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a Historical Graphical Analysis Method for Rigid Frames

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1.1 Rigid frames

Key feature:

- Rigid connections which are moment-resisting

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- structural redundancy

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- static indeterminacy

Analysis:

- static equilibrium of forces and moment

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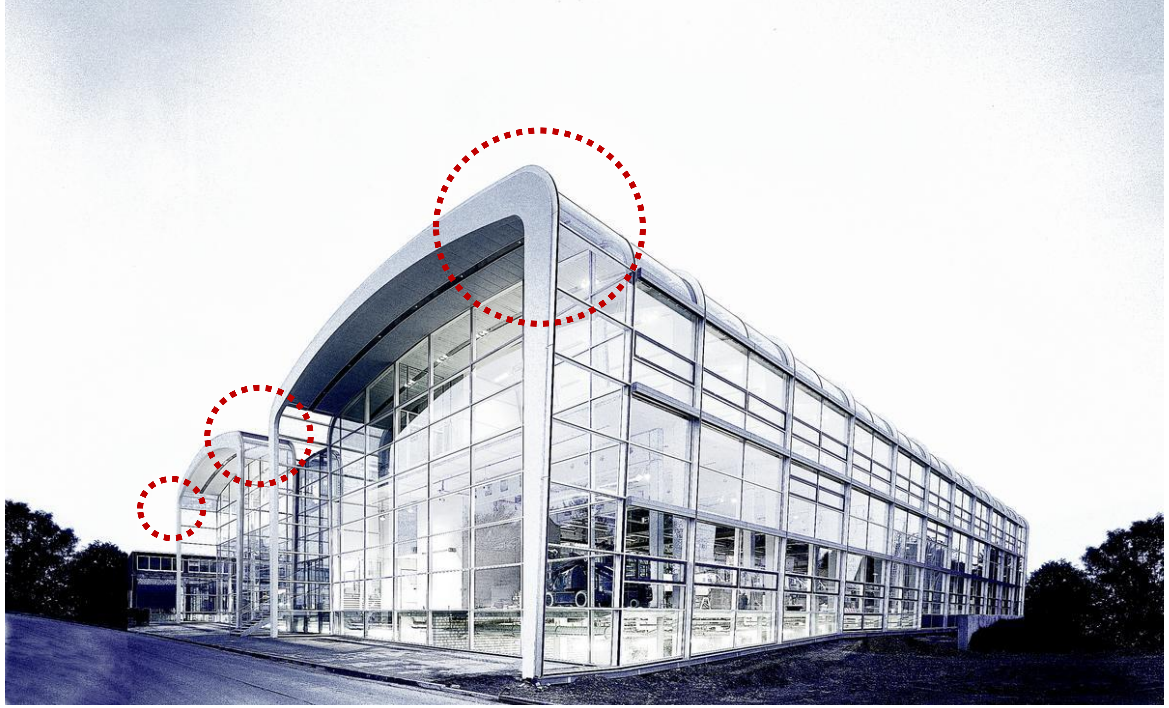
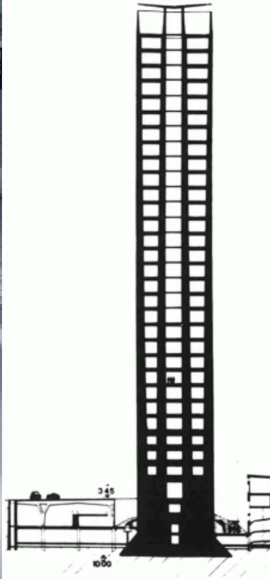
- strengthen of materials



1. GIRA production hall (Ingenhoven Architects)



2. Pirelli Tower (Giò Ponti,Pier Luigi Nervi)

source:1. GIRA Giersiepen GmbH & Co. KG , 2.BMS

1.2 Graphic Statics

Characters:

