

**MODELLING INTEREST RATES WITH A COINTEGRATED
VAR-GARCH MODEL**

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Abstract

We use a bivariate VAR model to model and predict the joint evolution of short term and long term interest rates. We introduce a GARCH effect on the innovations of the model in order to account for the changing volatility of the series. We test the cointegration of the two interest rates, which is implied by a present value relation between the rates. The cointegration test is done both with and without taking account of the GARCH effect. The empirical results for five countries (Belgium, Germany, France, Great Britain and the USA) point to the same conclusions: i) the incorporation of the GARCH part allows to conclude more clearly that a cointegration relation exists; ii) GARCH effects are quite present; and iii) the models are useful for short term predictions of interest rates.

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1 Introduction

Bivariate VAR models have been used for a long time in order to analyze the joint evolution of long and short run interest rates. These models offer a convenient framework to test hypotheses about the term structure of interest rates, such as the validity of the rational expectations theory: see e.g. Sargent (1979), Campbell and Shiller (1987). Sargent was able to detect the presence of unit roots in the series and so used differenced series. Campbell and Shiller showed that this unit root was common to the two rates and therefore used a cointegrated VAR model. In another type of research, the univariate analysis of interest rate series showed the presence of autoregressive conditional heteroskedasticity (ARCH), as initiated by Engle (1982). This observation calls for the introduction of ARCH in the joint modelling of interest rates, but a multivariate generalization of the univariate ARCH model turned out to involve difficulties (in particular of parametrization). One had to wait until 1995 to get a tractable formulation of multivariate ARCH models, as proposed by Engle and Kroner, who labelled it the “BEKK” formulation.

This paper combines the two types of models, i.e. the cointegrated VAR and the GARCH, in the modelling of the joint evolution of short and long run interest rates. The model is estimated using data for five countries, and is also used for prediction. In these empirical exercises, we test for cointegration within the VAR-GARCH model.

The paper is organized as follows: Section 2 presents the econometric model, Section 3 explains our specification strategy, Section 4 contains the empirical results and Section 5 concludes.

2 The econometric model for the interest rates

A present value model for the short term interest rate (r_t) and the long term one (R_t), states that R_t is a linear function of the present discounted value of the expected future r_t :

$$R_t = \beta(1 - \delta) \sum_{i=1}^{\infty} \delta^i E_t r_{t+i} + \mu \quad (1)$$

where β is the constant of proportionality, μ is the intercept, δ is the discount rate, and E_t denotes the expectation formed at time t conditionally on the available information (which includes R_t , r_t , and their past values). If both R_t and r_t are integrated of order 1, and if (1) holds, it can be shown that the quasi-spread

$$S_t = R_t - \beta r_t \quad (2)$$

is stationary (Campbell and Shiller, 1987; Phillips, 1988). Therefore R_t and r_t are cointegrated with cointegration vector $(1 \ - \ \beta)'$.

Campbell and Shiller (1987) derive and test the parameter restrictions implied by the present value relation on a VAR model for Δr_t (the first difference of r_t)

and S_t . We use a VAR model for the two rates and we test if cointegration holds. Since a look at the differenced time series of interest rates (see panels B of Figures 1 to 5) reveals that the volatility of the series is not constant, we enlarge the model with a multivariate GARCH effect on the innovations. Let y_t be the column vector $(R_t \ r_t)'$. We parametrise the VAR model in the error correction mechanism form:

$$\Delta y_t = \gamma + \Pi y_{t-1} + \sum_{i=1}^m \Pi_i \Delta y_{t-i} + \epsilon_t \quad (3)$$

where Π_i ($i = 1 \dots m$) and Π are square matrices of order 2 and γ is the vector of intercepts. With the cointegration relation (2) between the interest rates, the matrix Π is of rank one, and we include the constant in the cointegrating relation, so that

$$\Pi = AB', \quad \gamma = A\mu, \quad A = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ -\beta \end{pmatrix} \quad (4)$$

The error vector ϵ_t follows a GARCH(p,q) process, defined by

$$E(\epsilon_t \mid I_{t-1}) = 0 \quad (5)$$

$$E(\epsilon_t \epsilon_t' \mid I_{t-1}) = H_t \quad (6)$$

where I_{t-1} denotes the past information set and H_t has the BEKK formulation of Engle and Kroner (1995):

$$H_t = \Omega + \sum_{i=1}^p A_i' \epsilon_{t-i} \epsilon_{t-i}' A_i + \sum_{j=1}^q G_j' H_{t-j} G_j \quad (7)$$

Ω , A_i ($i = 1 \dots p$) and G_j ($j = 1 \dots q$) are square matrices of order 2. Estimation is done by the quasi-maximum likelihood method assuming that ϵ_t is Gaussian with moments given by (5) and (6). Tuncer (1994) and Bauwens and Vandeuren (1995) have shown that the QML estimator for this type of model is consistent under some “high-level” conditions. We assume that asymptotic normality holds, but it has not been proved, as far as we know.

3 Specification strategy

The VAR-GARCH model may involve a lot of parameters. With $p = q = 2$, there are 19 parameters in the GARCH part. Since we use monthly data, a reasonable value for the lag order (m) in the VAR system may be 5 or 6, in which case there are 24 or 28 parameters in the VAR part. The total number of parameters is thus of the order of 40 to 50. Estimation is then typically demanding in computing time. A sound specification strategy is to start with a “general” starting model, meaning that it is preferable to start with an overparameterised model rather than an

underparameterised one. Moreover the starting point of a simplification strategy should be checked for misspecification, in particular for residual autocorrelation. Autocorrelation tests can be performed as follows: i) one estimates the starting model and saves its residuals; ii) one adds lags of the two residuals (up to the order of autocorrelation to be tested) in both equations; and iii) one tests that all the added variables have zero coefficients. If the restriction is accepted, one can assume that the lag order of the starting model is sufficient. This method involves estimating more parameters at step ii) than in the starting model. It turned out to be practically unfeasible. Another test of autocorrelation is the multivariate version of the portmanteau test (see e.g. Lütkepohl, 1991, p. 152) which uses only the residuals of the model estimated under the null hypothesis.

We decided to introduce five lags of the two variables (ΔR_t and Δr_t) in both equations and to apply portmanteau tests to the residuals of the VAR model estimated without a GARCH part. As the test was passed (with a few exceptions), we proceeded with the estimation of the specified VAR model together with the GARCH effect. For practical purposes, we have considered at most GARCH(2,2) specifications. The residuals of the VAR-GARCH model were then submitted to the portmanteau test.

Obviously, before the specification search and the estimation of the VAR, we tested for the presence of a unit root in the two interest rate series, and we tested for cointegration between them. Cointegration can be tested by using the Engle-Granger test or the likelihood ratio test (i.e. Johansen's trace test). Likelihood ratio tests that take into account the GARCH effects can be computed after the estimations of the VAR model (3) with the GARCH part (5)-(7) under the three hypotheses that Π has a rank equal to two, to one -as in (4)-, or to zero. We decided to use the Johansen's trace test critical values even in the presence of GARCH effects, since we conjecture that they should remain valid asymptotically (given that Johansen's results rely on a functional central limit theorem). Moreover, Cheung and Lai (1993) studied the size of the trace test in finite samples under non-normality and conclude that it shows little bias in the presence of skewness or of excess kurtosis. As they did not specify the non-normal distribution of their Monte Carlo experiments as a GARCH process, other Monte-Carlo experiments should be performed to check if their conclusion holds in this case. As our estimation results seem to imply that the covariance matrix of the errors of the VAR does not exist, we also used critical values computed by Caner (1996), which seem to be relevant in such a case. Caner has computed several tables as the asymptotic distribution of the trace statistic depends in his setup on a nuisance parameter (the index of the stable law of the errors). As this parameter is unknown, we used the table corresponding to the smallest index, which gives critical values which are the most distant from Johansen's ones.

4 Empirical results

4.1 Data

We have used OECD data (source: Main Economic Indicators) of five countries: Belgium (BEL), Germany (DEU), France (FRA), the United Kingdom (GBR), and the USA. For all countries, the long term interest rate (R_t) is a yield of long term government bonds (Belgium: yield of long term government bonds; Germany: yield of 7-15 year public sector bonds; France: yield of public and semi-public sector bonds; UK: yield on 20 year central government bonds; USA: yield of long term government bonds). For France and the UK, the short term rate (r_t) is the call money rate; for the USA, it is the Federal Funds rate; for Belgium, it is the market rate of 3-month treasury certificates, and for Germany, it is the FIBOR). All the series consist of monthly data. The series are not strictly comparable between countries, as our choice was dictated by data availability over a long period. The period covered by the data and the number of observations are given in Table 1. The plots of the series and of the differenced series are shown in panels A and B of Figures 1 to 5. A look at these plots reveals that the individual series may well be $I(1)$, that there may be a common movement in the short term and the long term rates in each country, and that the differenced series exhibit the typical ARCH behaviour. We see also that the short term rate is much more volatile than the long term rate. This strong volatility tends to mask the possible presence of a common trend in the two interest rates.

