

Ethnic conflict, power dynamics and growth

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Discussion Paper 2014-8

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December 21, 2013

Abstract

I construct a successive generations model of appropriative competition between ethnicities where de facto power puts an upper-bound on the consumption of each clan. My aim is to derive the implications of the structure of the ethnic and political divisions in terms of economic prosperity. I expose a mechanism reducing negative externalities in consumption implying that long-run economic growth is larger when influence is concentrated rather than dispersed. I find that competition intensity sometimes is beneficial if it is above a threshold or if small groups initially have a large influence. I provide an argument in favour of the Ethno-Linguistic Fractionalization index over a polarization index when considering the impact of diversity on growth. This paper fills a gap in the literature by allowing for the possibility of more than two groups of different sizes in a dynamical setup and by being consistent with the evidence of proportionality between ethnic and political shares in sub-Saharan Africa.

JEL Classification: E21, O43, P48.

Keywords: Economic performance, de facto power dynamics, diversity, ethnic structure, appropriative competition.

*I thank David de la Croix for his help, advice and suggestions. I also thank Raouf Boucekkine, Frédéric Docquier, Frédéric Gaspard, Nippe Lagerlöf, Hélène Latzer, Fabio Mariani, Petros Sekeris and Chrysovalantis Vasilakis for their comments.

1 Introduction

Since Easterly and Levine (1997), there is a consensus among economists that *ethnic diversity* was an obstacle to *growth* in sub-Saharan Africa. However, the good economic performance experienced by Botswana is far away from that of the Democratic Republic of Congo or Mozambique and it seems very likely that political factors have played a major role there. The purpose of this paper is to answer the following question: What is the impact of the structure of the ethnic and political divisions in the relationship between diversity and development in weakly institutionalized societies? I focus on sub-Saharan Africa whose underdevelopment was explained by factors such as governance, natural resources abundance, colonial past and violent conflict¹.

So far, the existing models dealing with the impact of heterogeneity on development have not yet achieved to provide a microfounded framework. Most models integrate diversity in their analysis without neither explicitly considering the structure of the divisions nor the underlying mechanisms that make heterogeneous societies invest less in public good provision or function less efficiently (Ashraf and Galor, 2011). For instance, the model in Alesina and La Ferrara (2004) assumes that diversity enters production positively and the preference towards public goods negatively. Likewise the model presented in Ashraf and Galor (2013) assumes that diversity comes with higher total factor productivity but drives the economy further below the production possibility frontier.

A common theoretical answer to the question of why diverse societies grow less rapidly rests on the contest success function approach that assumes that the amount of resources devoted to the conflict by each side determines the probability of winning the prize (Skaperdas, 1996). Unfortunately, this is not consistent with the evidence of proportionality between political representation and demographic shares shown by the recent evidence gathered about the ethnic belonging of minister cabinet members in 15 African states (Rainer and Trebbi, 2012; Francois et al., 2012).

This paper allows for the possibility of having more than two groups and includes asymmetrical aspects in group sizes and initial strength. In general, many papers on conflicts consider the case of two sides like for example Esteban and Ray (2008) or Hirshleifer (1991). Like many, these models use a static approach. Benhabib and Rustichini (1991), Caselli and Tesei (2011), de la Croix and Dottori (2008) bring dynamical aspects in their bilateral analysis. Tangeras and Lagerlof (2009) is an exception associating a multilateral contest with a long-run perspective. Here, I explicitly include the ethnic structure in a successive generations framework with insecure property rights, an issue that has not been considered yet in that framework. So far, few papers have considered overlapping generations and insecure property rights and have adopted a different approach².

¹Acemoglu and Robinson (2005); Baland et al. (2009); Collier (2010b,a); Caselli and Tesei (2011); Nunn (2007); Collier and Hoeffler (2004); Fearon and Laitin (2003); Wimmer et al. (2009)

²Lagerlof (2012) has a distributive conflict over land among different political entities of a region. De la Croix and Dottori (2008) features a Nash bargaining over the crop and is the first to implement

In this paper, I relate the *structure of the divisions* (i.e. the number and relative magnitude of the groups) to economic growth in a society split in a number of groups of different sizes. I assume that the groups behave like representative agents that have a taste for consumption, dislike making an effort in the political contest and attach value to what they leave to the next generation of their kinship. I assume that the output is common and this gives rise to negative externalities in an investment game as the consumption of one group reduces what is left to the others. I describe a mechanism reducing these negative externalities which is based on a restriction of consumption by *de facto power* that curbs the consumption of the less influential groups and diminishes the number of groups effectively taking part in the strategic interaction. The restriction used here is inspired by the ideas of ethnic politics and patronage and I simply assume that the consumption of a clan cannot exceed a slice of the output equal to its fraction of *de facto power*.

I derive the Nash equilibrium of the appropriative competition of the model and the consequent long-run dynamics of the political shares. I then infer the implications in terms of economic performance and I analyse the impact of changing the ethnic structure, the parameters of the model and the initial conditions. This allows to study the impact of competition intensity on growth. I obtain the result that the mechanism reducing negative externalities is the strongest when power is concentrated in the hands of few dominant groups and therefore, investment and economic growth are higher when power is gathered rather than dispersed.

Another result is that the dynamics of the model imply that the influence share of each group tends to a certain limit value in the long-run. For that, I define the fair representation of a group as the amount of power at which it stops the political action and I distinguish two cases: when the sum of the fair representations of the groups is more or less than unity. In the first case, the limit value of each group is strictly smaller than the fair representation and appropriative efforts are positive in the long-run. Otherwise, the limit value is either strictly larger than or equal to the fair representation, depending on the relative starting position. I deduce from this that a higher competition intensity can lead to a better economic performance when smaller factions are initially proportionately weaker or when competition intensity is already above some threshold.

In the empirical literature on growth, the way to quantify diversity is a debated issue among proponents of the Herfindahl formula-based Ethno-Linguistic Fractionalization index³ and polarization measures⁴. I provide a numerical argument in favor of the former by revealing its monotonous negative link to economic performance seeing that the direction of the relationship with polarization depends on the structure

strategic fertility decisions. Artige (2004) determines the optimal extraction by an infinitely lived dictator facing overlapping generations of agents subject to a no-insurgency constraint. Bellettini and Ceroni (1997) studies transaction costs and Weikard (1997) an intergenerational distributive conflict. Finally, Dincer and Ellis (2005) considers predation activities.

³Easterly and Levine (1997); Collier (2001); Fearon (2003)

⁴Esteban and Ray (1994); Montalvo and Reynal-Querol (2005); Esteban et al. (2012)

of the divisions.

I take the stance to investigate a conflict between politically organized ethnic groups. A priori, the factions competing for resources could be any type of social or political groups. The reasons to focus on ethnicities come from the theory of ethnic conflicts developed following the seminal contribution by Horowitz (1985) that exposes the primordialist and instrumentalist motives.⁵ Bates (1983, 1988) adduce the geographical location of public goods and the cost effectiveness due to the shared language while Fearon (1999) and Caselli and Coleman (2013) refer to the exclusivist nature of ethnic categories. Esteban and Ray (2007, 2008) and Francois et al. (2012) describe the possibility that within-group inequalities result in the salience of ethnicities that are made of rich and poor individuals that are able to contribute funds and labour to the technology of conflict.

I use the concepts of influence and de facto power. Here, de facto power is used to attract resources through patronage and clientelism. De facto power is defined in opposition to de jure power and is what happens in practice or actuality, but not officially established. An extensive literature discusses the causes and consequences of these social phenomena.⁶ For instance, Bates (1983, 1988) and Bardhan (1999) describe the redistributive mechanisms like the marketing boards and the allocation of desirable government and state-owned enterprise jobs. The threat of coups is a reason why politicians rely on patronage by providing a cut of the rents to opposing factions so that the incentive of an attempt diminishes (Collier, 2010b; Rainer and Trebbi, 2012).

Incidentally, this paper is related to the institutional approach to economic development which is concerned with the fundamental causes of growth.⁷ Many authors point to the fact that insecure property rights reduce prosperity through a negative effect on private investment (Acemoglu and Robinson, 2010; Besley and Ghatak, 2009). Besides, the model of this paper explicitly captures the underlying mechanism connecting diversity and growth and is consistent with the proportionality observed in Rainer and Trebbi (2012).

The rest of the paper is organized as follows. Section 2 presents the model, characterizes the equilibrium and derives the resulting power dynamics along with the implications in terms of economic growth. Section 3 presents numerical simulations that are used to highlight the impact of the ethnic structure, of the parameters of the

⁵In the primordialist view, the success of the group has value per-se due to ancestral bonds whereas the instrumentalist view is related to the benefits that the ethnicity generates.

⁶In Besley and Persson (2010), patronage is modelled explicitly and redistribution happens through taxation and spending on a group specific public good. Posner (2005) argues that diversity leads to clientelism and favoritism and that it is natural to measure the distribution of influence around the ethnic aspect. In Padro I. Miquel (2007) the ruler taxes both sides and then returns patronage to the supporters. A similar mechanism is present in Acemoglu et al. (2004) where the ruler avoids challenges thanks to a threat of punishment and reward. (La Porta et al., 1999) mentions that there is costly rent-seeking and conflict over the provision of public goods when dominant clans use their supremacy to appropriate economic benefits.