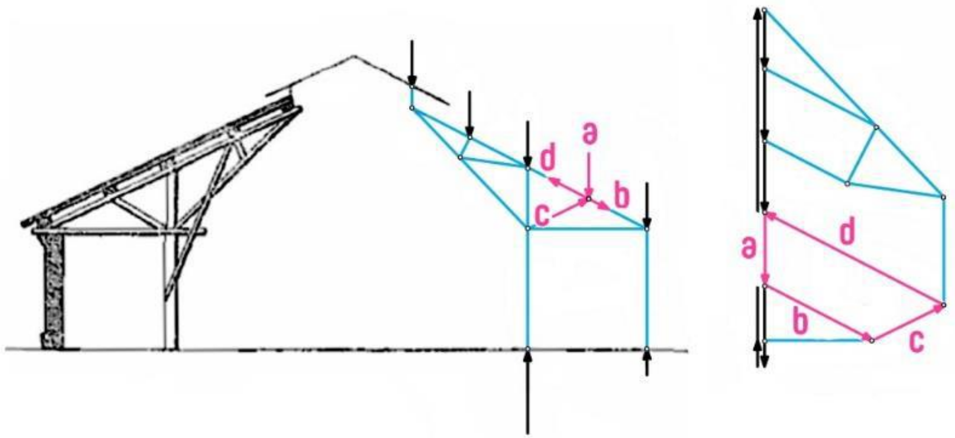
- A geometric-based approach
- Reciprocal diagrams

Advantages:

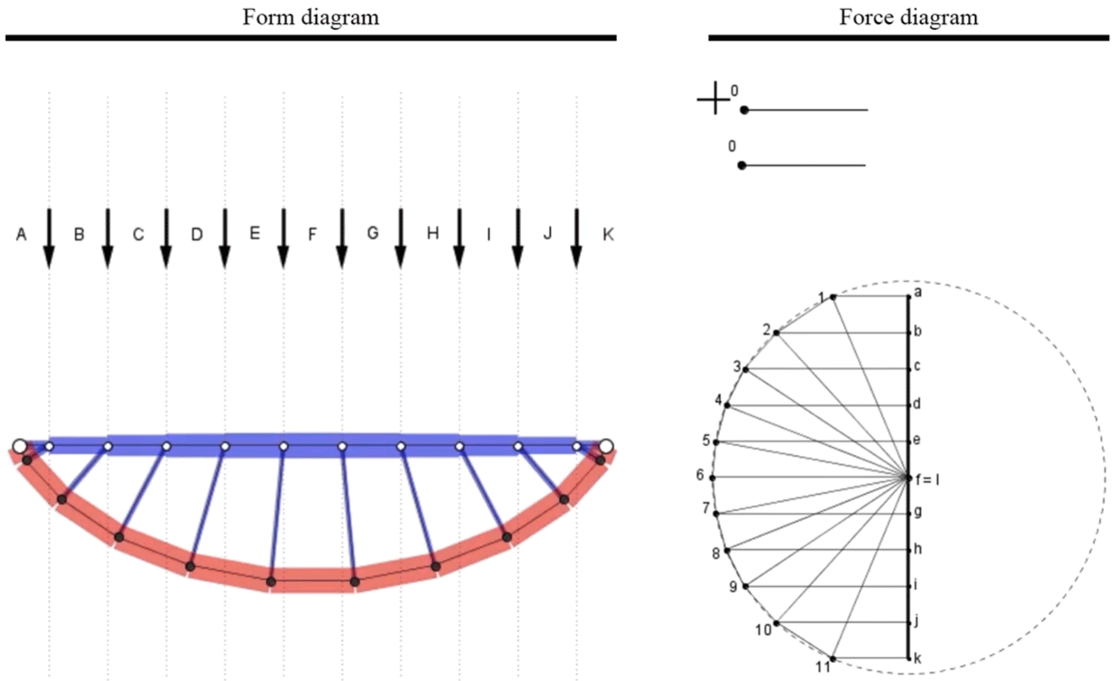
- Insightful into the relation between form and forces.
- Two-way approach
- Less demanding of mathematics knowledge

History:

- Popular around the turn of the nineteenth and twentieth century
- “Renaissance” in the recent two decades