Table 1: Period and number of observations

Country	(Code)	Starting date	Ending date	Number of observations
Belgium	(BEL)	60:01	95:07	427
Germany	(DEU)	60:01	95:07	427
France	(FRA)	60:01	95:07	427
UK	(GBR)	63:01	94:06	378
USA	(USA)	60:01	95:07	427

4.2 Unit root and cointegration tests

The graphical evidence suggesting unit roots and cointegration is not sufficient and must be supported by statistical tests. Unit root tests were performed using the augmented Dickey-Fuller τ test (DF), the weighted symmetric (WS) τ test (see Pantula et al, 1994), and the Phillips and Perron (PP) variation of the Dickey-Fuller Z -test. These tests are performed automatically by TSP, version 4.3 (see the COINT command for details). Results are summarised in Table 2. For the long

Table 2: P-values (in percent) of unit root tests

Country	Test	Short term rate	Long term rate
BEL	WS:	10	49
	DF:	14	40
	PP:	13	51
DEU	WS:	0.1	1.7
	DF:	0.3	6.4
	PP:	1.6	8.4
FRA	WS:	7.0	55
	DF:	11	41
	PP:	11	51
GBR	WS:	18	58
	DF:	12	23
	PP:	3.2	32
USA	WS:	2.0	58
	DF:	3.1	41
	PP:	9.8	53

A p-value lower than α percent means that the unit root hypothesis is rejected at the level α . WS is the weighted symmetric τ test (see Pantula et al, 1994), DF is the augmented Dickey-Fuller τ test, and PP is the Phillips-Perron Z test.

term rates, the tests indicate the presence of a unit root, except for Germany. For the short term rates, the evidence is rather opposite. The ARCH-type behaviour of the series has not been specifically taken into account in the unit root tests. According to the results of Kim and Schmidt (1993), the Dickey-Fuller test tends to overreject the unit root if the error variance follows a GARCH(1,1) process but the problem is very serious only in special parameter configurations that should not occur with our monthly time series. We notice that the results of the Dickey-Fuller tests in Table 2 are not clearcut for Belgium, France and the UK (with a p -value between 10 and 15 percent). No results are available on the size of the other tests in the presence of ARCH effects.

Cointegration tests using Johansen's critical values (see Table 3) clearly indicate that there is a cointegration relation between the two interest rates in Belgium, France, and the USA. For the United Kingdom, the tests that do not take into account the GARCH effect indicate that there is no cointegration, whereas the test

with the GARCH effect concludes in favour of cointegration. For Germany, the Johansen's trace test leads to conclude that the system (in levels) is stationary, and the corresponding test when the GARCH effect is present leads to conclude that the system is cointegrated. In general, the test with the GARCH model gives more clear results in favour of cointegration than when the GARCH effect is ignored. As mentioned in Section 3, we also used the critical values computed by Caner (1996, Table 4, $\alpha = 0.5$, left panel). The conclusions from these tests are qualitatively the same as in Table 3, except in the case of Germany: even with GARCH errors, the rank of cointegration seems to be equal to two (rather than one).

Table 3: Results of cointegration tests

Country	E-G test		Likelihood ratio test on VAR without GARCH			Likelihood ratio test on VAR with GARCH		
		r	$r \leq 0$	$r \leq 1$	r	$r \leq 0$	$r \leq 1$	r
BEL	0.4	1	$p < 1$	$50 < p < 75$	1	$p < 1$	$p > 50$	1
DEU	1.6	1	$p < 1$	$1 < p < 2.5$	2	$p < 1$	$25 < p < 50$	1
FRA	2.1	1	$2.5 < p < 5$	$p > 50$	1	$p < 1$	$25 < p < 50$	1
GBR	53	0	$p > 50$	$25 < p < 50$	0	$p < 1$	$15 < p < 20$	1
USA	28	0	$2.5 < p < 5$	$25 < p < 50$	1	$p < 1$	$p > 50$	1

Entries are p -values in percent (or bounds on the p -values), except in the columns headed by “r”, where we give our choice of the rank of matrix Π in (3). For the E-G (i.e. Engle-Granger) test, a p -value lower than α percent means that the cointegration hypothesis is accepted at the level α ; the dependent variable of the CI regression is R_t . For the likelihood ratio tests, a p -value lower than α percent means that the corresponding null hypothesis is rejected at the level α . Bounds on the p -values of the likelihood ratio tests (with and without GARCH) are obtained from Table 15.2 of Johansen (1995).

4.3 Estimation results

4.3.1 Cointegration and error-correction parameters

In Table 4, we report the estimates of the cointegrating vectors and of the coefficients of the error-correcting terms in the VAR, with and without the GARCH. Five lags of the first differences of the two rates were included in the two equations of the VAR, and the twelfth lag was also included for Germany (the corresponding estimates are not reported).

The cointegrating relation between the interest rates should have a slope equal to one if the present value relation holds, so that the spread $R_t - r_t$ is stationary

around the constant μ . Whether a GARCH effect is included or not, the hypothesis that $\beta = 1$ is rejected at usual significance levels for Belgium and Germany, and is accepted for the other countries (for the UK the test is not relevant in the model without GARCH effects, since cointegration is rejected in that case - see also the “aberrant” estimates of β and μ). Another feature of the results is a large negative correlation (at least -0.9) between the estimates of β and μ for all countries, implying that one could consider to fix β equal to one and let the constant adjust (it will become closer to zero).

Regarding the adjustment coefficients α_1 of the long term rate equation and α_2 of the short term rate equation, we notice that in the VAR-GARCH, the long term rate is not significantly error-correcting in Belgium, the USA, and perhaps Germany, so that the stochastic trend of the long term rate is driving the evolution of the system in these countries. The short term rate is adjusting to the error correcting term more quickly than the long rate in all countries.

Table 4: Results on VAR parameters

	BEL	DEU	FRA	GBR	USA
VAR without GARCH					
β	0.74 (0.08)	0.77 (14.2)	1.07 (66.7)	3.47 (0.00)	0.95 (88.1)
μ	2.83 (0.00)	2.61 (0.34)	0.57 (34.0)	-20.75(45.2)	1.07 (29.9)
α_1	-0.02 (10.7)	-0.01 (29.5)	-0.01 (27.2)	0.00 (41.9)	-0.02 (19.9)
α_2	0.13 (0.02)	0.11 (0.00)	0.08 (0.55)	0.01 (44.5)	0.04 (2.23)
VAR with GARCH					
β	0.73 (0.00)	0.59 (0.04)	1.07 (18.4)	0.47 (21.1)	1.35 (24.6)
μ	2.92 (0.00)	3.80 (0.00)	1.31 (0.00)	6.50 (3.70)	-1.25 (23.7)
α_1	-0.00 (41.7)	-0.03 (7.67)	-0.01 (5.73)	-0.02 (2.84)	0.00 (47.7)
α_2	0.16 (0.00)	0.08 (0.01)	0.16 (0.18)	0.04 (18.9)	0.03 (1.84)

β and μ are the parameters of the cointegration relation $R_t = \mu + \beta r_t$; α_1 and α_2 are the coefficients of the error correction term in the VAR equations for ΔR_t and Δr_t , respectively. Entries are parameter estimates and p -values (in percent) of the corresponding t-statistics (for $\beta - 1$ in the case of β , for nullity in the case of the other coefficients). The t-statistics are based on standard errors evaluated by the quasi-maximum likelihood formula. A p -value lower than α percent means that the null hypothesis is rejected at the level α .

Table 5: Results on GARCH parameters

	BEL	DEU	FRA	GBR	USA
Diagonality test: p -value (in percent)					
	0.0	0.0	0.2	0.0	55
Moduli of eigenvalues of estimate of (8)					
λ_1	1.04	0.98	1.52	1.01	1.15
λ_2	0.84	0.96	1.02	0.84	1.09
λ_3	0.80	0.37	0.07	0.84	1.02
λ_4	0.77	0.32	0.02	0.74	1.01

Diagonality test: the null hypothesis is that the matrices A_1 , A_2 , G_1 , and G_2 are diagonal. A p -value lower than α percent means that the null hypothesis is rejected at the level α .

4.3.2 GARCH parameters

A GARCH(2,2) model was assumed as the most general specification. For the USA, it could be reduced to a diagonal model with $G_1 = 0$, and for Germany to a GARCH(1,2) ($A_2 = 0$). With a GARCH(2,2) specification, there are sixteen parameters to be estimated in the GARCH part, in addition to the constant matrix Ω , see (7).

Rather than the estimates of all the coefficients and their standard errors, we report the times series plots of the estimated conditional standard deviations and correlation (see panels C and D of Figures 1-5). The conditional correlations fluctuate quite a lot around country-specific positive values (0.27 for Belgium, 0.42 for Germany, 0.22 for France, 0.07 for the UK, and 0.23 for the USA). Anyway, the plots suggest that a constant correlation GARCH model is not appropriate for the analysed series. The profiles of the conditional standard deviations show the much higher volatility of the short term rate. They suggest that there may be a common factor in the two volatilities; the correlation coefficient between the series of conditional standard deviations is positive (0.65 for Belgium, 0.49 for Germany, 0.40 for France, 0.84 for the UK, and 0.54 for the USA) and highly significant. One can also see the effects of some general or country-specific events (e.g. for the USA, the high increase of the volatility of the short term rate around 1980).

In Table 5, we give the outcome of the diagonality tests of the GARCH, i.e. that the matrices A_1 , A_2 , G_1 , and G_2 (if included in the specification) are diagonal matrices. The hypothesis is rejected except for the USA. We also report in Table 5 the moduli of the eigenvalues of the estimate of the matrix

$$A_1 \otimes A_1 + A_2 \otimes A_2 + G_1 \otimes G_1 + G_2 \otimes G_2 \quad (8)$$

The moduli must be less than one to ensure the covariance stationarity of the GARCH process on the errors of the VAR (see Proposition 2.7 of Engle and Kroner,

1995). For all countries, there is at least one eigenvalue that is larger than one in modulus, implying that the unconditional covariance matrix of the errors does not exist (an exception to this occurs in the case of Germany, but when we use the call money rate instead of the FIBOR as the short rate, there is also one eigenvalue that is larger than one). The non-existence of the unconditional second moments is probably a consequence of the occurrence of at least one spell of extreme volatility of the short term rate in every country (in particular around 1980, see observations around the time period 250, on panels B of the figures). If we compare the means of the estimated conditional standard deviations and correlations (as shown in panels C and D of the figures) to their empirical counterparts (standard deviations and correlation of the residuals of the VAR without GARCH), we find that they match almost perfectly in the case of the standard deviation of the long term rate and of the correlation; for the standard deviation of the short term rate, the means of the conditional estimates are a little too low (0.51 versus 0.54 for Belgium; 0.97 versus 1.03 for the UK; 0.47 versus 0.56 for the USA) except for Germany (0.18 versus 0.18) and for France (0.63 versus 0.55).