⁷North et al. (2009); Acemoglu and Robinson (2012); Besley and Persson (2010)

model and of the starting conditions and to reveal the connection between economic growth and the usual measures of diversity. The conclusion contains a final discussion while all proofs are relegated in the appendix.

2 The Model

Time is discrete and infinite, indexed by t ($t \geq 0$). I consider a successive generations framework with agents living for a single period. I assume that the society is divided into N clans, the set of which is denoted $\mathcal{N}$, of respective size n^i with $\sum_{i \in \mathcal{N}} n^i = 1$. The set of states is infinite and denoted by $\mathcal{S} = \mathbb{R}^+ \times [0, 1]^N$ with $K_t \in \mathbb{R}^+$, a stock of common capital at time t and $P_t = (P_t^i)_{i \in \mathcal{N}} \in [0, 1]^N$, a vector of shares of de facto power of the different clans at time t ⁸.

The production function is

$$Y_t = AK_t.$$

This output is subject to an appropriative contest. The parameter A is the total factor productivity, as usual.⁹ Preferences are logarithmic, additively separable and represented by the utility function

$$U_t^i(C_t^i, E_t^i) = \log(C_t^i) + \delta \log(n^i - E_t^i) + \beta \log(P_{t+1}^i Y_{t+1}) \quad (1)$$

C_t^i is total consumption of group i at time t and E_t^i is the total political action of group i . Utility comes from consumption, the leisure fraction of time and the discounted de facto part of production $P_{t+1}^i Y_{t+1}$ left to the following generation of the kinship which is the product of future de facto power P_{t+1}^i times future production Y_{t+1} . The parameter δ measures the cost of effort while β is the discount factor. I assume that the groups do not suffer from the collective action problem.¹⁰

Traditionally, in overlapping generations models, the consumption when old enters the preferences. However, this possibility must be rejected here because the absence of property rights implies that there are no ways to save privately.

I discussed the politics of patronage above. To model this formally, I assume that the total consumption of clan i which is C_t^i cannot exceed the part of production $P_t^i Y_t$. That is to say, an ethnicity is never able to attract more resources than the fraction corresponding to its influence.

$$\forall i \in \mathcal{N} : C_t^i \leq P_t^i Y_t \quad (2)$$

This constraint driven by the appropriative efforts made in the previous period fits in the political economy approach. It is inspired by Benhabib and Rustichini (1991)

⁸With $\sum_{i \in \mathcal{N}} P_t^i = 1$.

⁹This functional form abstracts from labour and helps to focus on allocation and growth issues.

¹⁰Tilly (1978); Olson (1965); Esteban and Ray (1999). They act collectively pursuing their joint interest because the elite of each ethnicity governs the action and redistributes the gains.

and Grossman and Mendoza (2003).¹¹ I interpret the portion of de facto power of a clan in my model as the share of executives in the ministerial cabinet with reference to Rainer and Trebbi (2012) and Francois et al. (2012)¹². I conceptualize this variable as continuous by assuming that a seat in any organization like the natural resources extraction industries or the state-owned and private firms provides de facto power.

Furthermore, the maximum number of people engaging in the political contest is equal to the number of people in the lineage, or

$$0 \leq E_t^i \leq n^i. \quad (3)$$

The changes in power shares and capital are determined by the consumption and political actions of the agents. I assume that capital fully depreciates and that the groups jointly consume the common output. Thus, the law of motion of capital is expressed as follows:

$$K_{t+1} = Y_t - \sum_{i \in \mathcal{N}} C_t^i. \quad (4)$$

K_{t+1} , the stock of capital of the next period is equal to what remains after the present consumption of all groups. Here, property rights are insecure because what each side invests is not private but is subject to the appropriative contest. In other words, savings are pooled together at each period.

Power is used to attract resources and valued per se and the altruistic agents can choose to carry on an effort to increase the future influence of their co-ethnics. This is expressed in the law of motion of power:

$$P_{t+1}^i = \begin{cases} 0 & \text{if } P_t^i + \gamma(E_t^i - \frac{\sum_{j \neq i} E_t^j}{N-1}) \leq 0 \\ P_t^i + \gamma(E_t^i - \frac{\sum_{j \neq i} E_t^j}{N-1}) & \text{if } P_t^i + \gamma(E_t^i - \frac{\sum_{j \neq i} E_t^j}{N-1}) \in]0, 1[\\ 1 & \text{if } P_t^i + \gamma(E_t^i - \frac{\sum_{j \neq i} E_t^j}{N-1}) \geq 1 \end{cases} \quad (5)$$

where γ is the efficiency of the political action. The specification in (5) expresses that the change in de facto power of a group is positively affected by its political effort and negatively by the average effort of the other groups nevertheless staying between

¹¹Benhabib and Rustichini (1991) provides a game theoretic model of conflict between social groups with a distinction between attempted and effective consumption and present an allocation rule that states what each group gets in the case that attempted consumption is above available output. They specify the fraction of revenue that goes to each side. I endogenize this fraction in the form of de facto power. Grossman and Mendoza (2003) presents a model of appropriative competition where resource scarcity engenders a higher amount of time used in the appropriative competition. My model of appropriative competition is comparable to Grossman and Mendoza (2003) in the sense that output is split by an appropriative competition mechanism.

¹²They presents novel information about the ethnic affiliations of the executive cabinet members of 15 African states. These authors listed the names of the cabinet members for the years since independence, and then for each name, they went through different sources of information to attribute the member to an ethnicity.

zero and unity. It reflects the symmetry assumption that all opposing groups are treated equally in the competition.

$$\gamma \left(E_t^i - \frac{\sum_{j \neq i} E_t^j}{N-1} \right)$$

is net flow of power to group i from all the other sides. With this specification, a change in power is persistent because P_t^i is a key determinant of P_{t+1}^i .¹³

I make the following assumptions about the parameters of the model, the sizes of the groups and the initial influence shares. Assumptions 1 to 5 are sufficient to prove the uniqueness of the equilibrium in Proposition 1. Assumption 1 is natural and requires that the parameters are positive and smaller than one. Assumption 2 involves the discount factor and the taste for leisure and asks that these parameters are sufficiently large.

Assumption 1 $\beta, \delta, \gamma \in [0, 1]$.

Assumption 2 $\beta + \frac{\delta}{N-1} > 1$.

Assumption 3 states that at least one group is underrepresented and thus gets rid of the possibility that no group makes a political effort at the equilibrium. This condition is needed to prove the uniqueness in Proposition 1. I define here the underrepresentation of a group as

$$u_t^i = n^i \beta - \frac{\delta P_t^i}{\gamma}$$

and the fair representation

$$P_{\text{fr}}^i = n^i \frac{\beta \gamma}{\delta}.$$

A group is underrepresented if its influence is less than its fair representation and I denote the set of underrepresented groups at $t = 0$ as $\mathcal{U}_0$. Underrepresentation is proportional to the deficit of power to attain the fair representation: $u_t^i = \frac{\delta}{\gamma} P_{\text{fr}}^i - P_t^i$.

Assumption 3 *The initial set of underrepresented groups $\mathcal{U}_0$ is nonempty.*

Assumption 4 is about the initial influence that each group has which must be sufficiently large so that it stays strictly positive in the long run. It expresses the fact that the initial influence of each group is greater than what the other groups are going to take away.¹⁴

¹³The idea of persistence of political power is very present in the literature (Acemoglu and Robinson, 2006, 2008; Acemoglu et al., 2010; Amegashie, 2008).

¹⁴In this paper, I do not consider social exclusion that is absent under the following assumption. Social exclusion is when the de facto power of an ethnicity becomes zero. Byrne (2005) and Sen et al. (2000) discuss social exclusion. An excluded group is deprived of the fruits of modernity and relegated into subsistence.

Assumption 4 $\forall i \in \mathcal{N} : P_0^i > \frac{1}{N-1} \sum_{j \in \mathcal{U}_0} u_t^j \frac{\gamma}{\delta}$.

Assumption 5 involves the number and sizes of the groups and the parameters of the model and requires that no group is either too small nor too large. It guarantees the feasibility of the steady state efforts and the limit power shares when $\beta\gamma > \delta$.¹⁵

Assumption 5 $\forall i \in \mathcal{N} : \frac{1}{N} \left(1 - \frac{\delta}{\beta\gamma}\right) < n^i < \frac{\delta}{\beta\gamma} + \frac{1}{N} \left(1 - \frac{\delta}{\beta\gamma}\right)$.

The model can be solved under the assumptions 1 to 5. (C_t, E_t) is an equilibrium at time t whenever it is such that each group chooses (C_t^i, E_t^i) to maximize utility given by (1) subject to the constraints (2) and (3) and the laws of motion (4) and (5) when the other groups choose (C_t^{-i}, E_t^{-i}) .