1. The form diagram and force diagram of a truss



2. Parameterized graphic statics for a truss

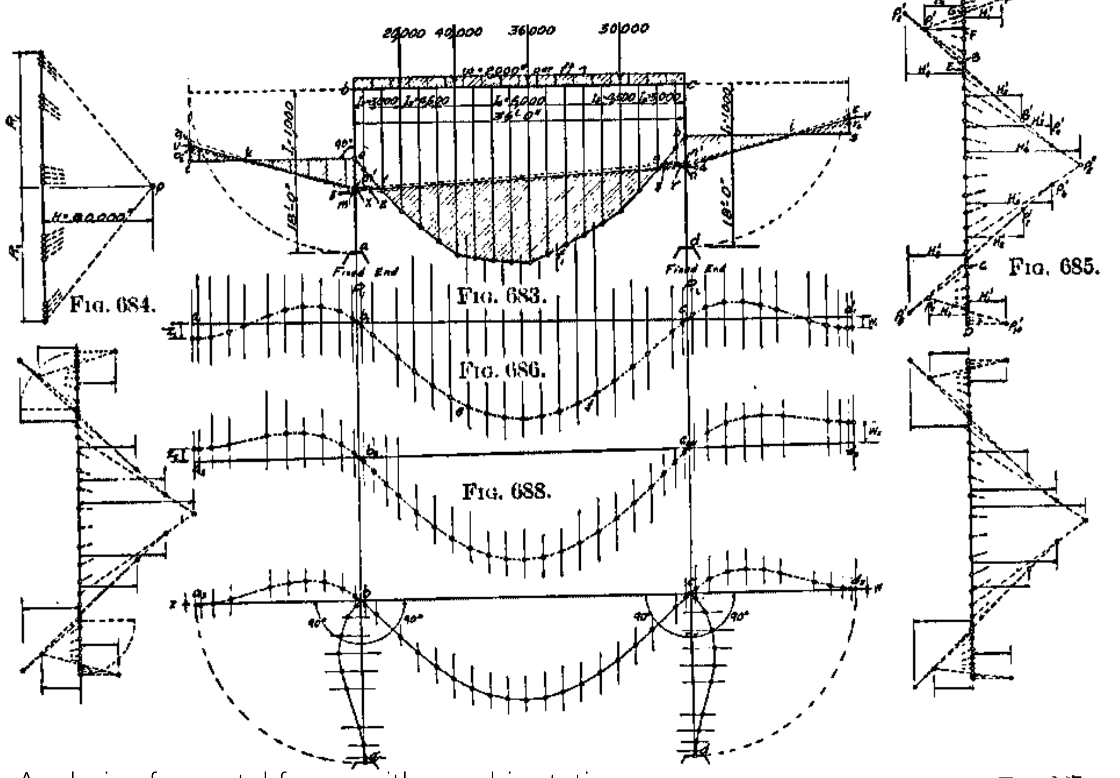
Source: 1. Fivet, Corentin et al,2018 2. Block Research Group official website

1.3 Problem statement:

- Rigid frames can be and have been analyzed with graphic statics
- These theories and techniques comprise a substantial part of the arsenal of graphic statics
- Currently these methods for rigid frames have received limited attention

1.4 Aims and outline of the paper:

- The history of the geometric-based method for rigid frames
- explanation of the techniques
- A discussion on the special values and drawbacks of the methods

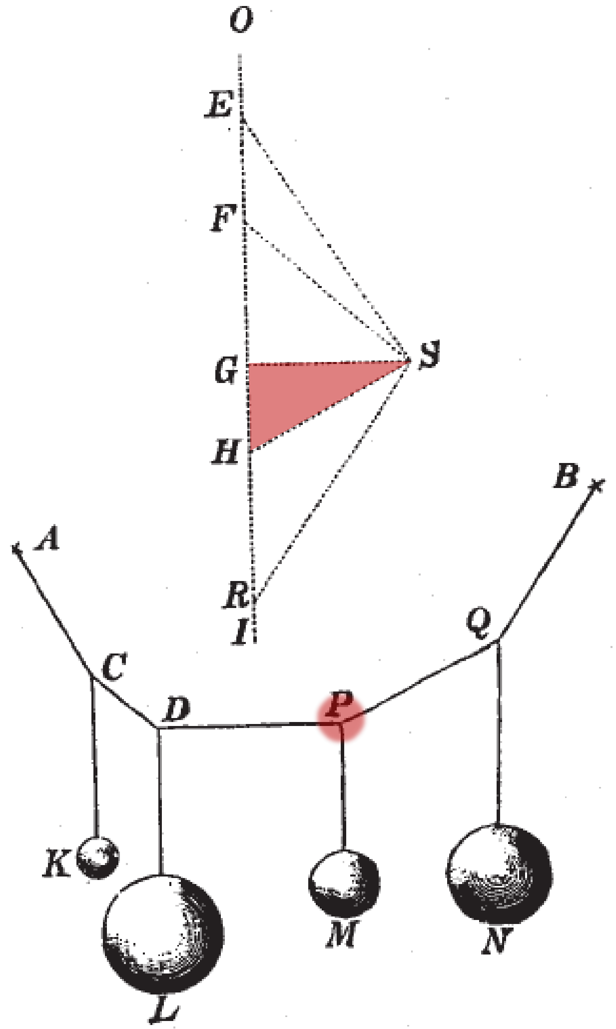


1. Analysis of a portal frame with graphic statics


2. A portal frame bridge designed with graphic statics by Robert Maillart

Source: 1. Wofe 1921, 2. photos by author BA (taken in 2022)