4.4 Misspecification tests

Autocorrelograms of the residuals of the VAR-GARCH models are shown in panels E and G of Figures 1-5. Multivariate portmanteau tests reveal that there remains some significant autocorrelation in the models. Compared to the model without GARCH effect, this is due to the change of the estimates of the coefficients of the VAR, induced by the estimation of the GARCH parameters. It is therefore not clear that this problem can be solved by adding more lags in the VAR part. Moreover, the residual autocorrelograms show that there are some large autocorrelations at some rather long lags. It is practically unfeasible to extend the VAR specification to include more lags in the VAR part given the number of parameters to be estimated. A possible approach to tackle this issue is to consider the inclusion of moving average terms on the errors.

Another issue is the normality of the errors. Skewness and kurtosis coefficients (not reported) of the standardized residuals reveal that they are far from being normal. Since we use QML estimation, non-normality is not crucial, since standard errors are adjusted to take account of the possible non-normality.

4.5 Prediction results

Predictions of the interest rates were computed using the models estimated on the data until July 1995, except for the UK where the data for estimation ends in June 1994 (because the call money rate for the UK is not available after 1995 in the OECD MEI database and we wanted to keep 18 observations for prediction evaluation). The prediction for each month after the estimation period is a three-month dynamic prediction (e.g. for predicting December 1996, we used the observed data until September 1996, and predicted October, November and December, keeping

Table 6: Evaluation of predictions

	BEL		DEU		FRA		GBR		USA	
VAR without GARCH										
MAE	0.56	0.26	0.47	0.31	0.70	0.29	0.80	0.26	0.35	0.39
RMSE	0.60	0.34	0.58	0.36	0.93	0.37	1.05	0.33	0.44	0.45
THEIL	0.17	0.05	0.16	0.06	0.21	0.06	0.18	0.04	0.08	0.07
BIAS	0.85	0.36	0.62	0.22	0.02	0.27	0.43	0.07	0.01	0.00
VAR.	0.05	0.02	0.18	0.00	0.69	0.00	0.01	0.01	0.04	0.01
COVAR.	0.10	0.62	0.20	0.78	0.29	0.73	0.56	0.92	0.95	0.99
VAR with GARCH										
MAE	0.74	0.22	0.21	0.33	1.13	0.28	0.61	0.19	0.18	0.36
RMSE	0.80	0.30	0.25	0.38	1.43	0.38	0.86	0.26	0.24	0.43
THEIL	0.23	0.05	0.07	0.06	0.32	0.06	0.15	0.03	0.04	0.06
BIAS	0.85	0.11	0.04	0.24	0.02	0.21	0.07	0.02	0.00	0.00
VAR.	0.09	0.04	0.09	0.02	0.78	0.01	0.06	0.01	0.16	0.01
COVAR.	0.06	0.85	0.87	0.74	0.20	0.78	0.87	0.98	0.84	0.99

For each country, the left column gives the results for the short term rate, and the right column is for the long term rate. MAE is the mean absolute deviation of the predictions. RMSE is the root mean squared error. THEIL is Theil's inequality coefficient, which is equal to zero when predictions are perfect. It is decomposed into BIAS, which is the fraction of the error in predictions due to bias, VAR., which is the fraction due to different variations, and COVAR. which is the remaining fraction. VAR. and especially BIAS should be close to zero, to avoid "systematic errors" in predictions. For more details, see Pyndick and Rubinfeld (1991, pp 336-342).

the value predicted for December for comparison with the observed value). Panels F and H of the figures plot the predictions and the observations (at the end of the prediction period, one or two observed values were not available when we did the predictions). One set of predictions is made with the VAR model without GARCH, and another is made using the VAR-GARCH model (although the GARCH part in itself plays no role in the predictions of the interest rates themselves). In Table 6, we report some usual criteria on the quality of the predictions: the mean absolute error, the root mean squared error, and Theil's inequality coefficient with its decomposition into three sources of variations (see footnote of Table 6).

In general, the differences in the quality of the two sets of predictions are neither systematic nor big. For the short term rates, the predictions are of better quality (in terms of the three criteria) in three countries (Germany, Great Britain, and the

USA) with the VAR- GARCH than with the VAR without GARCH. For Germany, the improvement due to the introduction of the GARCH model is impressive (see also Figure 3-F). For the long term rates, the predictions are better in all countries with the VAR-GARCH, except in the case of Germany where they are marginally better with the VAR without GARCH.

According to the three criteria, the predictions of the long term rate are generally of better quality than those of the short term rate (except in the case of Germany). Generally speaking, the short term rate, being more volatile, can be expected to be more difficult to predict than the long term rate. We did not estimate standard errors or confidence intervals for the predictions because they would be too large to be useful in practice, as usual in such exercises.

Overall, the quality of the predictions is neither very good, nor very bad. It improves by doing one-month or two-month ahead predictions, and it deteriorates if one does more long run predictions. We can conclude from other unreported results that our models are not useful for more long run predictions than about three months.

5 Conclusion

This paper has shown the feasibility of the bivariate modelling of interest rates using a VAR-GARCH model. The empirical findings of the papers are that:

- (i) cointegration is not rejected by the data, this conclusion being stronger in the presence than in the absence of the GARCH part;
- (ii) there are important GARCH effects, and in particular a GARCH(2,2) model is relevant for our data;
- (iii) a diagonal GARCH model seems to be the exception rather than the rule;
- (iv) the unconditional covariance matrix of the GARCH model may not exist;
- (v) there may be a common factor driving the dynamics of the conditional second moments;
- (vi) the VAR-GARCH model can be useful for predicting the interest rates, and seems to predict better long term interest rates than a pure VAR model.

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Figure 1: Belgium

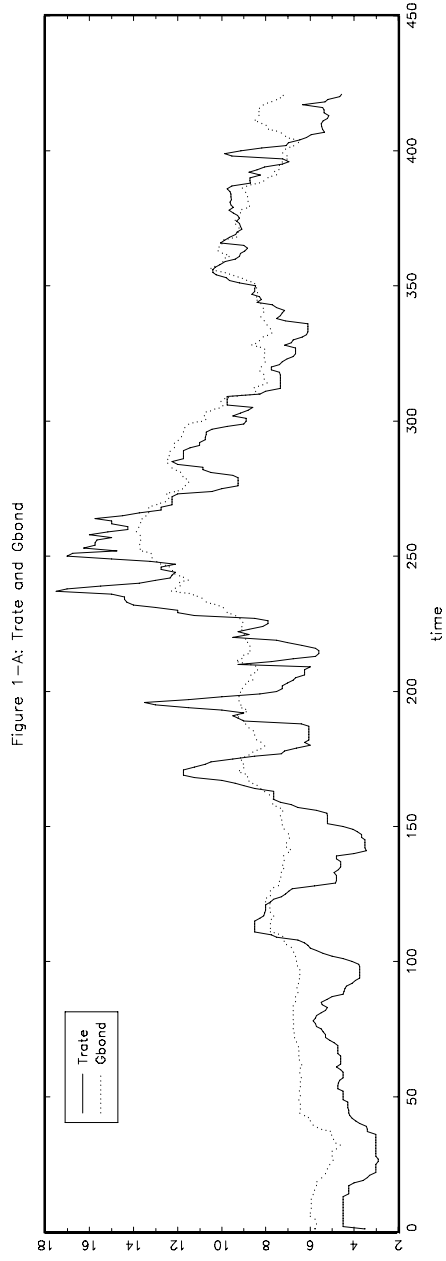
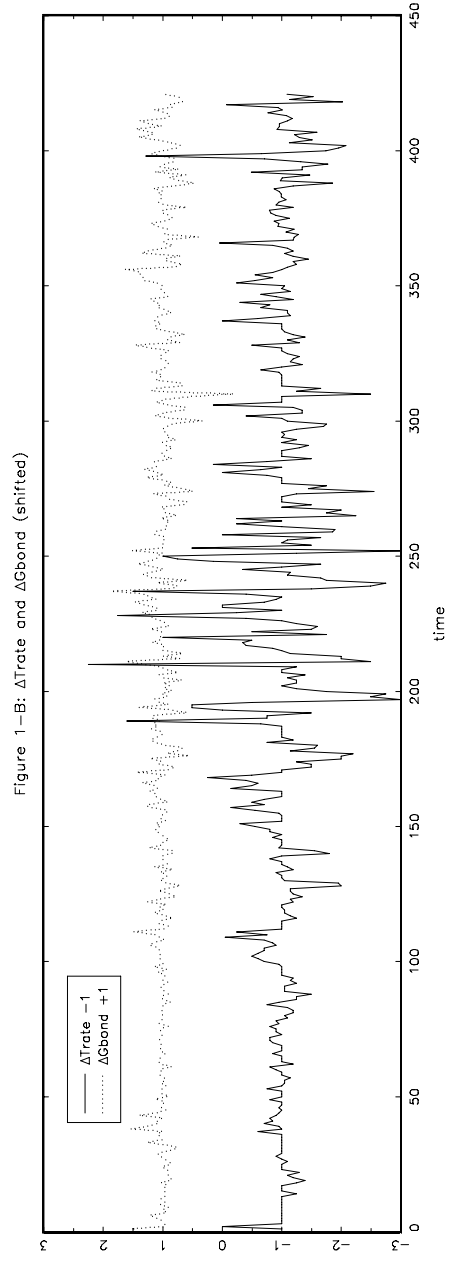