Thanks to its log-linearity, the utility function (1) can be rewritten as

$$U_C(C_t^i, C_t^{-i}) + U_E(E_t^i, E_t^{-i}) + \text{constant}$$

where $U_C(C_t^i, C_t^{-i}) = \log(C_t^i) + \beta \log(K_{t+1})$, $U_E(E_t^i, E_t^{-i}) = \delta \log(n^i - E_t^i) + \beta \log(P_{t+1}^i)$ and $\text{constant} = \beta \log(A)$ so that the optimal consumption and political effort rules can be investigated separately. Using the above, equivalently, (C_t, E_t) is an equilibrium at time t whenever C_t is such that each group chooses C_t^i to maximize

$$U_C(C_t^i, C_t^{-i}) = \log(C_t^i) + \beta \log(K_{t+1}) \quad (6)$$

subject to $0 \leq C_t^i \leq P_t^i Y_t$ and to the law of motion (4) given the optimal values for all other groups C_t^{-i} and E_t is such that each group chooses E_t^i to maximize

$$U_E(E_t^i, E_t^{-i}) = \delta \log(n^i - E_t^i) + \beta \log(P_{t+1}^i) \quad (7)$$

subject to $0 \leq E_t^i \leq n^i$ and to the law of motion (5) given the optimal values for all other groups E_t^{-i} . I derive the first order necessary conditions and complementary-slackness conditions of the investment game given in (6) and the appropriative competition given in (7) in appendix A. From this, I obtain the best response functions.

The following proposition states that a temporary equilibrium exists and is unique and then characterizes it. The sets $\mathcal{I}$ and $\mathcal{O}$ denote respectively whether the concerned constraint of a group is loose or tight¹⁶.

Proposition 1 *Under the assumptions 1 to 5,*

(i) *A pure strategy temporary equilibrium exists.*

(ii) *The pure strategy temporary equilibrium is unique.*

¹⁵and is always true when $\beta\gamma < \delta$.

¹⁶indicated with a c (for $C_t^i \leq Y_t^i$) or e (for $E_t^i \geq 0$) subscript

(iii) At the pure strategy temporary equilibrium,

(a.) there is a partition $(\mathcal{I}_c, \mathcal{O}_c)$ of $\mathcal{N}$ with $M = \#(\mathcal{I}_c)$ such that

$$C_t^i = \begin{cases} \frac{Y_t(1-\sum_{j \in \mathcal{O}_c} P_t^j)}{M+\beta} & \text{for } i \in \mathcal{I}_c \\ Y_t P_t^i & \text{for } i \in \mathcal{O}_c \end{cases}$$

(b.) and there is a partition $(\mathcal{I}_e, \mathcal{O}_e)$ of $\mathcal{N}$ with $L = \#(\mathcal{I}_e)$ such that

$$E_t^i = \begin{cases} \frac{(N-1)(\beta+\delta)}{(N-1)\beta+N\delta} \left(u_t^i + \delta \frac{\sum_{j \in \mathcal{I}_e} u_t^j}{(N-1)\beta+(N-L)\delta} \right) & \text{for } i \in \mathcal{I}_e \\ 0 & \text{for } i \in \mathcal{O}_e. \end{cases}$$

Proof The proof is in appendix B. ■

Using Rosen (1965), the existence of the equilibrium follows from the fact that the strategy sets are nonempty, compact and convex and also that the utility functions are continuous and quasiconcave. The diagonal-strict-concavity of the utility function is sufficient to demonstrate the uniqueness of the equilibrium of the investment game given in (6). Each time, the equilibrium is described using a partition of the groups into two sets.

The characterization of the consumption decisions is given in part (iii.a) of the proposition. The M groups with more influence have a nonbinding resource constraint. Strategically, they consume a fraction of the output not yet consumed by the groups in $\mathcal{O}_c$ inversely proportional to M , the number of ethnicities in $\mathcal{I}_c$. The $N - M$ weaker groups have a binding resource constraint. They consume as much as the resource constraint allows.

The characterization of the political effort decisions is given in part (iii.b) of the proposition where L is the number of ethnicities in the set $\mathcal{I}_e$. The more underrepresented groups make a positive political effort and the less underrepresented groups make a zero effort. From the best response function if a group is underrepresented, it makes a positive effort.

Steady State of the Power Dynamics

I have defined the fair representation of a group as

$$P_{fr}^i = n^i \frac{\beta\gamma}{\delta}$$

which is the amount of influence at which a group stops the political action if the other sides are not acting either (see the best response function (12)). This fair representation is proportional to n^i , the size of the group. It increases with β and γ and decreases with δ which is sensible because β is the discount factor, γ is the efficiency of the effort in the contest and δ is the cost of political effort.

The following proposition describes the long-run dynamics of the influence shares. It distinguishes the cases where $\beta\gamma$ is smaller or larger than δ .

Proposition 2 *Under the assumptions 1 to 5, the power dynamics are such that P^i tends to*

- (i) *A value larger than P_{fr}^i (from above) or equal to P_{fr}^i (from below) when $\beta\gamma < \delta$.*
- (ii) *A steady state influence share $P_{ss}^i = P_{fr}^i - \frac{1}{N} \left(\frac{\beta\gamma}{\delta} - 1 \right) < P_{fr}^i$ when $\beta\gamma > \delta$.*
- (iii) *n^i when $\beta\gamma = \delta$.*

for each $i \in \mathcal{N}$.

Proof The proof is in appendix C. ■

When $\beta\gamma < \delta$, the dynamics of P^i follow one out of three potential cases. The first possibility is that the initial strength of group i is below its fair representation. Then, the lobbying effort of the group is positive and P_t^i tends asymptotically from below to P_{fr}^i . A second possibility is that the initial strength is above the fair representation. Then, the lobbying effort is zero and P_t^i tends asymptotically from above to a value strictly greater than P_{fr}^i . This value is equal to

$$P_0^i - \frac{1}{N-1} \sum_{j \in \mathcal{I}_e} (P_{fr}^j - P_0^j)$$

and depends on how much total effort the underrepresented groups make.¹⁷ A third pattern is possible where the influence first decreases and then increases towards the fair representation.¹⁸ Under the condition $\beta\gamma < \delta$, the efforts always decrease asymptotically towards zero.

At the opposite, when $\beta\gamma > \delta$, the dynamics of P^i are decreasing when $P^i > P_{ss}^i$ and asymptotically increasing towards the steady state influence share P_{ss}^i when $P^i < P_{ss}^i$. At the steady state all sides are underrepresented and make an equal positive political effort of

$$e_{ss} = \frac{1}{N} \left(1 - \frac{\delta}{\beta\gamma} \right).$$

Therefore, larger groups have relatively more power. The value of $\frac{\beta\gamma}{\delta}$ determines how more influential bigger ethnicities relatively are. Finally, when $\beta\gamma = \delta$, the power shares tend to the demographic percentages and the political efforts tend to zero.

¹⁷The first and second possibilities are illustrated by the evolution of P^1 and P^2 in Figure 1b and by the evolution of P^1 in Figure 1a respectively.

¹⁸The initial strength of group i is above the fair representation. The political effort is zero and P_t^i decreases to a value strictly lower than P_{fr}^i in a first movement. After that, P_t^i increases asymptotically to P_{fr}^i like in the second possibility.

Steady State Power Configuration, Investment and Growth

Proposition 1 implies that the total amount invested is equal to

$$I_t = \frac{\beta}{M + \beta} Y_t (1 - \sum_{j \in \mathcal{O}_c} P_t^j) \quad (8)$$

when the power configuration is P_t . M is the number of groups making a positive investment and $\sum_{j \in \mathcal{O}_c} P_t^j$ is the total influence of the groups who are not investing. The implication concerning the impact of the power configuration on growth is that a change in the power configuration that leaves $\mathcal{I}_c$ unchanged but decreases $\sum_{j \in \mathcal{O}_c} P_t^j$ boosts investment and economic expansion. A larger share of the pie going to the groups who do a positive investment and a smaller number of these groups positively determine growth in this model.¹⁹

Insecure property rights come with negative externalities in consumption that are reduced here by the resource constraints. In this case, it happens for two reasons that equation (8) highlights. The first is because constrained groups consume less. The second acts through a reduction of M , the number of effective groups taking part in the investment game as investment is proportional to $\frac{\beta}{\beta + M}$. In this model, progress is possible when the accumulation implied by the power configuration resulting from the ethnic composition, the initial strength distribution and the power dynamics is sufficiently high. Evidently from (8), total factor productivity A promotes economic growth.²⁰ Additionally, the number of divisions has a negative effect on income rise.

The discussion above exposes that growth is higher when de facto power is concentrated rather than dispersed. I analyze here the implications of the value of the parameters β and $\frac{\gamma}{\delta}$ on the long-run economic performance when $\beta\gamma < \delta$ (i) and then when $\beta\gamma > \delta$ (ii). β has a direct positive effect on investment through $\frac{\beta}{M + \beta}$ in the investment function (8) and an indirect effect through the power distribution attained while $\frac{\gamma}{\delta}$ has an indirect effect only. When studying long-run economic success here, it is not useful to disentangle the effect of γ and δ from their ratio because these parameters come in the formula of the fair representation and steady state power only in this form.

¹⁹It should also be noticed that a variation in the power configuration that changes the equilibrium partition has a continuous impact on investment and on growth. To do so, imagine that the change causes a cluster that was formerly in $\mathcal{I}_c$ to belong to $\mathcal{O}_c$. This variation puts two effects at work. First, M decreases by one which has a positive effect on investment. Second, $\sum_{j \in \mathcal{O}_c} P_t^j$ increases by the value of the influence of the cluster that has switched membership which has a negative effect on growth. Those two effects exactly offset because

$$\frac{\beta}{M + \beta} (1 - \sum_{j \in \mathcal{O}_c} P_t^j) = \frac{\beta}{M - 1 + \beta} (1 - \sum_{j \in \mathcal{O}_c} P_t^j - P_x)$$

when $P_x = \frac{1}{M + \beta} (1 - \sum_{j \in \mathcal{O}_c} P_t^j)$ which is the threshold value at which cluster x switches membership to $\mathcal{O}_c$.