2.1. Varignon’s determination of funicular polygon with force diagram



force polygon

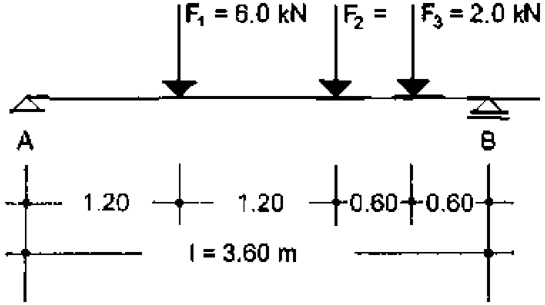
form diagram/ funicular polygon

Varignon's introduction of the polygon of forces and funicular polygon

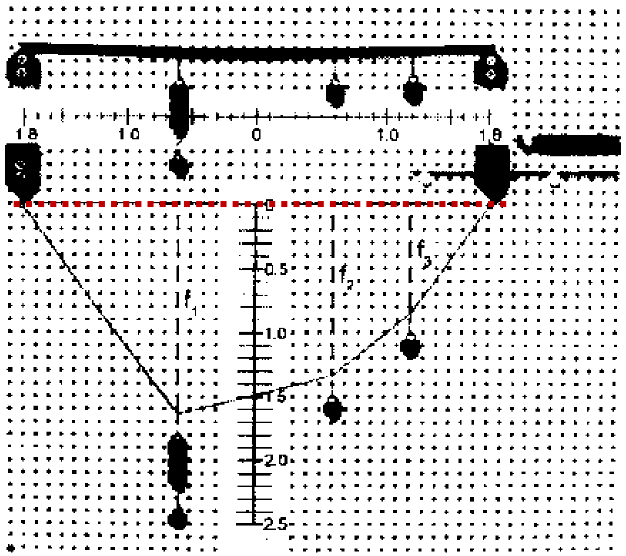
 Source: Varignon, 1725

2.2 Culmann's visualization of the bending moment of a statically determinate beam with graphic statics

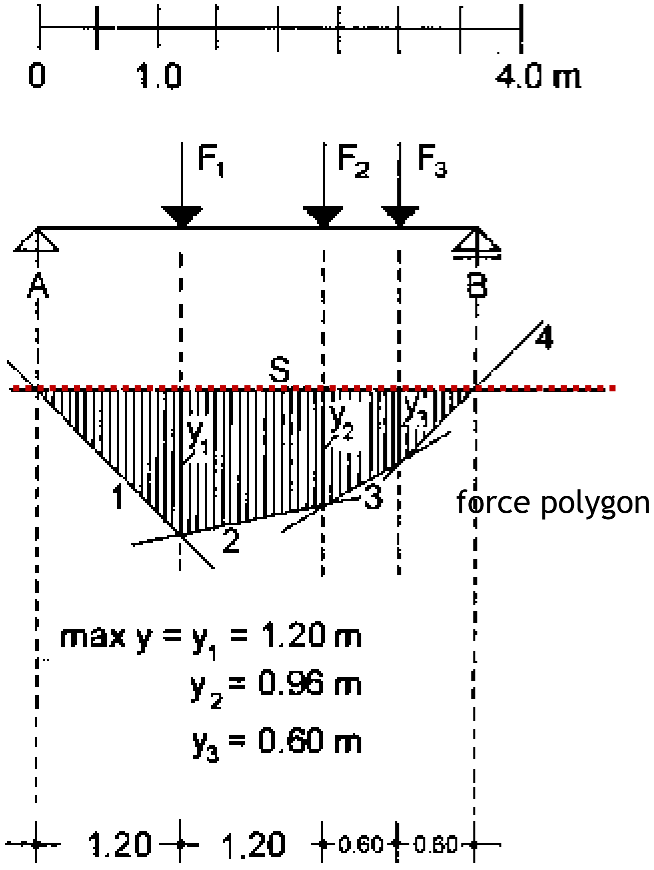
Carl Culmann's method for bending moment of a statically determinate beam
(demonstrated by Gerhardt with physical modes and drawings)