Figure 2: Belgium

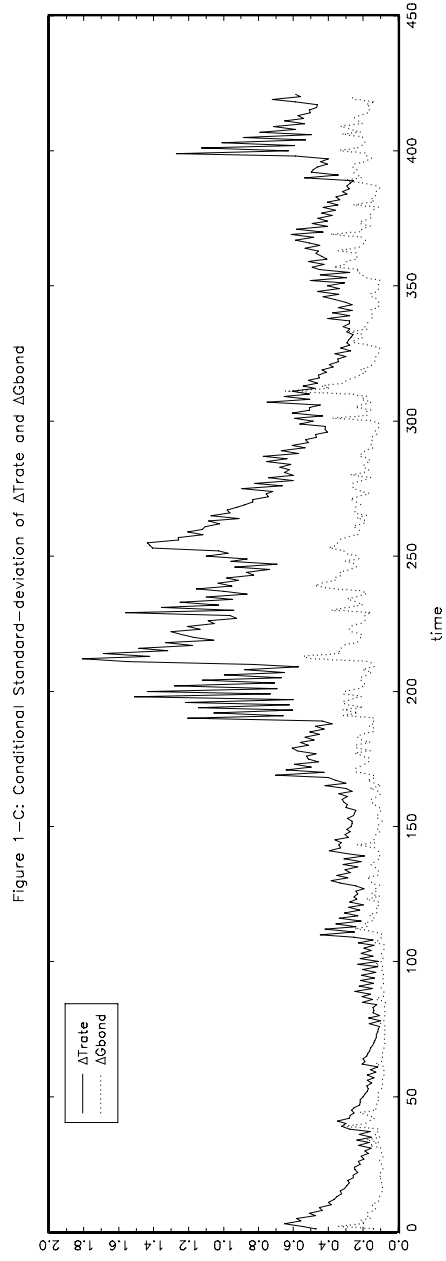
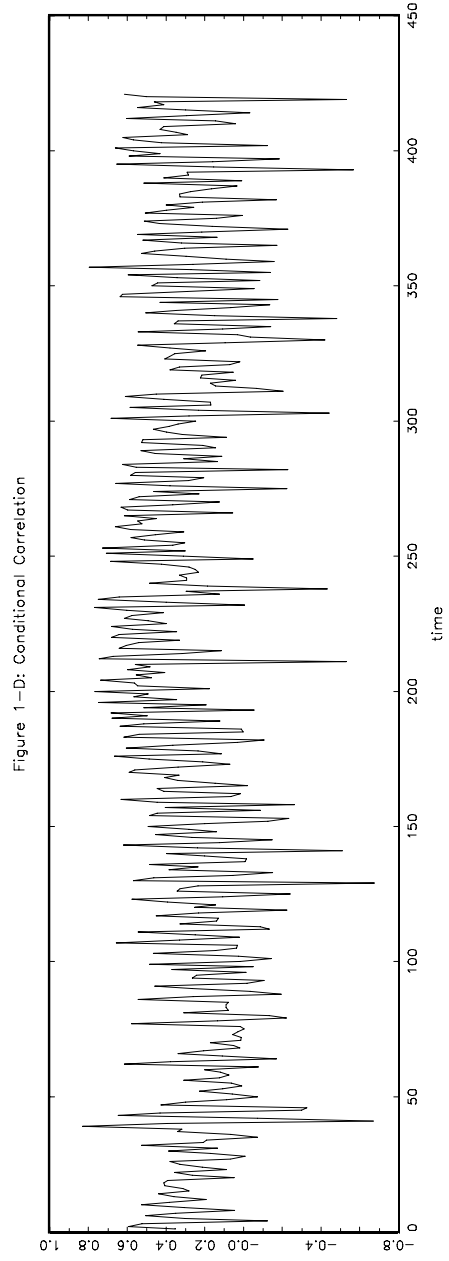


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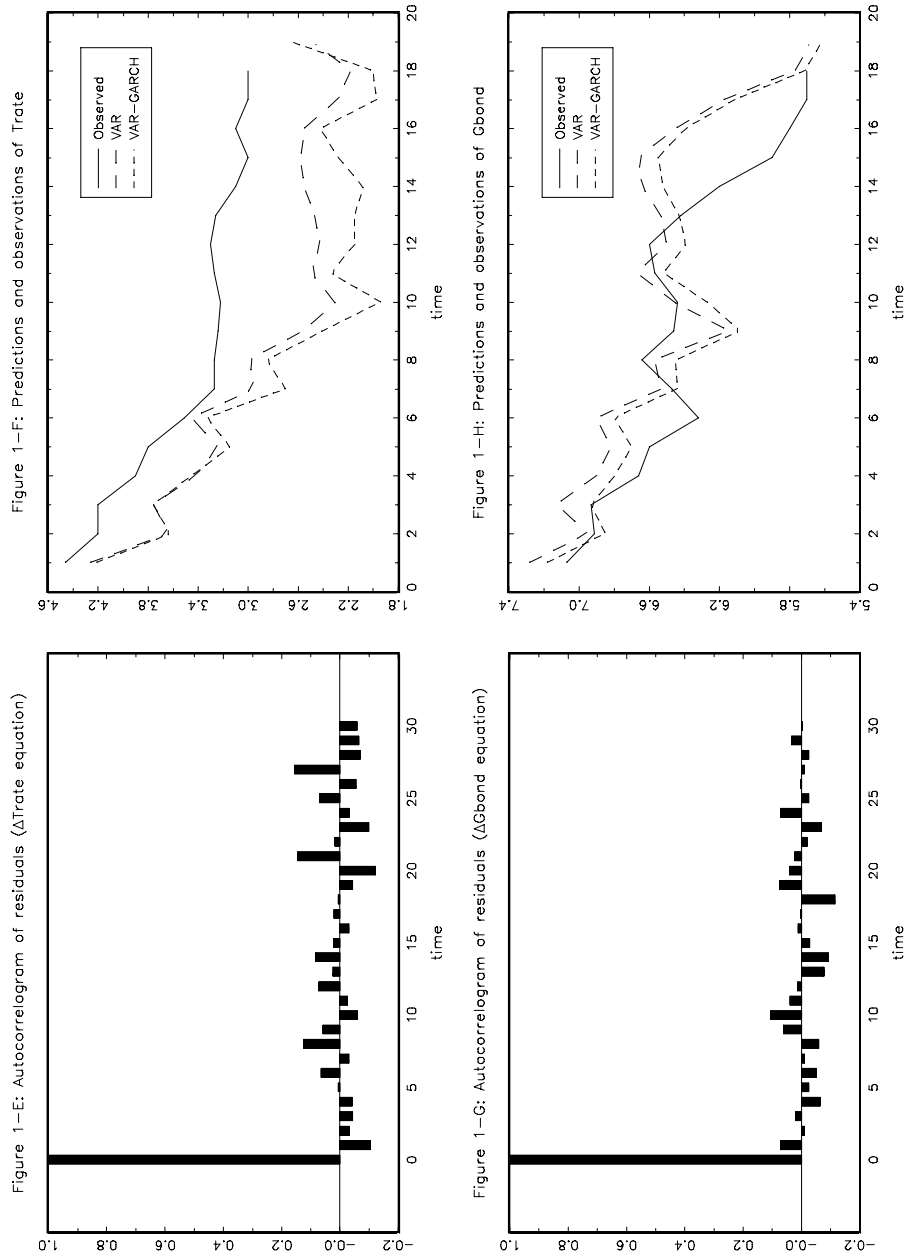


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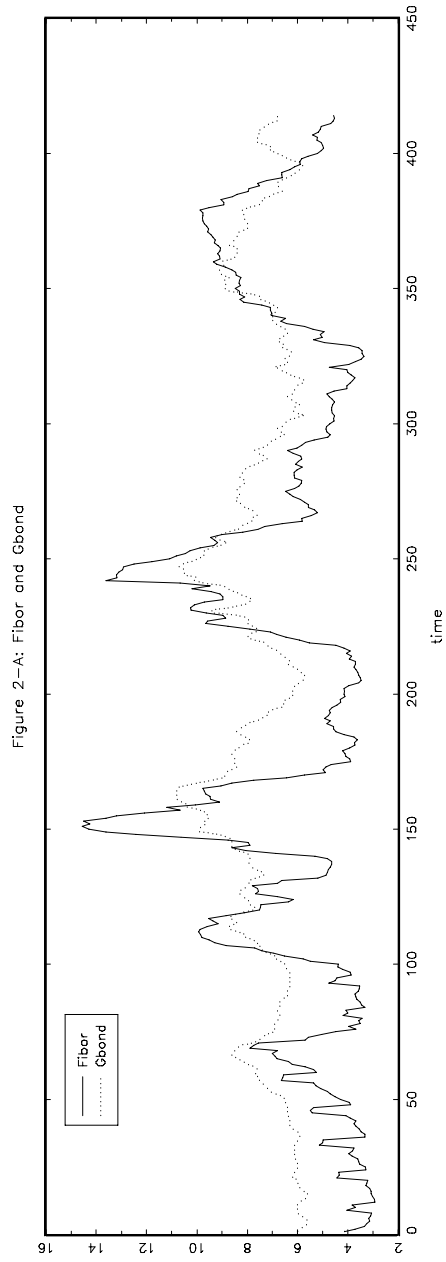
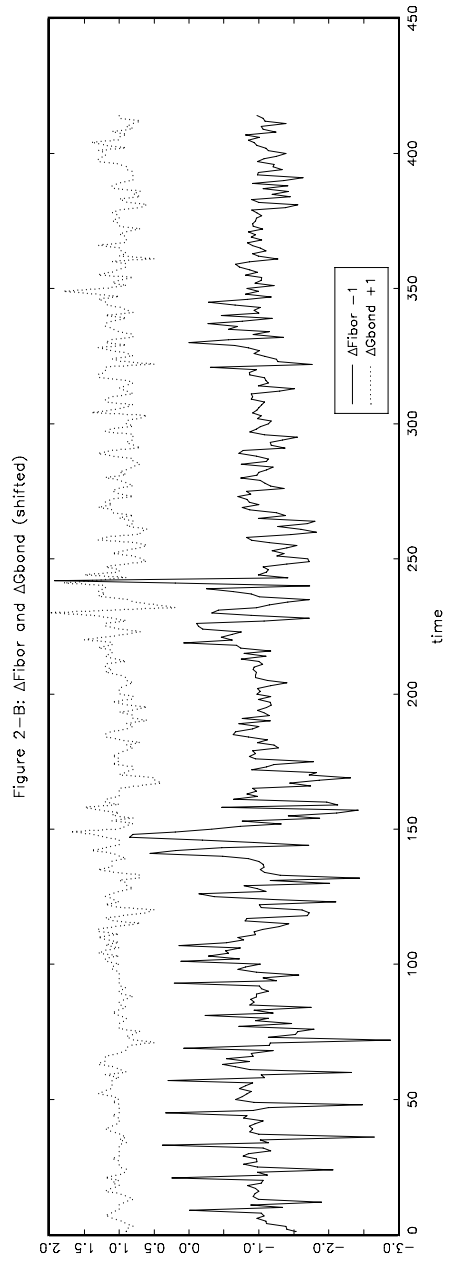