²⁰There is no voracity effect like in Tornell and Lane (1999) or in Strulik (2009) here.

When $\beta\gamma < \delta$, the long-run distribution of power attained is described by Proposition 2 part (i). The indirect effects of β and $\frac{\gamma}{\delta}$ depend on the size of the groups relative to their initial strength because this determines if the influence share tends to the fair representation or stays strictly above it.

When small groups have a starting power larger than their fair representation, the effect of β and $\frac{\gamma}{\delta}$ can go both directions due to the positive impact of the greater fair representation of the bigger groups and the negative impact of the reduction of underrepresentation which is the driving factor of the decrease of the power of the smaller groups. At the opposite, when small groups are weak and have an initial strength less than their fair representation, the higher β and $\frac{\gamma}{\delta}$ are, the less is long-run growth because it relaxes the consumption constraint of the smaller groups. These issues are investigated numerically in the sensitivity analysis of the next section.

When $\beta\gamma > \delta$, the long-run distribution of power attained is described by Proposition 2 part (ii). Again, there is an indirect effect of β and $\frac{\gamma}{\delta}$. Now however, it is because the value of $\frac{\beta\gamma}{\delta}$ determines the extent of relative excess of the influence of the larger groups seeing the steady state power of a clan is equal to the fair representation, proportional n^i minus the steady state political action, constant across groups. As long as the consumption constraint of some groups is tight, a greater value of $\frac{\beta\gamma}{\delta}$ engenders more investment. Combining the direct and indirect effects, when $\beta\gamma > \delta$, β and $\frac{\gamma}{\delta}$ have an unambiguously positive effect on economic performance.

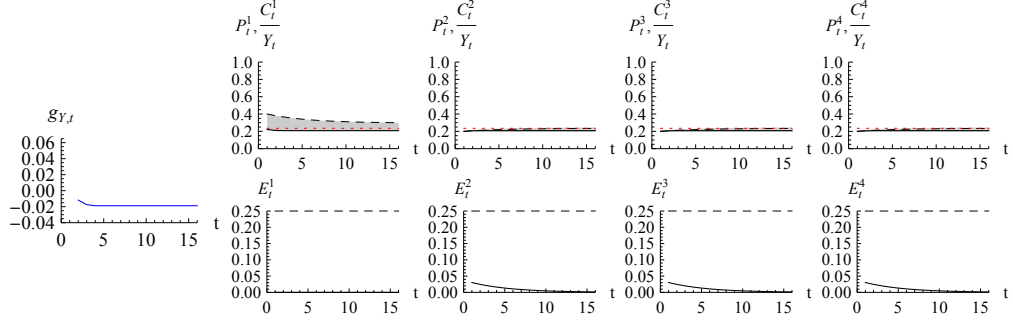
3 Numerical Simulations

3.1 Effect of the ethnic structure on economic growth

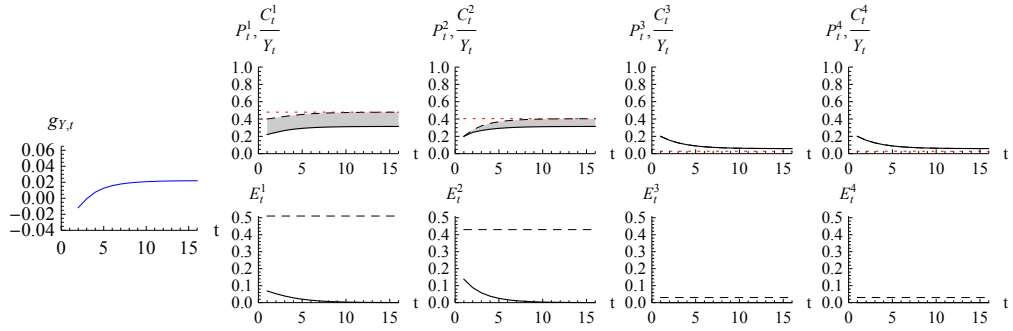
I show in appendix D that it is possible to compute the equilibrium and the long-run dynamics of the model numerically. In Figures 1 and 2, I present the evolution over time of the equilibrium strategies, the influence shares and economic growth in six numerical simulations for two sets of parameter values.

I concentrate on the case of four different sides which is enough to illustrate the mechanisms at work. I assume that the starting strength distribution is inherited from a transition, possibly the end of the colonial era or the end of an internal conflict situation. I assume the existence of a more influential or historically leading ethnicity that has a larger initial strength. Apart from this, I suppose that power is split evenly and I take the starting strength configuration (0.4, 0.2, 0.2, 0.2).

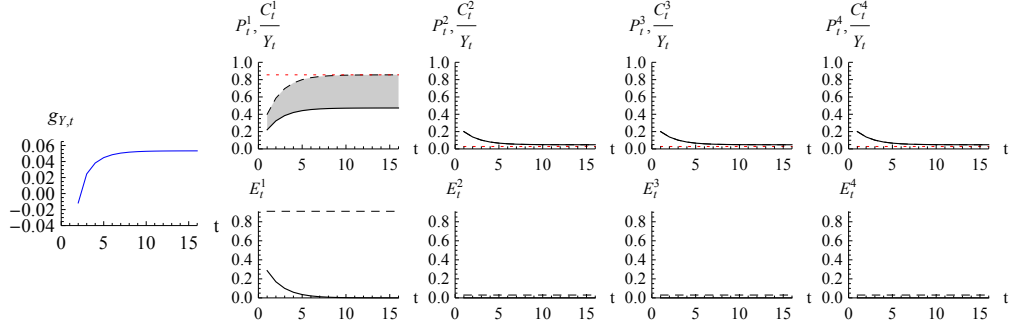
The benchmark parametrization is inspired by the literature (Lucas, 1990; Kydland and Prescott, 1982). The value of the preference parameter of leisure δ comes from Lucas (1990). The value of total factor productivity is such that progress arises for investment rates higher than 10%. The discount factor is chosen to match an annual discounting equal to 0.99 and periods of 20 years (King and Rebelo, 1999). I vary the ethnic structure among three typical patterns referred to as fractionalization, polarization and homogeneity.



(a) Fractionalized ethnic structure. $n_1 = 0.25, n_2 = 0.25, n_3 = 0.25, n_4 = 0.25$



(b) Polarized ethnic structure. $n_1 = 0.51, n_2 = 0.43, n_3 = 0.03, n_4 = 0.03$

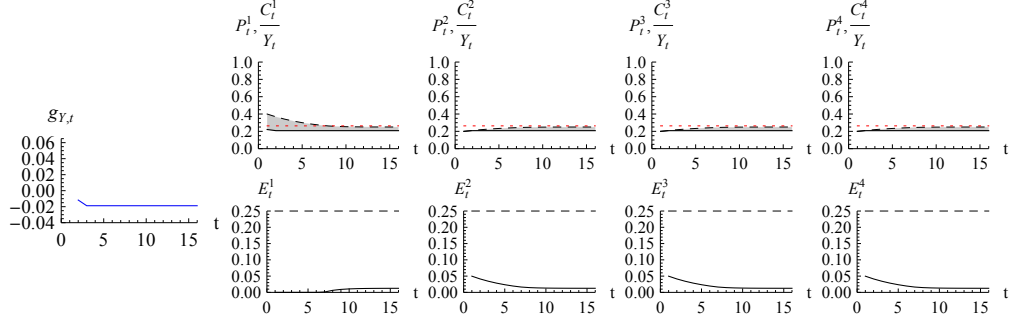


(c) Homogeneous ethnic structure. $n_1 = 0.91, n_2 = 0.03, n_3 = 0.03, n_4 = 0.03$

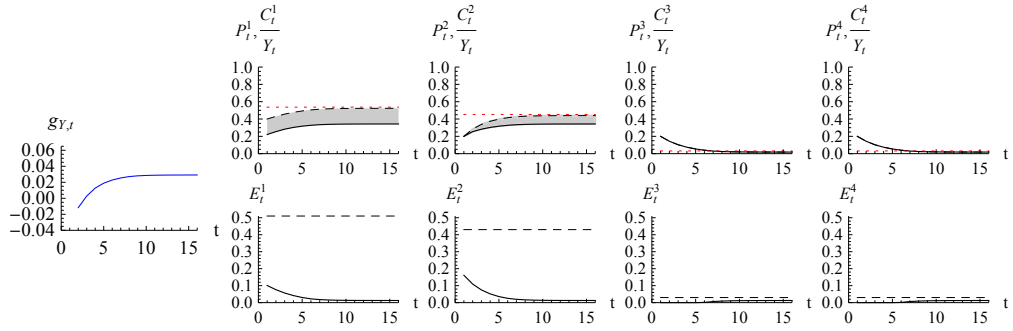
Figure 1: Evolution over time of the economic growth rate together with the influence share, consumption as a share of output and political effort of each group. Parameter values are $A = 10$, $\beta = 0.81$, $\delta = 0.56$ and $\gamma = 0.65$. ((i) $\beta\gamma < \delta$ in Proposition 2.) A different ethnic structure is considered in each panel (a) to (c).

In the top line: the dashed horizontal red line is P_{fr}^i , the fair representation of group i , the thick dashed line is P_t^i , the thick solid line is the consumption of group i as a fraction of output and the shaded vertical distance is the investment by group i as a fraction of total output.

