syst)
110     syst = kwant.Builder()
111     syst[lat.shape(myshape,(L,0))] = onsite
112     syst[lat.neighbors()] = hopping
113
114     # build the leads and attach them to the quantum ring-
    like structure
115
116     def lead_shape(pos):
117         x,y = pos
118         return abs(y) < (W*(407/300)/2)
119
120     sym=kwant.TranslationalSymmetry(lat.vec((-a,0)))
121     lead =kwant.Builder(sym)
122     lead[lat.shape(lead_shape,(0,0))] = 4*t # ! no disorder
    in the lead
123     lead[lat.neighbors()] = hopping
124     syst.attach_lead(lead)
125     syst.attach_lead(lead.reversed())
126
127     syst = syst.finalized()

```

```

128         # Magnetic field orthogonal to the 2DES
129
130
131     B = 0 # (T)
132     phi= B*(a**2)*(10**-18)/(h/e) #(-)
133
134     # energy (E/t) at which the conductance is computed
135     density = 1*10**16 # typical density in InGaAs 2DES
136     energy = 2*pi*density*(a**2)*(10**-18)
137
138     # Tip-induced potential parameters (Lorentzian shape
139     parameter)
140     xtip = -800
141     ytip = 0
142     Rtip = 150 # Half of the Full Width at Half
143     Maximum (FWHM) of the potential distribution
144     Vtip = energy*0.4 # The sign of Vtip define the
145     electrostatic coupling ! + : depletion ; - : accumulation
146
147     #####
148     # 2D mappings of the conductance G #
149     #####
150
151     Npts_xtip=301
152     Npts_ytip=301
153
154     xtips = np.linspace(-1.6*L, 1.6*L, Npts_xtip)
155     ytips = np.linspace(-1.6*L, 1.6*L, Npts_ytip)
156
157     file=open("ABS_%s.dat"% os.environ["SLURM_ARRAY_TASK_ID
158     "],"w")
159
160     G = np.zeros((Npts_xtip, Npts_ytip))
161
162     i = int(os.environ["SLURM_ARRAY_TASK_ID"])
163
164     ytip = ytips[i]
165     for j, xtip in enumerate(xtips):
166         params = SimpleNamespace(phi=phi, Vtip=Vtip,
167         xtip=xtip, ytip=ytip, Rtip=Rtip)
168         smat = kwant.smatrix(syst, energy=energy, args
169         =[params])
170         G[j,i]=smat.transmission(1, 0)
171         print(xtip, ytip, G[j, i])
172         file.write("%0.7f %0.7f %0.7f \n" % (xtip, ytip, G[j

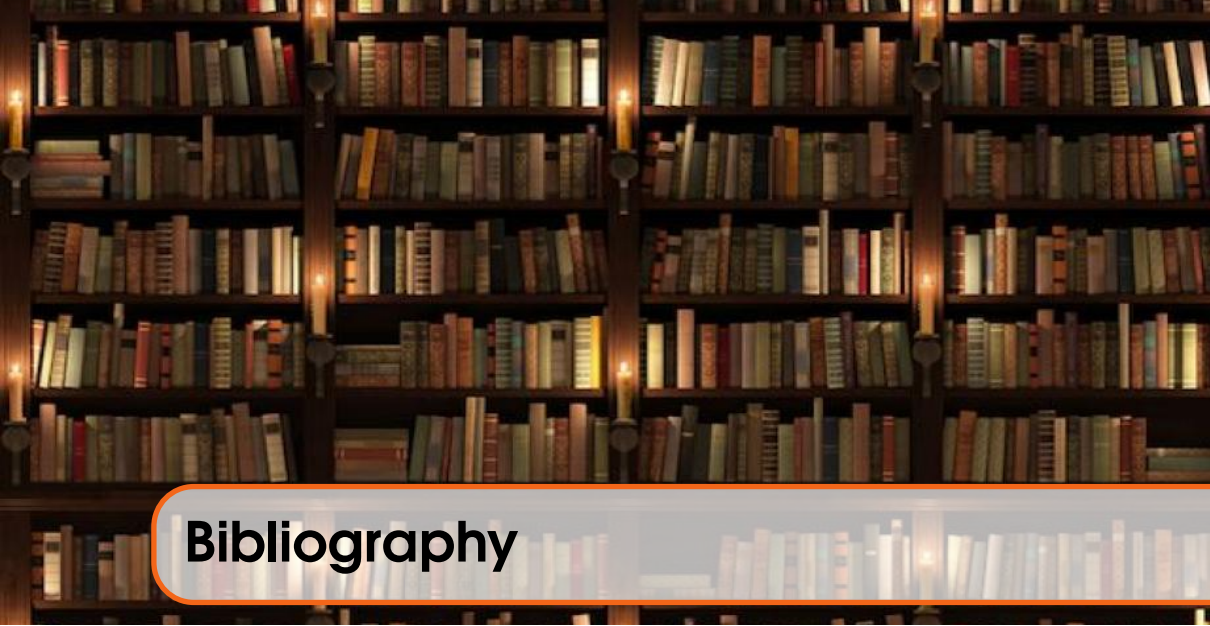
```



```
168         , i ] ) ) # writing data in a .dat file
169                 file . flush ( )
170 if __name__ == '__main__':
171     main ( )
```

The disorder map (see Fig.2.4d) included in the code is generated by simulating 2.36.

In order to realise rough computations to orient the research, we used the open-source software *Jupyter*. Jupyter (developed initially for JULia, PYthon and R, hence the name) is based on the principle of *notebooks*. In a notebook, are present cells that can be executed one at a time (enabling easier debugging). Notebooks can be shared between researchers via *Git* (another open source project). In the Anaconda 3 environment, many advanced features of data processing and data visualisation are accessible (thanks to open sources libraries such as NumPy (vastly based on BLAS and LAPACK), Matplotlib etc.). In addition, it can be directly installed on a local server.



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