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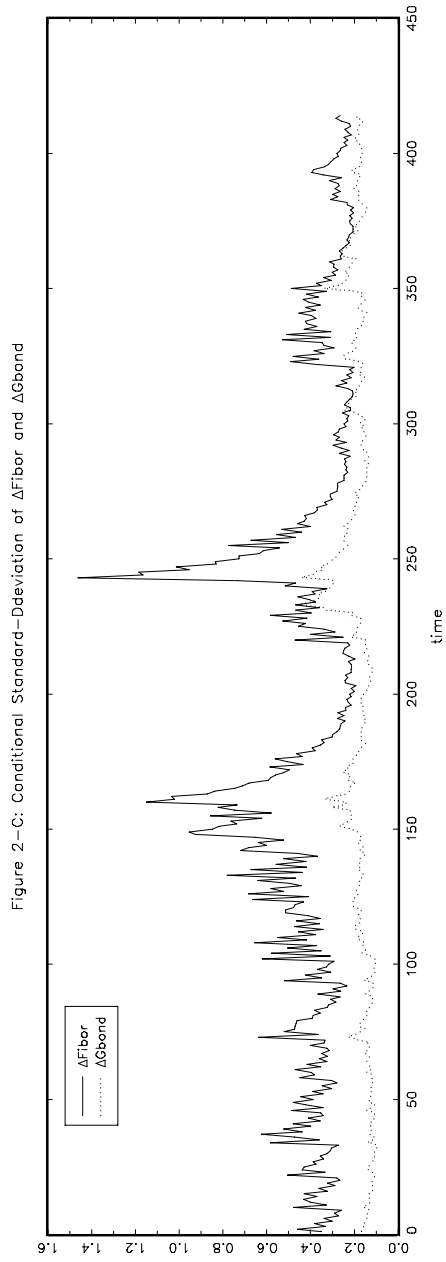
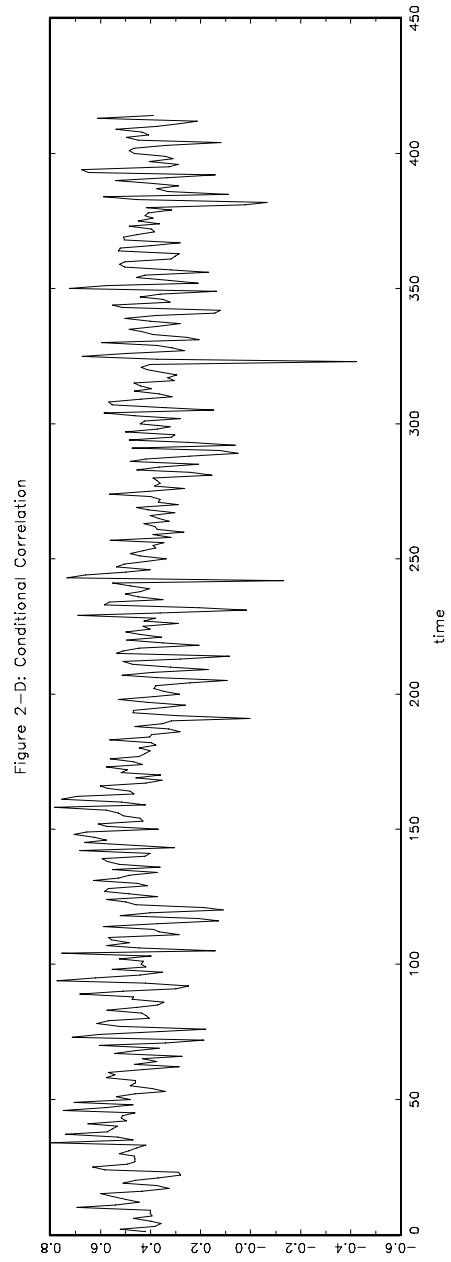


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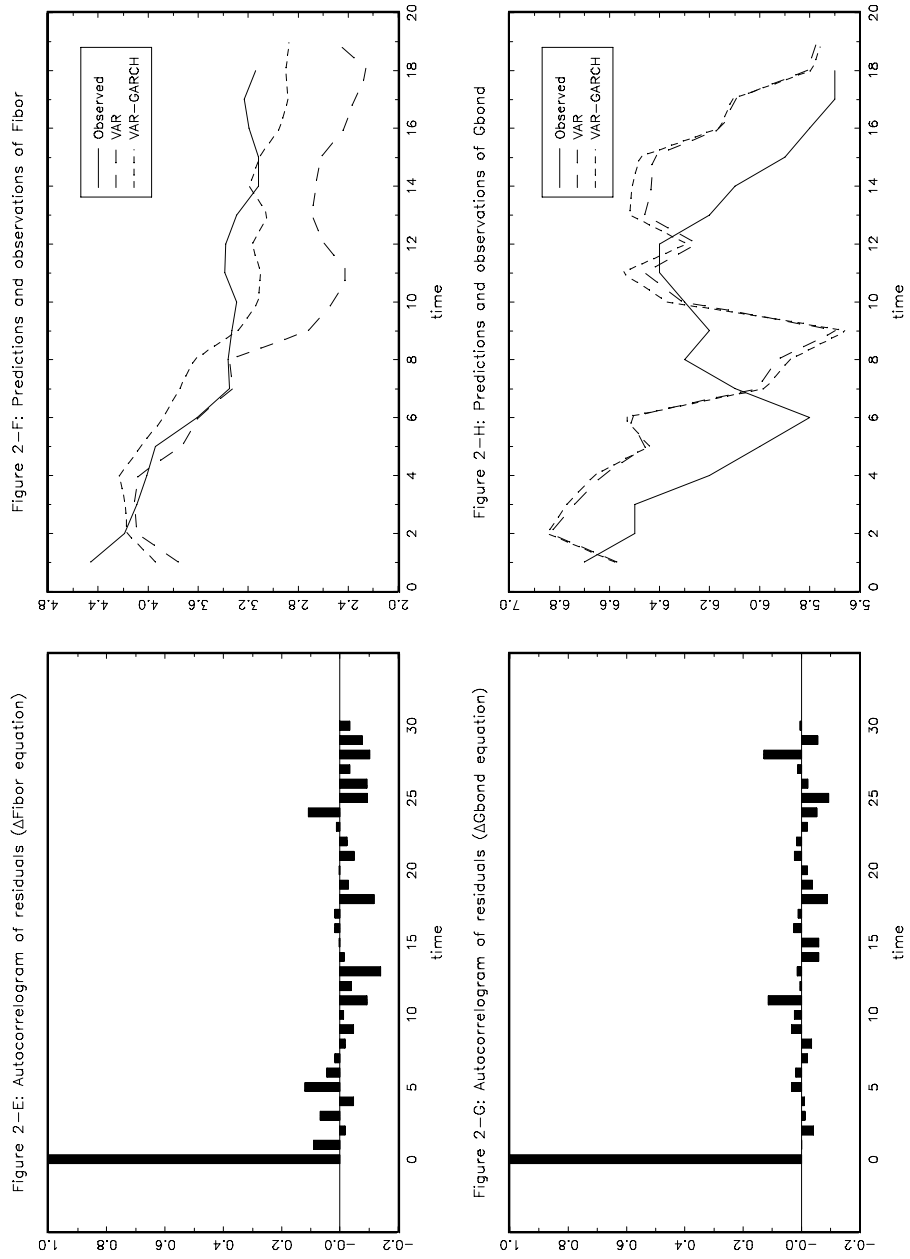