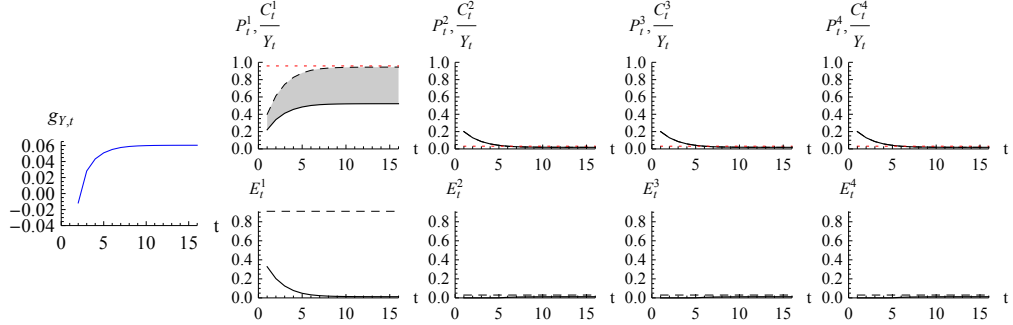
In the bottom line, the horizontal dashed line is n^i and the solid line is E_t^i .



(a) Fractionalized ethnic structure. $n_1 = 0.25, n_2 = 0.25, n_3 = 0.25, n_4 = 0.25$



(b) Polarized ethnic structure. $n_1 = 0.51, n_2 = 0.43, n_3 = 0.03, n_4 = 0.03$



(c) Homogeneous ethnic structure. $n_1 = 0.91, n_2 = 0.03, n_3 = 0.03, n_4 = 0.03$

Figure 2: Evolution over time of the economic growth rate together with the influence share, consumption as a share of output and political effort of each group. Parameter values are $A = 10$, $\beta = 0.81$, $\delta = 0.5$ and $\gamma = 0.65$. ((ii) $\beta\gamma > \delta$ in Proposition 2.)

A different ethnic structure is considered in each panel (a) to (c).

In the top line: the dashed horizontal red line is P_{fr}^i , the fair representation of group i , the thick dashed line is P_t^i , the thick solid line is the consumption of group i as a fraction of output and the shaded vertical distance is the investment by group i as a fraction of total output.

In the bottom line, the horizontal dashed line is n^i and the solid line is E_t^i .

When $\beta\gamma < \delta$ in Figure 1, the political efforts are either positive or sometimes zero. Here, the fair representation is less than the demographic share. All groups with an initial influence below the fair representation do a positive political effort and their influence share increases towards the fair representation. At the opposite, the groups with an initial influence above the fair representation do a zero political effort and their influence decreases however staying strictly above the fair representation.

When $\beta\gamma > \delta$ in Figure 2, the political efforts are sometimes zero but tend to a strictly positive steady state value in the long run. Here the fair representation is more than the demographic share and the steady state power shares are strictly below the fair representations.

Economic growth is larger when the ethnic structure is closer to homogeneity. This is due to the reduction of M , the number of effective group taking part in the investment game. When the ethnic structure is changed, this number goes down from 4 for fractionalization to 2 for polarization and then to 1, for homogeneity. The shaded vertical distance is the investment as a share of output by each group. In panel (a) in both figures, each group makes a minor investment whereas in panel (c), only the largest group invests but this investment is substantial.

3.2 Diversity measures and economic growth

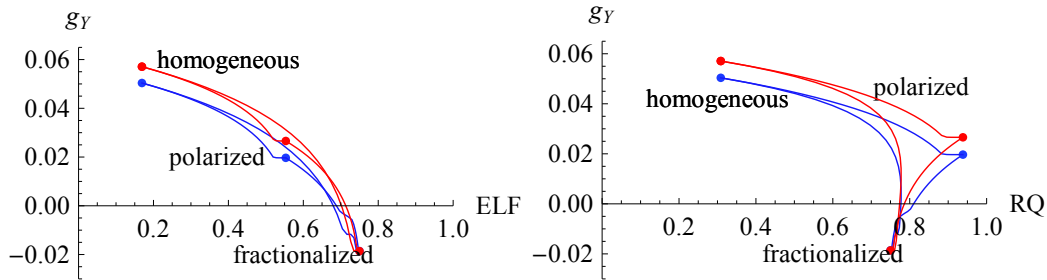


Figure 3: Relationship between long-run growth and ELF and between long-run growth and RQ.

Blue curves for parametrization (a) $A = 10, \beta = 0.81, \delta = 0.56, \gamma = 0.65$.

Red curves for parametrization (b) $A = 10, \beta = 0.81, \delta = 0.5, \gamma = 0.65$.

I now turn to the question of the shape of the relationship between economic performance and heterogeneity measures in my model. In Figure 3, I present the results of three experiments where I leave the parametrization and the starting conditions as in the numerical simulations of Figures 1 and 2. To carry out this, I take all possible pairs of ethnic compositions $\mathbf{n}_A$ and $\mathbf{n}_B$ among the ones used before and I replicate the computation of the model with an ethnic composition $\mathbf{n} = \alpha\mathbf{n}_A + (1 - \alpha)\mathbf{n}_B$ for a grid of values of α with $0 \leq \alpha \leq 1$. Each time, I record the value of the long-run growth rate, the ELF index and the RQ index. The values of the indices for each

structure are computed using

$$ELF \text{ index} = \sum_{i \in \mathcal{N}} 1 - n^i{}^2$$

and

$$RQ \text{ index} = 1 - \sum_{i \in \mathcal{N}} \left(\frac{0, 5 - n^i}{0, 5} \right)^2 n^i.$$

Figure 3 shows the obtained relationships between the ELF index and growth and between the RQ index and growth. The study of this figure reveals that the RQ index is linked to economic performance either positively or negatively depending on the structure of the divisions whereas the ELF is always negatively related. This is because growth and RQ both increase when changing the ethnic structure from a fractionalized to a polarized one. This finding suggests that, among these two possibilities, heterogeneity measured by the ELF formula is a better predictor of economic performance, at least with respect to the mechanism studied in this paper.

3.3 Competition intensity and economic growth

I study the sensitivity to changing the value of the parameters on the economic growth rate in Figures 4 and 5. I use the three reference ethnic compositions and consider corresponding initial strength configurations where the smaller clusters are either strong or weak i.e. have a starting power larger or smaller than their fair representation.²¹ It is important to notice that when $\frac{\beta\gamma}{\delta}$ becomes larger than one, the implications in term of growth vary continuously because the power dynamics collapse to Proposition 2 part (iii) from both sides. This threshold is indicated by the vertical dashed lines in Figures 4 and 5. Each time, to the left of this line, $\beta\gamma < \delta$ and the dynamics of influence follow part (i) of Proposition 2 and to the right of this line, $\beta\gamma > \delta$ and the dynamics of influence follow part (ii) of Proposition 2.

Figure 4 reports the results of the sensitivity analysis for a change in β that takes values in the range $[0.60, 0.91]$. The values of $\gamma = 0.65$ and $\delta = 0.5$ are fixed. The ethnic composition considered is indicated below each part (a) to (f) of the figure as well as whether small groups are initially strong or weak i.e. have a starting power larger or smaller than their fair representation. In Figure 4, the positive effect of β appears. This is quite sensible in reference to the investment function (8). However, in part (d), with a fractionalized structure and weak smaller groups, the negative indirect effect gives rise to a S-shaped relationship.

Figure 5 reports the results of the sensitivity analysis for a change in $\frac{\gamma}{\delta}$ that takes values in the range $[0.68, 1.32]$. The value of $\beta = 0.81$ is fixed. The parts of the figure are organized as previously. Here, the effect of contest intensity on performance happens only indirectly through the power configuration. It is negative when the structure is fractionalized in (a) and (d) and when $\beta\gamma < \delta$ for all subcases except (b)

²¹To respect the condition, I change the initial power configuration if necessary.

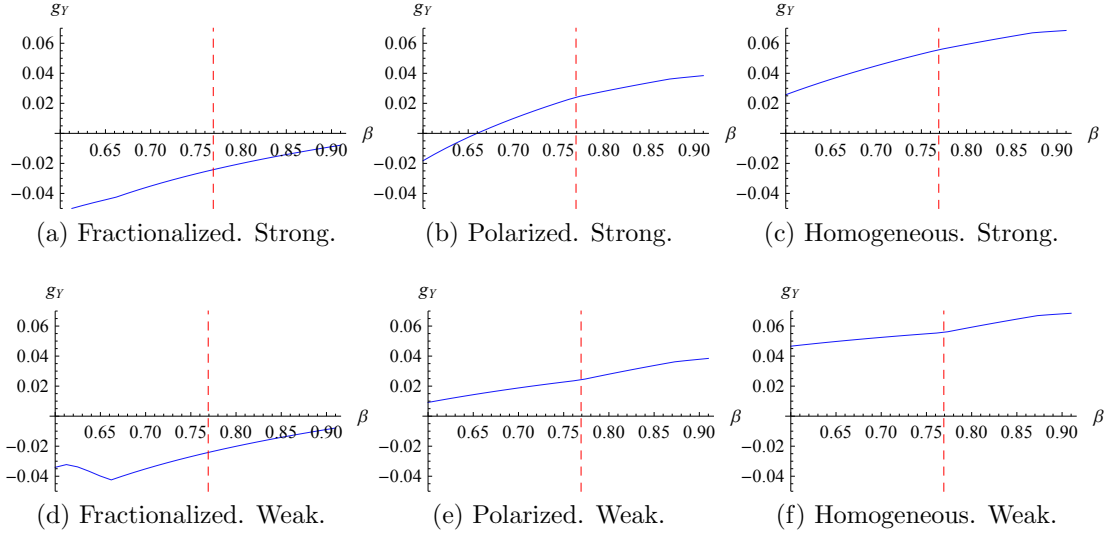


Figure 4: Sensitivity analysis: effect the discount factor β on the growth rate. β takes values in the range $[0.60, 0.91]$, $\gamma = 0.65$ and $\delta = 0.5$. The smaller groups are initially either strong or weak.