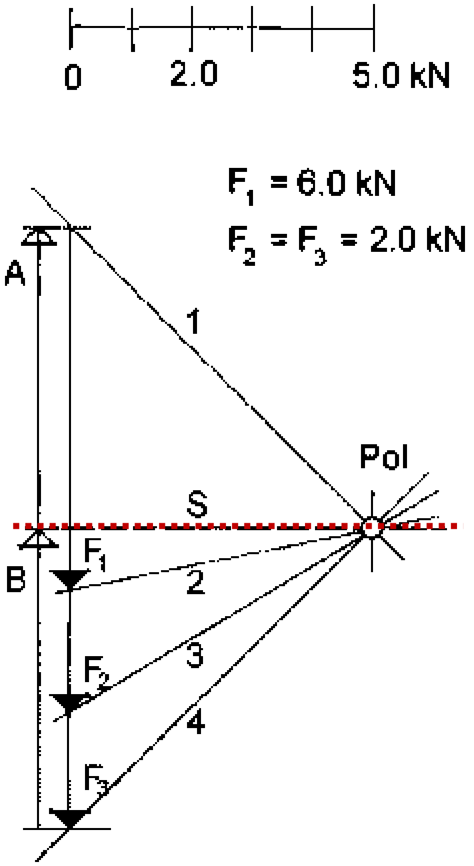
a statically determinate beam



The physical mode of loaded beam and corresponding funicular polygon



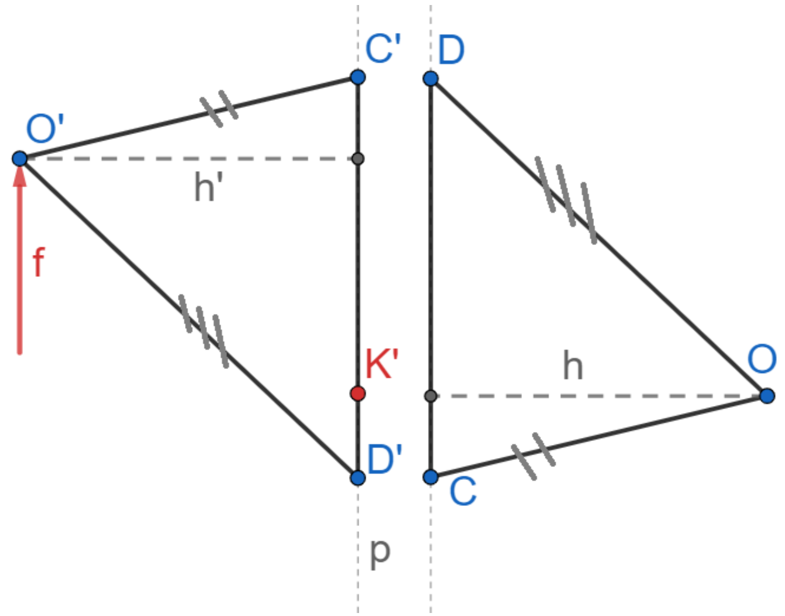
form diagram/ funicular polygon/ bending moment diagram