Figure 7: France

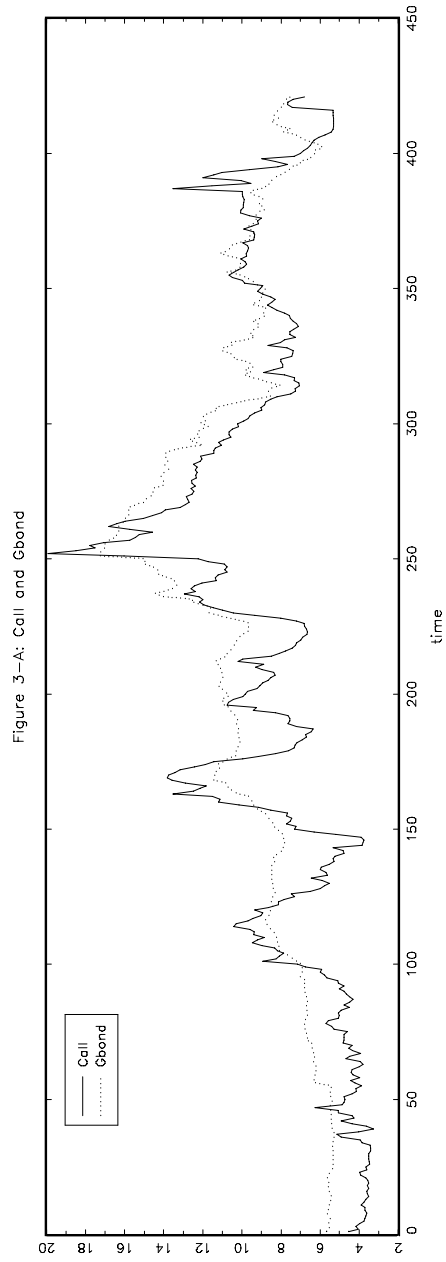
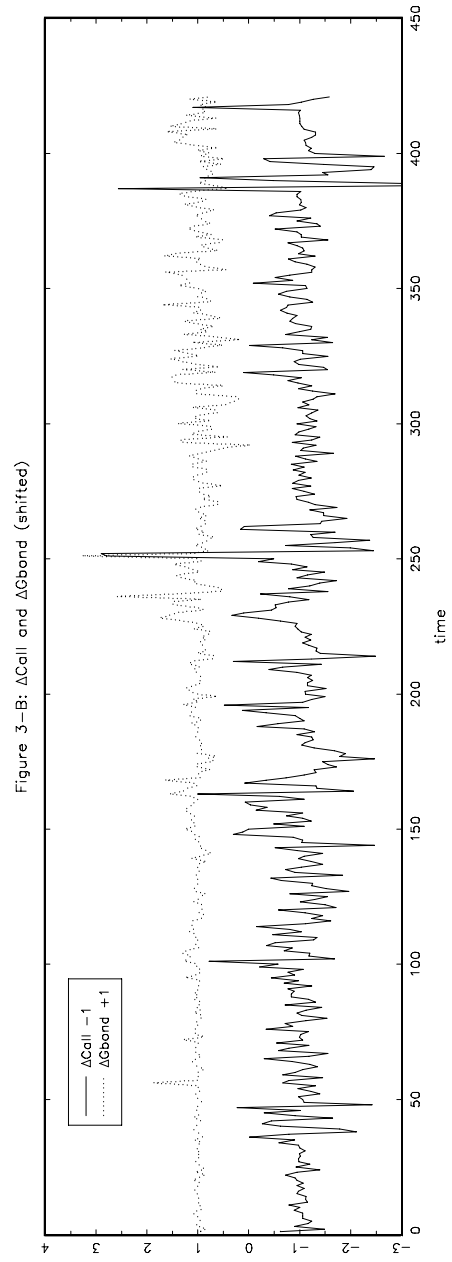


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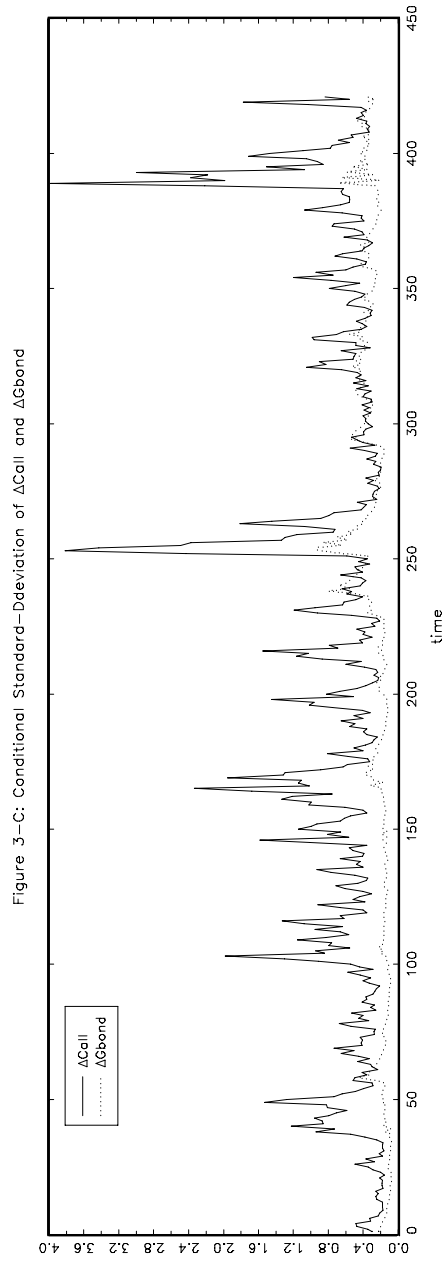
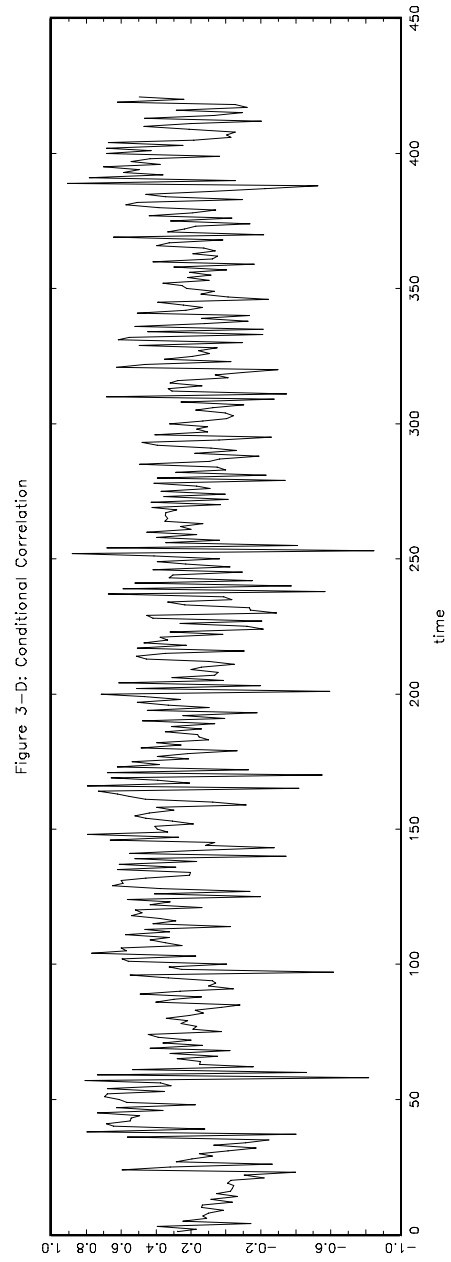


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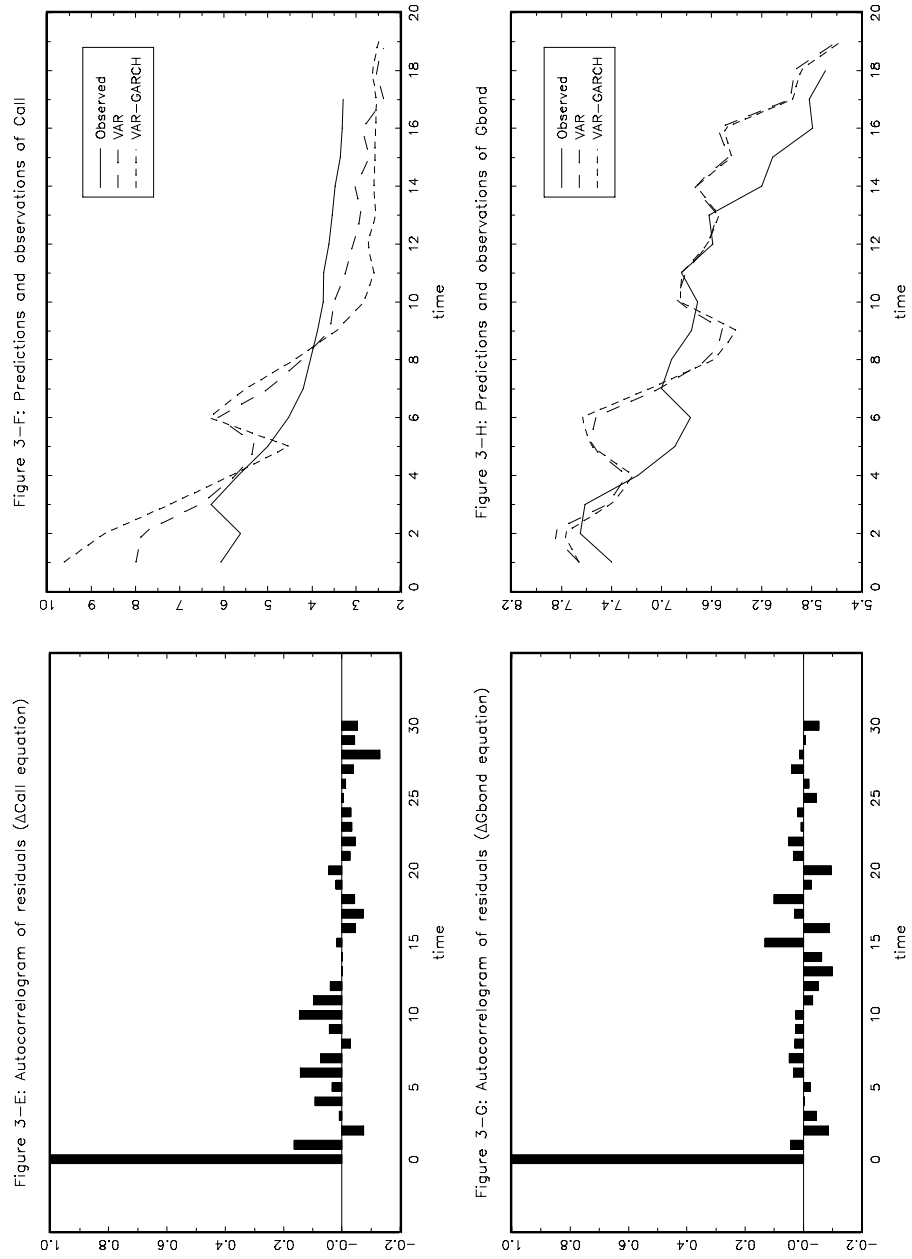