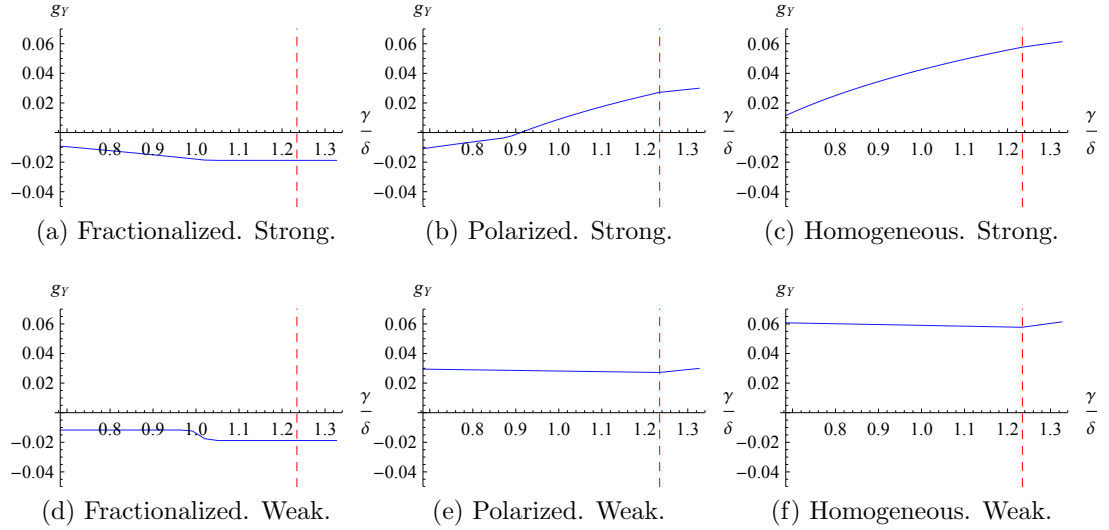


Figure 5: Sensitivity analysis: effect the competition intensity $\frac{\gamma}{\delta}$ on the growth rate. $\beta = 0.81$, $\gamma = 0.65$, $\frac{\gamma}{\delta}$ takes values in the range $[0.68, 1.32]$. The smaller groups are initially either strong or weak.

and (c) where it is positive. Besides, when $\beta\gamma > \delta$ to the right of the red vertical dashed line, the effect is positive.

There is a mechanism that makes higher contest intensity lead to higher economic growth. It works through a greater influence of the ethnicities making a positive contribution and lower influence of the ethnicities with a binding resource constraint. This happens under two conditions. First, if $\beta\gamma > \delta$, Proposition 2 part (ii) implies that higher contest intensity makes the steady state power of bigger ethnicities relatively larger because the steady state power is the fair representation, proportional to the population percentages, less the steady state political effort which is constant across groups.²² Second, if $\beta\gamma < \delta$ and if the little groups originally have a strength greater than their fair representation, higher contest intensity determines by how much the power of the larger ethnicities is going to increase and thus by how much the power of the little group will decrease.²³

4 Conclusion

The central point that motivated this paper is the evidence that African polities are characterized by a high degree of proportionality between political representation in the minister cabinet and demographic shares. Thus far, the existing theories that relate diversity to economic growth have not worked out to match this regularity. Nonetheless, this paper adjusts this by constructing a model with power sharing and proportional allocation of the rents, thus contrasting with the contest success function approach. Besides my approach brings a novel aspect to this literature by explicitly including the ethnic structure in a successive generations model.

This paper constructs a successive generations model of appropriative competition between ethnicities and highlights a mechanism reducing negative externalities in consumption thanks to a constraint driven by de facto power and patronage. I expose the conditions under which the equilibrium exists and is unique and then deduce the implications in terms of the impact that diversity has on economic prosperity.

I obtain the following results. The composition of influence attained through the appropriative competition determines the outcome of the strategic interaction in the investment game. This is the main mechanism of this paper and it implies that long-run economic growth is larger when de facto power is concentrated rather than dispersed. Furthermore, I compare the appropriateness of two diversity measures as determinants of economic growth applying my model and prefer the Herfindahl formula-based ELF index because of the monotonicity of the link between these variables. Of course, I find that total factor productivity and the discount factor have a positive impact on growth. Surprisingly however, competition intensity sometimes has a positive effect. This happens when a higher intensity creates large ethnicities that are more contentious, control a bigger fraction of the pie and invest more.

²²The result can be seen in the right part of Figure 5 parts (b), (c), (e) and (f).

²³The result can be seen in the left side of Figure 5 parts (b) and (c).

In future work, this last conjecture could be tested using a proxy variable approach using oil, diamonds, aid or the insecurity of property rights as possible substitutes for contest intensity. This could give an insight over the decisive conditions and have policy implications about when it is the most useful to promote democracy and fair elections for instance. In certain ethnic configurations, the consequences of the decrease of competition intensity can sometimes be bad for growth. In this paper, I make an assumption that sidesteps the possibility of social exclusion. The analysis can be extended by relaxing this assumption and by studying how initial conditions then affect income rise, considering the fact that some less influential groups could become excluded from the formal economy. Including asymmetry in the cost of political effort is another potential extension of the model.

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Appendix

A: First order conditions

Here are the first order necessary and complementary-slackness conditions of the investment game given in (6). A Lagrange multiplier should be associated with the constraint $C_t^i \geq 0$ but because of the log-utility, this multiplier is 0 and $C_t^i > 0$. After substituting K_{t+1} from (4), the Lagrangian of (6) becomes

$$\mathcal{L}(C_t^i, C_t^{-i}, \lambda_t^i) = \log(C_t^i) + \beta \log(AK_t - \sum_{j \in \mathcal{N}} C_t^j) - \lambda_t^i (C_t^i - P_t^i AK_t)$$

Differentiating $\mathcal{L}$ with respect to C_t^i and λ_t^i gives the first order conditions

$$\frac{1}{C_t^i} - \frac{\beta}{AK_t - \sum_{j \in \mathcal{N}} C_t^j} - \lambda_t^i = 0 \quad (9)$$

for $i \in \mathcal{N}$ and the complementarity-slackness conditions

$$\begin{aligned} C_t^i &\leq P_t^i Y_t \\ \lambda_t^i &\geq 0 \\ (C_t^i - P_t^i Y_t) \lambda_t^i &= 0 \end{aligned}$$

for $i \in \mathcal{N}$.

Here are the first order necessary conditions of the appropriative competition (7). Again, a Lagrange multiplier should be associated with the constraint $E_t^i \leq n^i$ but because of the log-utility, this multiplier is 0 and $E_t^i < n^i$. After substituting the interior value of P_{t+1}^i from (5)²⁴, the Lagrangian of (7) is

$$\mathcal{M}(E_t^i, \mu_t^i) = \delta \log(n^i - E_t^i) + \beta \log(P_t^i + \gamma (E_t^i - \frac{\sum_{j \neq i} E_t^j}{N-1})) + \mu_t^i \cdot E_t^i$$

Differentiating $\mathcal{M}$ with respect to E_t^i and λ_t^i gives the first order conditions

$$\frac{-\delta}{n^i - E_t^i} + \frac{\beta \gamma}{P_t^i + \gamma (E_t^i - \frac{\sum_{j \neq i} E_t^j}{N-1})} + \mu_t^i = 0 \quad (10)$$

for $i \in \mathcal{N}$ and the complementarity-slackness conditions

$$\begin{aligned} 0 &\leq E_t^i \\ \mu_t^i &\geq 0 \\ \mu_t^i E_t^i &= 0 \end{aligned}$$

²⁴This can be done thanks to Assumption 4.

for $i \in \mathcal{N}$.

By setting the multipliers equal to zero in the first order conditions (9) and (10), I obtain the interior best response functions

$$C_{t,\text{br}}^i = \frac{Y_t - \sum_{j \neq i} C_t^j}{\beta + 1}. \quad (11)$$

and

$$E_{t,\text{br}}^i = \frac{u_t^i}{\beta + \delta} + \frac{1}{\beta + \delta} \frac{\delta}{N - 1} \sum_{j \neq i} E_t^j. \quad (12)$$

B: Proof of Proposition 1

(i) Existence

Rosen (1965) states that the necessary conditions for existence of a pure strategy Nash equilibrium for the strategic form game $\langle \mathcal{I}, (S_i), (u_i) \rangle$ is that the strategy sets S_i are nonempty, convex, and compact sets. $u_i(s)$ must be continuous in s , and $u_i(s_i, s_{-i})$ must be quasiconcave in s_i . The investment game in (6) and the appropriative competition in (7) are such games and respect all these conditions. As a consequence, a pure strategy Nash equilibrium exists.