force polygon

Source: Gerhardt, 2002

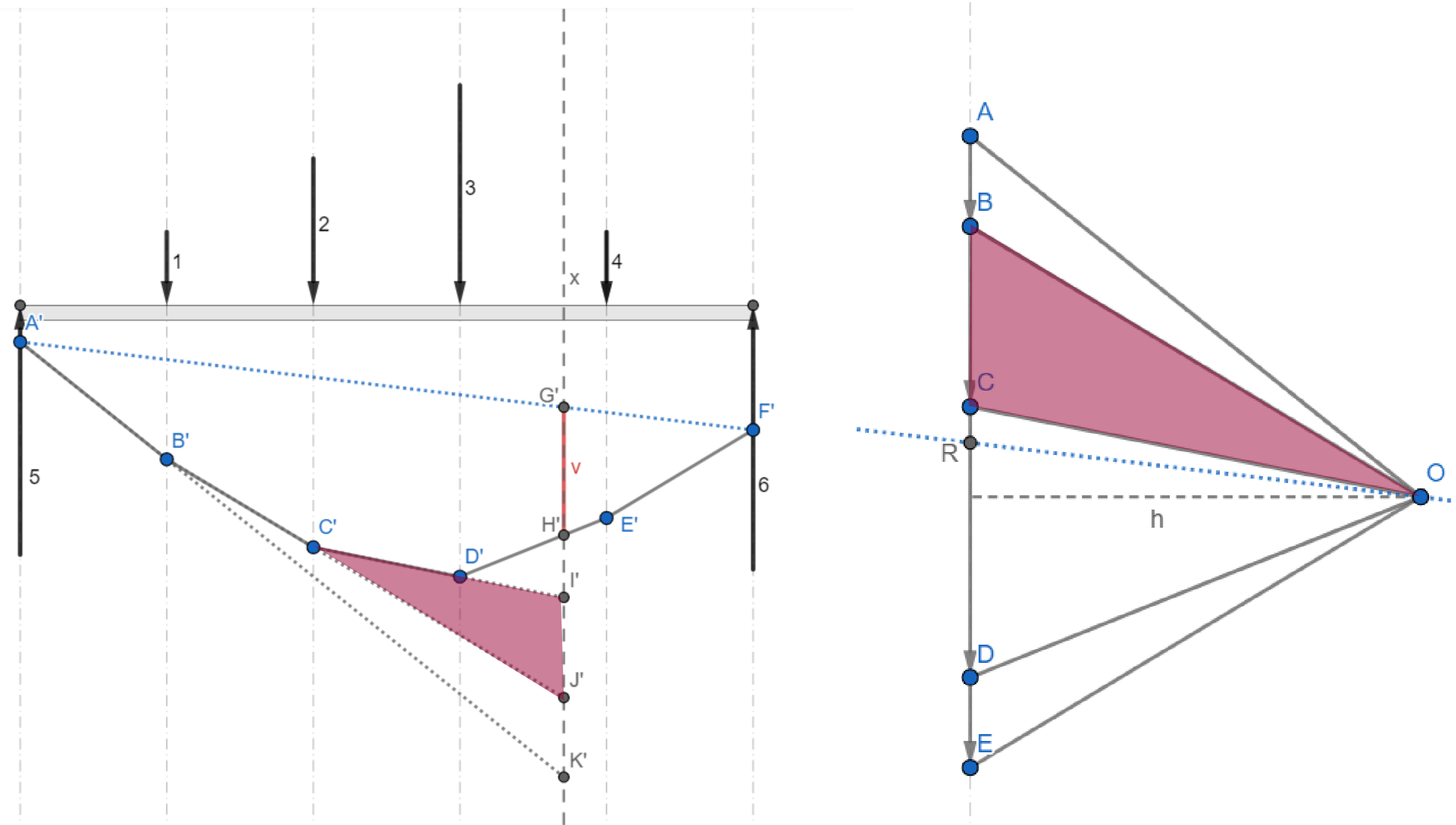
The rationale of Culmann’s method for bending moment