Figure 10: Great Britain

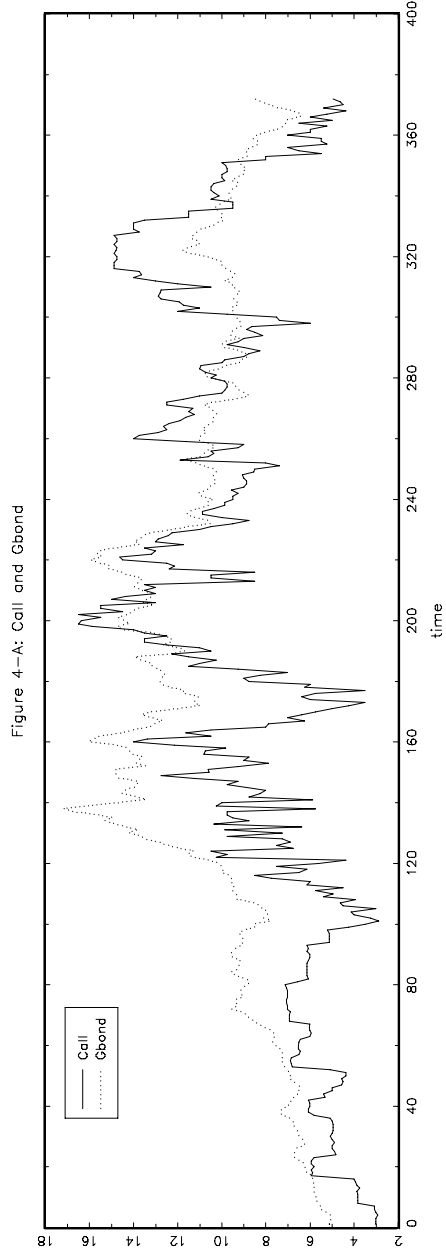
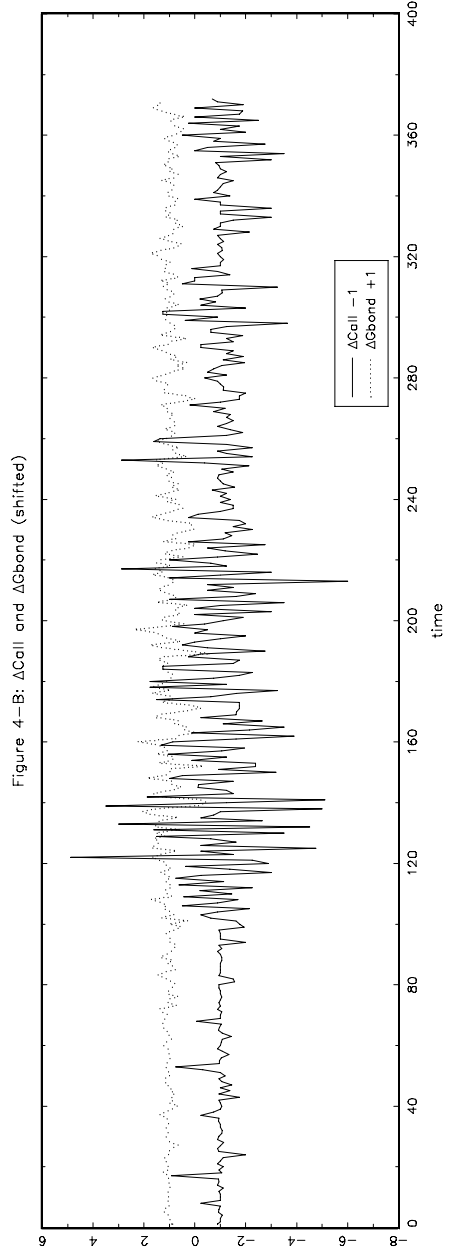


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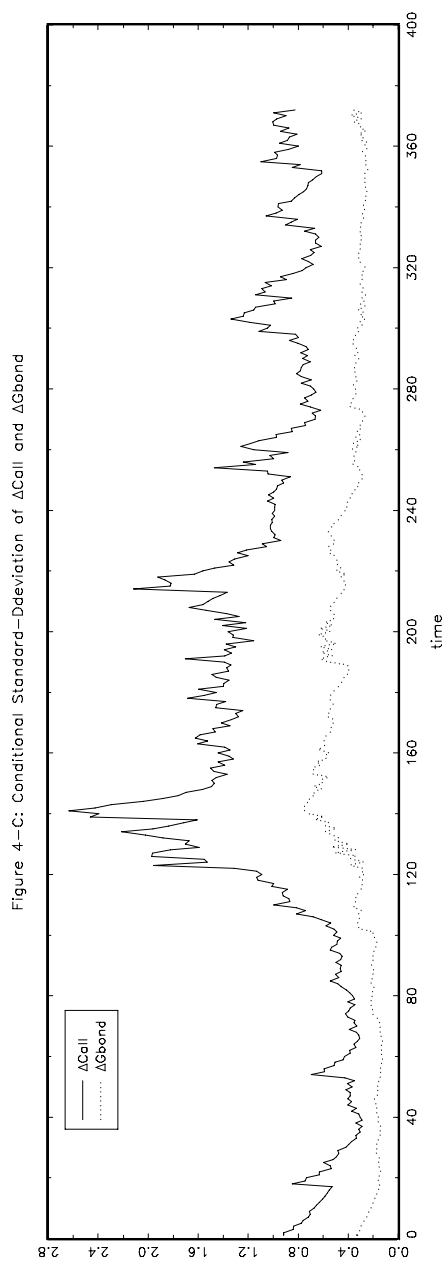
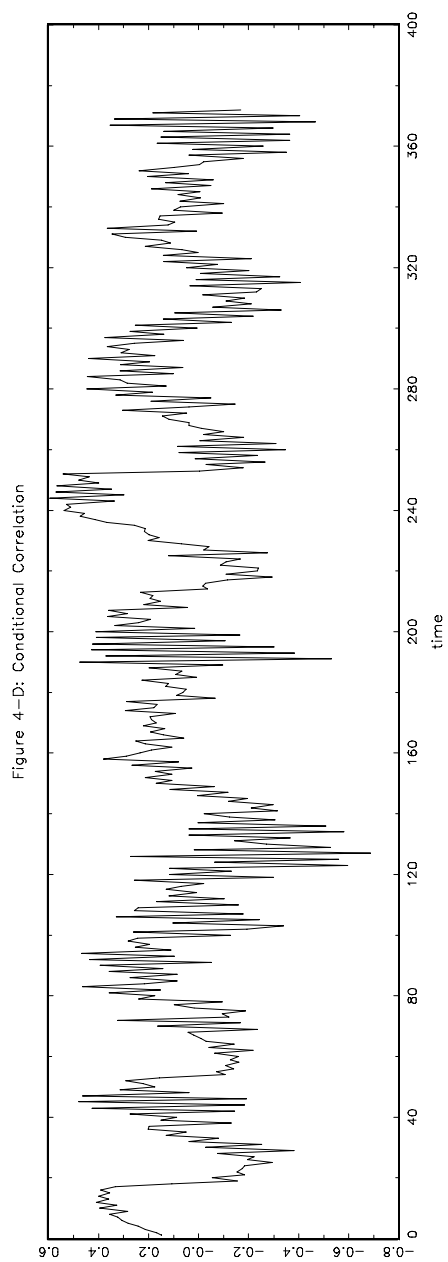