(ii) (ii.a) Uniqueness of a pure strategy Nash equilibrium of the investment game given in (6).

To prove uniqueness, following Rosen (1965), I need to show that the sufficient condition for uniqueness of a pure strategy equilibrium is met. This condition requires that the pay-off functions are diagonally strictly concave. The notation $\nabla u(x)$ denotes $\left[\frac{\partial u_1}{\partial x_1}, \dots, \frac{\partial u_N}{\partial x_N} \right]^T$. The payoff functions $(u_1, \dots, u_N)$ are diagonally strictly concave for $x \in S$ if for every $x^*, \bar{x} \in S$, $(\bar{x} - x^*)^T \nabla u(x^*) + (x^* - \bar{x})^T \nabla u(\bar{x}) > 0$. The investment game in (6) respects this condition because when

$$\bar{x} = \begin{pmatrix} \bar{C}^1 \\ \vdots \\ \bar{C}^N \end{pmatrix} \quad x^* = \begin{pmatrix} C^{1*} \\ \vdots \\ C^{N*} \end{pmatrix},$$

I have that

$$\begin{aligned} & \begin{pmatrix} \bar{C}^1 - C^{1*} \\ \vdots \\ \bar{C}^N - C^{N*} \end{pmatrix}^T \begin{pmatrix} \frac{1}{C^{1*}} - \frac{\beta}{Y - \sum_{i=1}^N C^{i*}} \\ \vdots \\ \frac{1}{C^{N*}} - \frac{\beta}{Y - \sum_{i=1}^N C^{i*}} \end{pmatrix} + \begin{pmatrix} C^{1*} - \bar{C}^1 \\ \vdots \\ C^{N*} - \bar{C}^N \end{pmatrix}^T \begin{pmatrix} \frac{1}{\bar{C}^1} - \frac{\beta}{Y - \sum_{i=1}^N \bar{C}^i} \\ \vdots \\ \frac{1}{\bar{C}^N} - \frac{\beta}{Y - \sum_{i=1}^N \bar{C}^i} \end{pmatrix} \\ &= \sum_{i=1}^N \frac{(\bar{C}^i - C^{i*})^2}{C^{i*} \bar{C}^i} + \frac{\beta}{\left(Y - \sum_{i=1}^N C^{i*} \right) \left(Y - \sum_{i=1}^N \bar{C}^i \right)} \sum_{i=1}^N ((\bar{C}^i - C^{i*})^2) > 0. \end{aligned}$$

(ii.b) Uniqueness of a pure strategy Nash equilibrium of the appropriative competition given in (7). This proof makes use of the best response function of group i for $i \in \mathcal{I}_e$ which is (12) or

$$E^i = \frac{u^i}{\beta + \delta} + d \sum_{j \neq i} E^j$$

neglecting the subscripts and denoting with $d = \frac{1}{\beta + \delta} \frac{\delta}{N-1}$.

I derive some preliminary results first. Whenever the partition $\{\mathcal{I}_e, \mathcal{O}_e\}$ is known, there is a single equilibrium candidate found by inverting a nonsingular matrix.

Lemma 1 *There is only one equilibrium candidate associated with any partition $\{\mathcal{I}_e, \mathcal{O}_e\}$.*

The solution to the $L \times L$ system

$$E^i - d \sum_{j \neq i, j \in \mathcal{I}_e} E^j = \frac{u^i}{\beta + \delta} \text{ for } i \in \mathcal{I}_e$$

is unique because the values of d that nullify the determinant of

$$\begin{pmatrix} 1 & -d & \cdots & -d \\ -d & 1 & & \vdots \\ \vdots & & \ddots & -d \\ -d & \cdots & -d & 1 \end{pmatrix}$$

are -1 ($L - 1$ times) and $\frac{1}{L-1}$. The matrix has a nonzero determinant because $0 < d = \frac{1}{\beta + \delta} \frac{\delta}{N-1} < \frac{1}{N-1} \leq \frac{1}{L-1}$. The matrix is invertible and the solution to the system is unique.

This next lemma states that the more underrepresented groups belong to $\mathcal{I}_e$.

Lemma 2 $u^l > u^k$ and $k \in \mathcal{I}_e \Rightarrow l \in \mathcal{I}_e$.

$E^l > E^k$ because

$$\begin{aligned} E^l &= \frac{u^l}{\beta + \delta} + d \sum_{j \neq l, k} E^j + d \cdot E^k \\ E^k &= \frac{u^k}{\beta + \delta} + d \sum_{j \neq l, k} E^j + d \cdot E^l \end{aligned}$$

has the solution

$$\begin{aligned} E^l &= \frac{1}{1-d^2} \left(\frac{u^l + du^k}{\beta + \delta} + d \sum_{j \neq l, k} E^j \right) \\ E^k &= \frac{1}{1-d^2} \left(\frac{u^k + du^l}{\beta + \delta} + d \sum_{j \neq l, k} E^j \right) \end{aligned}$$

and that $u^l + du^k > u^k + du^l$ or equivalently that $u^l(1-d) > u^k(1-d)$ with $0 < d < 1$.

This next lemma expresses the sum of the equilibrium efforts of the groups in $\mathcal{I}_e$ as a function of the sum of the underrepresentations of the groups by summing the equilibrium values over all the elements of $\mathcal{I}_e$.

Lemma 3 $\sum_{i \in \mathcal{I}_e} E^i = \frac{(N-1)(\sum_{j \in \mathcal{I}_e} u^j)(\beta + \delta)}{(N-1)\beta + (N-L)\delta}$

$$\begin{aligned} \sum_{i \in \mathcal{I}_e} E^i &= \frac{(N-1)(\beta + \delta)}{((N-1)\beta + N\delta)((N-1)\beta + \delta(N-L))} \times \\ &\quad \left(((N-1)\beta + (N-L+1)\delta) \left(\sum_{j \in \mathcal{I}_e} u^j \right) + \delta(L-1) \left(\sum_{j \in \mathcal{I}_e} u^j \right) \right) \\ &= \frac{(N-1)(\beta + \delta)}{((N-1)\beta + N\delta)((N-1)\beta + (N-L)\delta)} \times \\ &\quad \left((-1+L) \left(\sum_{j \in \mathcal{I}_e} u^j \right) \delta + \left(\sum_{j \in \mathcal{I}_e} u^j \right) ((N-1)\beta + (N-L+1)\delta) \right) \\ &= \frac{(N-1) \left(\sum_{j \in \mathcal{I}_e} u^j \right) (\beta + \delta)}{(N-1)\beta + (N-L)\delta} \end{aligned}$$

I prove that the sum of the underrepresentations of the groups in $\mathcal{I}_e$ is positive.

Lemma 4 $\sum_{i \in \mathcal{I}_e} u^i > 0$

Consider h , the element of $\mathcal{I}_e$ with the smallest u^i .

Whenever $u^h > 0$, $u^i > 0 \forall i \in \mathcal{I}_e$.

Whenever $u^h < 0$, using Lemma 3 the best response $u^h + d' \left(\sum_{i \neq h, i \in \mathcal{I}_e} u^i \right) > 0$ because $h \in \mathcal{I}_e$. After substituting the equilibrium efforts, $d' \sum_{i \in \mathcal{I}_e} u^i > (1 - d')u^h + d' \sum_{i \in \mathcal{I}_e} u^i > 0$ where $0 < d' = \frac{\delta(\beta + \delta)}{(N-1)\beta + (N-L+1)\delta} < 1$.

Whenever a group is not in $\mathcal{I}_e$, it has a negative underrepresentation.

Lemma 5 $k \in \mathcal{O}_e \Rightarrow u^k < 0$

This follows directly from the best response function (12).

Lemma 6 *A nonempty set $\mathcal{X}_e \subset \mathcal{O}_e \Rightarrow \sum_{i \in \mathcal{X}_e} u^i < 0$*

This follows directly from Lemma 5.

After these preliminary results, the proof of Proposition 1.2.b starts here. ϵ_0 is an equilibrium associated with the partition $\{\mathcal{I}_e, \mathcal{O}_e\}$. Let's suppose that there is another equilibrium ϵ_X associated with the partition $\{\mathcal{I}_e \cup \mathcal{X}_e, \mathcal{O}_e \setminus \mathcal{X}_e\}$ where $\mathcal{X}_e$ is a nonempty subset of $\mathcal{O}_e$. Then, the equilibrium effort of group $k \in \mathcal{X}_e$ at ϵ_X is

$$E_{\epsilon_X}^k = \frac{(N-1)(\beta + \delta)}{((N-1)\beta + N\delta)((N-1)\beta + (N-L-J)\delta)} \times \left(\left(\sum_{j \in \mathcal{I}_e} u^j \right) \delta + \left(\sum_{j \neq k, j \in \mathcal{X}_e} u^j \right) \delta + u^k((N-1)\beta + (N-J-L+1)\delta) \right) \quad (13)$$

where $J = \#(\mathcal{O}_e)$. The fact that ϵ_0 is an equilibrium implies that

$$\frac{1}{\beta + \delta} \left(u^k + \frac{\delta \sum_{j \in \mathcal{I}_e} u^j}{(N-1)\beta + (N-L)\delta} \right) < 0$$

or that

$$\sum_{j \in \mathcal{I}_e} u^j < -\frac{u^k((N-1)\beta + (N-L)\delta)}{\delta(\beta + \delta)} \quad (14)$$

I want to show that $E_{\epsilon_X}^k < 0$ by replacing $\sum_{j \in \mathcal{I}_e} u^j$ from (14) in the right-hand side of (13). I obtain that $E_{\epsilon_X}^k < \bar{e} < 0$ where $\bar{e}$ is

$$\begin{aligned} & \frac{(N-1) \left(\sum_{j \neq k, j \in \mathcal{X}_e} u^j \right) \delta (\beta + \delta)}{((N-1)\beta(N-J-L)\delta)((N-1)\beta + N\delta)} \\ & + \frac{(N-1)u^k(\beta + \delta) \left((N-1)\beta(N-J-L+1) + \delta - \frac{(N-1)\beta + (N-L)\delta}{\beta + \delta} \right)}{((N-1)\beta + (N-J-L)\delta)((N-1)\beta + N\delta)} \end{aligned}$$

A sufficient condition for $\bar{e} < 0$ is that

$$((N-1)\beta + (N-J-L+1) + \delta) - \frac{(N-1)\beta + (N-L)\delta}{\beta + \delta} > 0$$

or that

$$\frac{(N-1)\beta + (N-J-L+1)\delta}{(N-1)\beta + (N-L)\delta} > \frac{1}{\beta + \delta}.$$

The left-hand side is the smallest when J takes its maximum value $N-L$ and when L takes its minimum value ²⁵. The condition reduces first to $\frac{\beta + \frac{\delta}{N-1}}{\beta + \frac{(N-L)}{(N-1)}\delta} >$

$\frac{1}{\beta + \delta}$ and then to $\frac{\beta + \frac{\delta}{N-1}}{\beta + \frac{N-1}{N-1}\delta} > \frac{1}{\beta + \delta}$ or that $\beta + \frac{\delta}{N-1} > 1$ which is Assumption 2.

(iii) Characterization.

a. The unique solution to the $N \times N$ system

$$\begin{cases} (\beta + 1)C^i + \sum_{j \in \mathcal{I}_c, j \neq i} C^j = Y - \sum_{j \in \mathcal{O}_c} C^j & \text{for } i \in \mathcal{I}_c \\ C^i = YP^i & \text{for } i \in \mathcal{O}_c \end{cases}$$

is

$$\begin{cases} C^i = \frac{Y(1 - \sum_{j \in \mathcal{O}_c} P^j)}{M + \beta} & \text{for } i \in \mathcal{I}_c \\ C^i = YP^i & \text{for } i \in \mathcal{O}_c \end{cases}$$

b. The unique solution to the $N \times N$ system

$$\begin{cases} E^i - d \sum_{j \neq i, j \in \mathcal{I}_e} E^j = \frac{u^i}{\beta + \delta} & \text{for } i \in \mathcal{I}_e \\ E^i = 0 & \text{for } i \in \mathcal{O}_e \end{cases}$$

is

$$E^i = \begin{cases} \frac{(N-1)(\beta + \delta)}{(N-1)\beta + N\delta} \left(u^i + \delta \frac{\sum_{j \in \mathcal{I}_e} u^j}{(N-1)\beta + (N-L)\delta} \right) & \text{for } i \in \mathcal{I}_e \\ 0 & \text{for } i \in \mathcal{O}_e \end{cases}$$

C: Proof of Proposition 2

- (i) $\beta\gamma < \delta$ implies that $P_{\text{fr}}^i < n^i$ and thus $\sum_i P_{\text{fr}}^i < 1$. As a consequence, at least some groups are overrepresented. Assumption 3 implies that some groups are underrepresented. The dynamics of P^i for $i \in \mathcal{I}_e$ are such that some groups are still underrepresented at $t+1$. The total power increase obtained by summing the equilibrium effort over $i \in \mathcal{I}_e$ is less than the amount of power to compensate

²⁵Due to Assumption 3.

for the underrepresentation.

$$\begin{aligned}
\sum_{i \in \mathcal{I}_e} \Delta P^i &= \gamma \left(\sum_{i \in \mathcal{I}_e} E^i - \frac{L-1}{N-1} \sum_{i \in \mathcal{I}_e} E^i \right) \\
&= \gamma \left(1 - \frac{L-1}{N-1} \right) \frac{(N-1)(\beta + \delta)}{(N-1)\beta + (N-L)\delta} \sum_{i \in \mathcal{I}_e} u^i \\
&= \gamma \left(\frac{N-L}{N-1} \right) \frac{(N-1)(\beta + \delta)}{(N-1)\beta + (N-L)\delta} \sum_{i \in \mathcal{I}_e} u^i \\
&= \gamma \frac{(N-L)(\beta + \delta)}{(N-1)\beta + (N-L)\delta} \sum_{i \in \mathcal{I}_e} u^i < \frac{\gamma}{\delta} \sum_{i \in \mathcal{I}_e} u^i
\end{aligned}$$

because

$$\delta \frac{(N-L)(\beta + \delta)}{(N-1)\beta + (N-L)\delta} < 1$$

and this implies that

$$\begin{aligned}
\sum_{i \in \mathcal{I}_e} \Delta P^i &< \sum_{i \in \mathcal{I}_e} \left(\frac{\beta\gamma}{\delta} n^i - P^i \right) = \sum_{i \in \mathcal{I}_e} (P_{\text{fr}}^i - P^i) \\
\sum_{i \in \mathcal{I}_e} P^i + \Delta P^i &< \sum_{i \in \mathcal{I}_e} P_{\text{fr}}^i
\end{aligned}$$

The dynamics of P^i are such that the total underrepresentation of the groups in $\mathcal{I}_e$ is decreasing.

$$\sum_{i \in \mathcal{I}_e} \Delta P^i = \gamma \frac{(N-L)(\beta + \delta)}{(N-1)\beta + (N-L)\delta} \sum_{i \in \mathcal{I}_e} u^i > 0 \quad (15)$$

where I use Lemma 4, $\sum_{i \in \mathcal{I}_e} u^i > 0$. This is true even though marginally a formerly overrepresented group could change membership and belong to $\mathcal{I}_{e,t+1}$ because the underrepresentation gained by this group is a part of the underrepresentation lost by groups in $\mathcal{I}_{e,t}$ and thus is not enough to compensate the total decrease.

The total underrepresentation is a strictly decreasing and positive suite and thus has a limit. From equation (15), this limit is 0 because the change in power is proportional to the total underrepresentation. This completes the proof of proposition 2 part (i).

- (ii) $\beta\gamma > \delta$ implies that $P_{\text{fr}}^i > n^i$ and thus $\sum_i P_{\text{fr}}^i > 1$. As a consequence, at least some groups are underrepresented and make a positive political effort. In the long run, no group is overrepresented because the positive political efforts of the underrepresented groups make their power decrease.

At the steady state all groups have $P^i < P_{fr}^i$ and make a strictly positive political effort. The steady state is P_{ss}^i and e_{ss} for all i . From the symmetry of the law of motion (5), at the steady state, the political efforts of all the groups are equal. Setting this in the interior best response functions (12), I obtain that

$$e_{ss}(1 - d(N - 1)) = \frac{\beta n^i - \frac{\delta}{\gamma} P_{ss}^i}{\beta + \delta}$$

or that

$$P_{ss}^i = \frac{\gamma}{\delta} (\beta n^i - e_{ss}(1 - d(N - 1))(\beta + \delta)). \quad (16)$$

Summing over i and because $\sum_i P^i = 1$ and $\sum_i n^i = 1$, I obtain that

$$1 = \frac{\gamma}{\delta} (\beta - N e_{ss}(1 - d(N - 1))(\beta + \delta))$$

that can be solved for the value of the steady state effort,

$$e_{ss} = \frac{\beta\gamma - \delta}{N\beta\gamma} = \frac{1}{N} \left(1 - \frac{\delta}{\beta\gamma} \right).$$

By replacing e_{ss} in (16), I obtain the steady state power of group i which is

$$\frac{\beta\gamma}{\delta} n^i + \frac{1}{N} - \frac{1}{N} \frac{\beta\gamma}{\delta}.$$

(iii) $P_{fr}^i = P_{ss}^i = n^i$ when $\beta\gamma = \delta$.

D: Convergence of the best response iteration process

I showed in Proposition 1 that the equilibrium of the strategic interaction exists and is unique. The characterization of the equilibrium rests on the partitions of the groups $(\mathcal{I}_c, \mathcal{O}_c)$ and $(\mathcal{I}_e, \mathcal{O}_e)$ but even though there is no a priori way to determine these partitions of $\mathcal{N}$, the equilibrium can be computed numerically because the process obtained by iterating the best response functions converge to the Nash equilibrium. The reasons for this are that first, the investment game in (6) is an exact potential game following the definition of Monderer and Shapley (1996) where the potential function is

$$\Phi(C_t^1, \dots, C_t^N) = \sum_{i \in \mathcal{N}} \log(C_t^i) + \beta \log(Y_t - \sum_{i \in \mathcal{N}} C_t^i).$$

Thus here, best response dynamics converge to the Nash equilibrium. In potential games, all equilibria are local maximizers of the potential and since best response adjustment processes increase potential, all equilibria are locally stable (Sandholm, 2001). Second, the appropriative competition given in (7) is a supermodular continuous game. If the Nash equilibrium of a supermodular game is unique, best-response dynamics converge to it (Milgrom and Roberts, 1990; Vives, 2008).

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