Figure 12: Great Britain

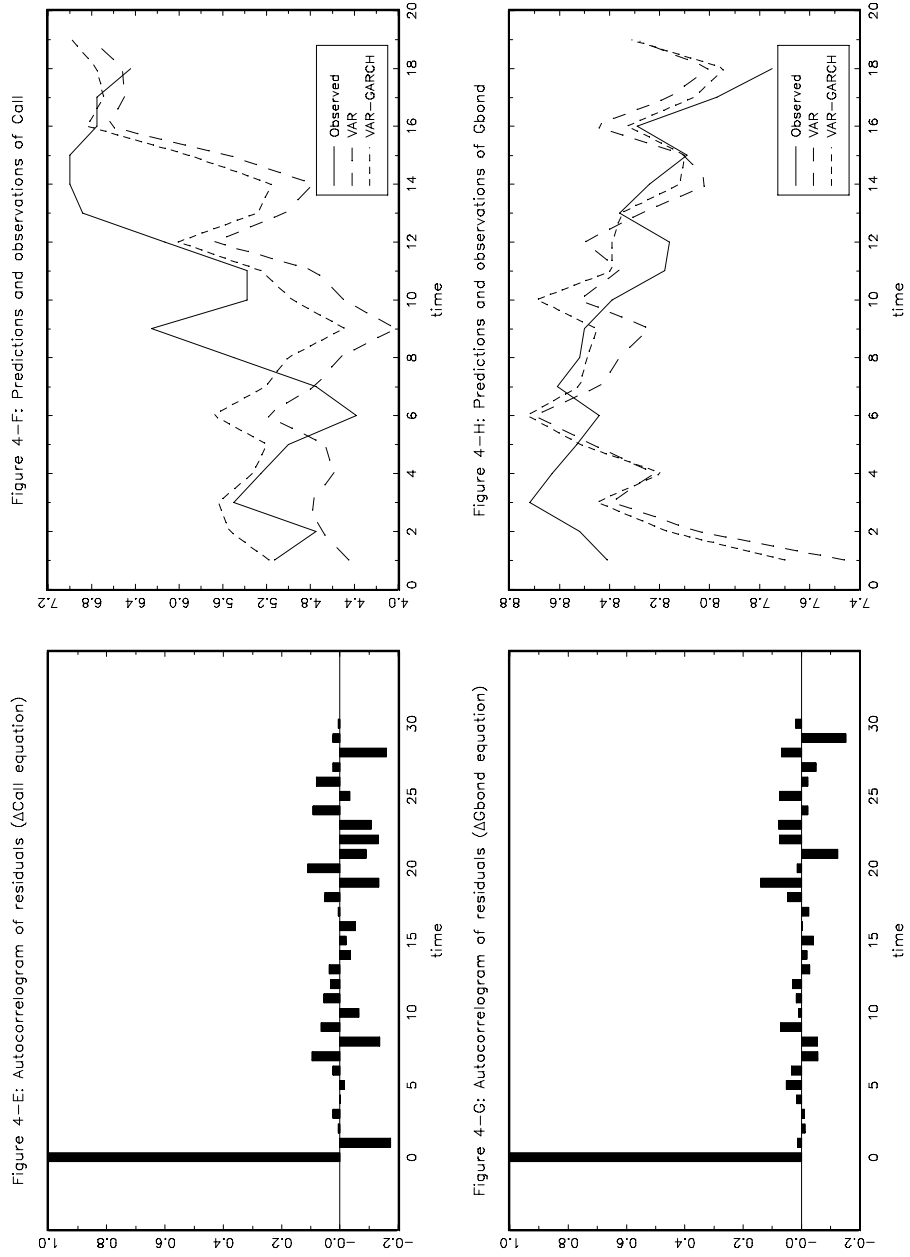


Figure 13: USA

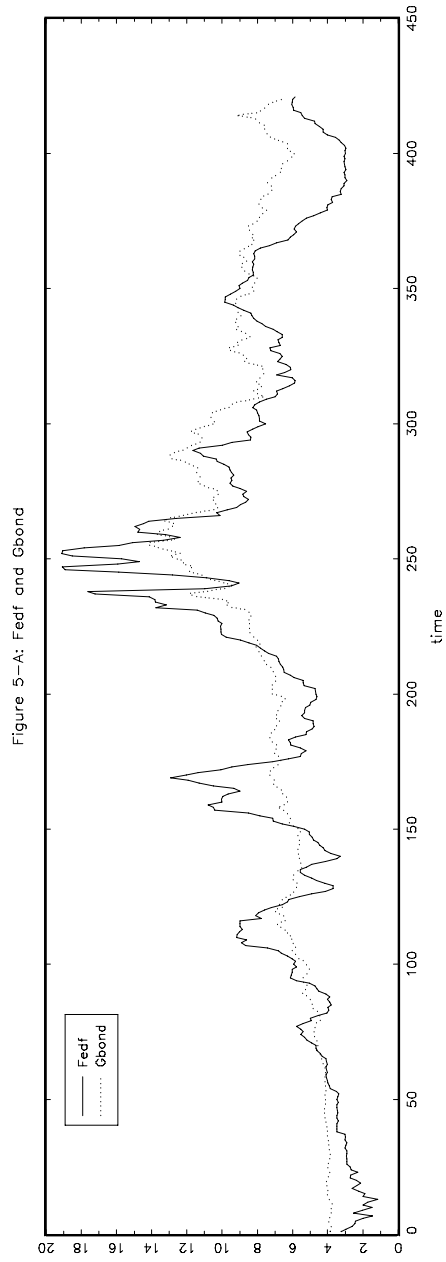
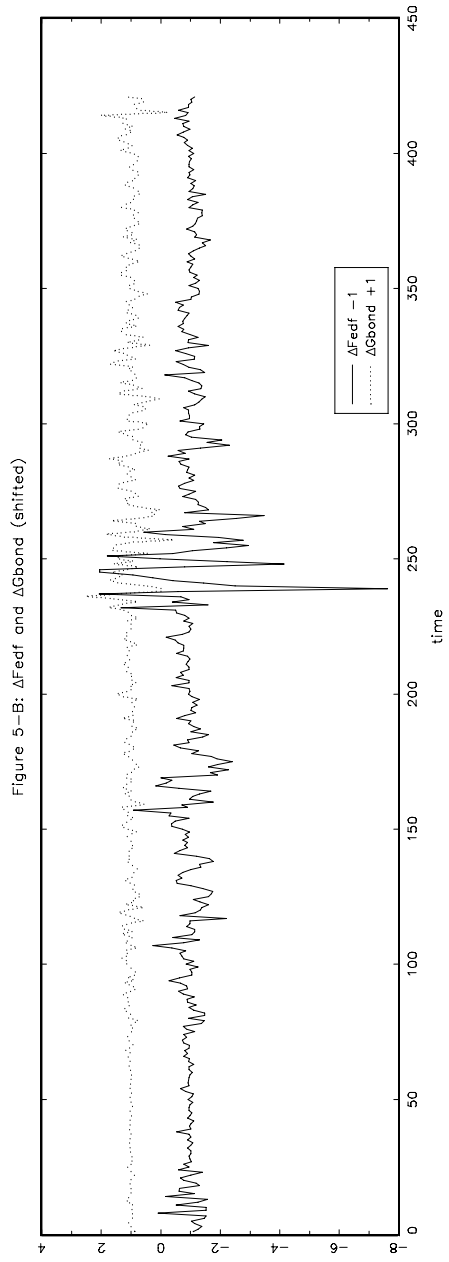


Figure 14: USA

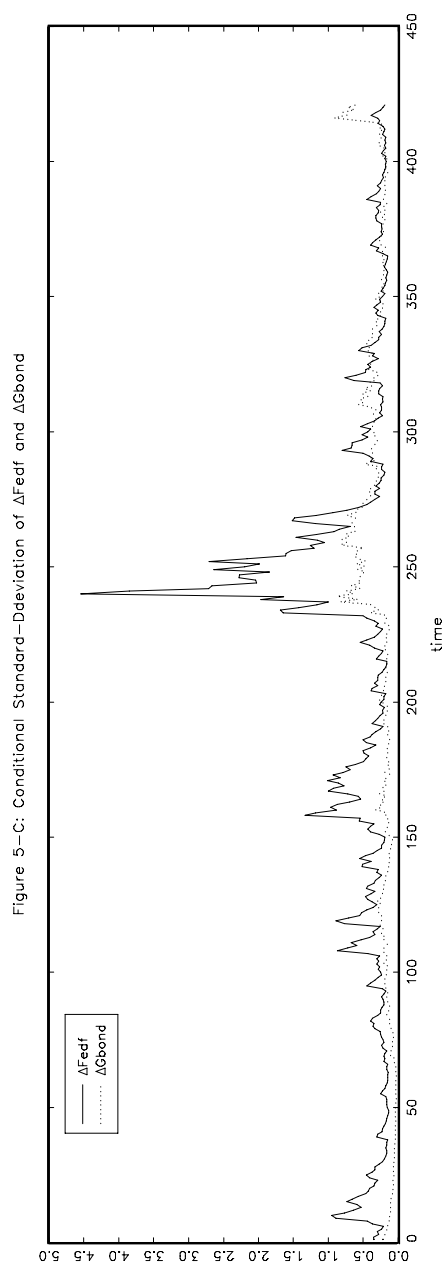
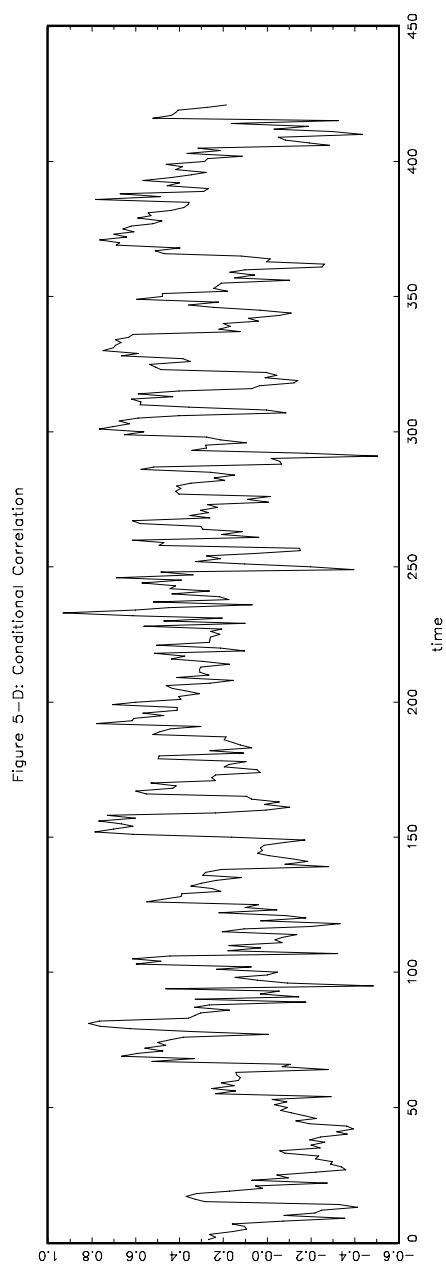


Figure 15: USA