form polygon

force polygon

1. graphical solution of the moment of force f

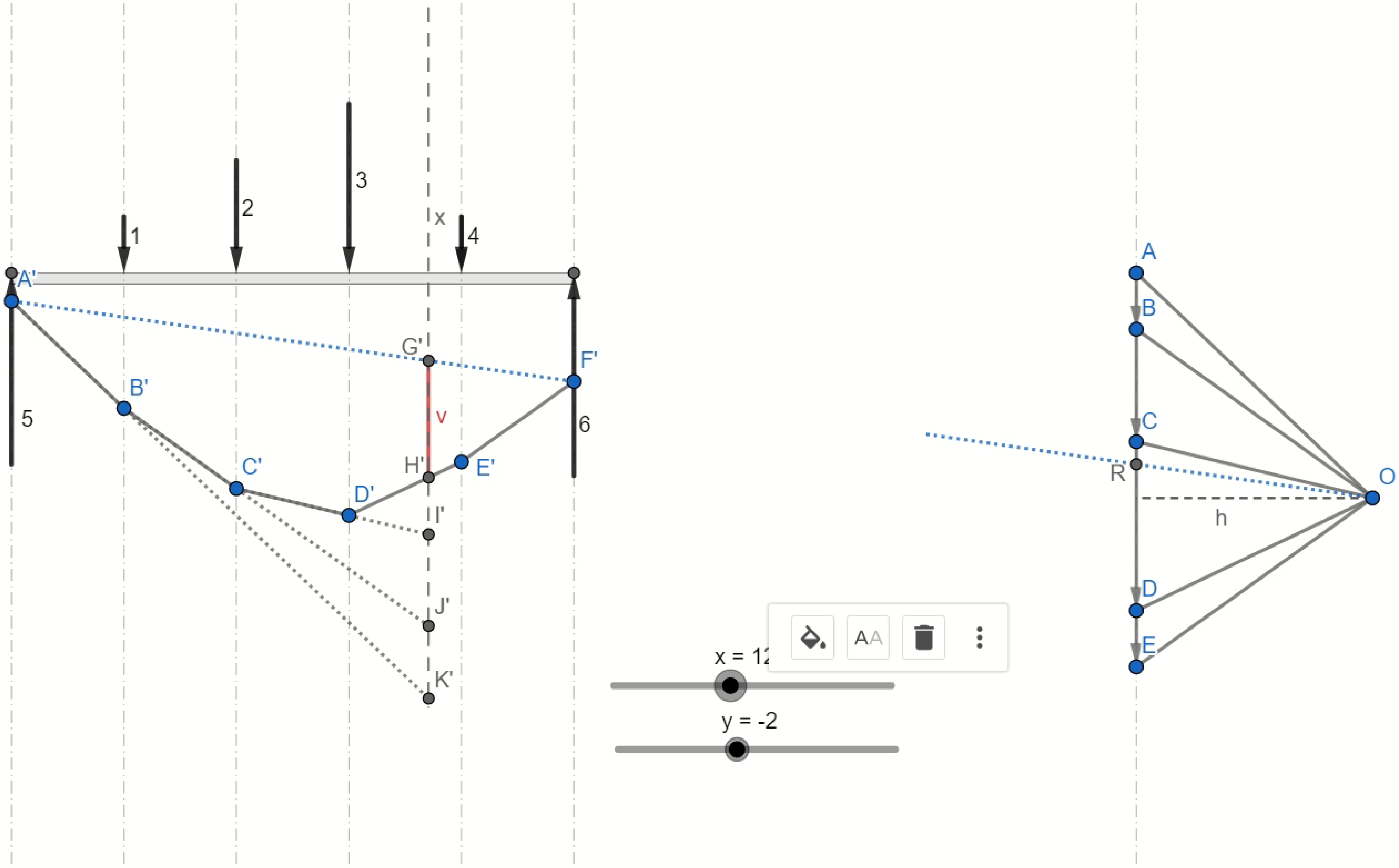


form diagram/ funicular polygon/bending moment diagram

force polygon

$$M_x = h \cdot G'K' - h \cdot J'K' - h \cdot I'J' - h \cdot H'I' = h \cdot \mathrm{v}$$

2. graphical solution of the moment of force 1-4



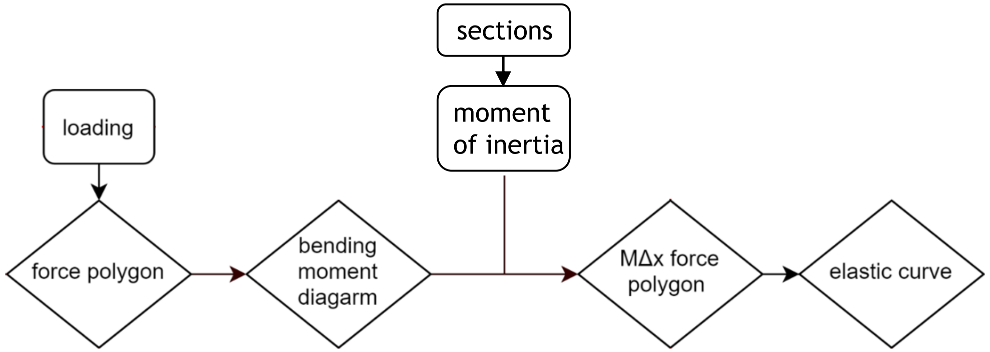
- The position of the pole won’t affect bending moment
- v is in reverse ratio to h

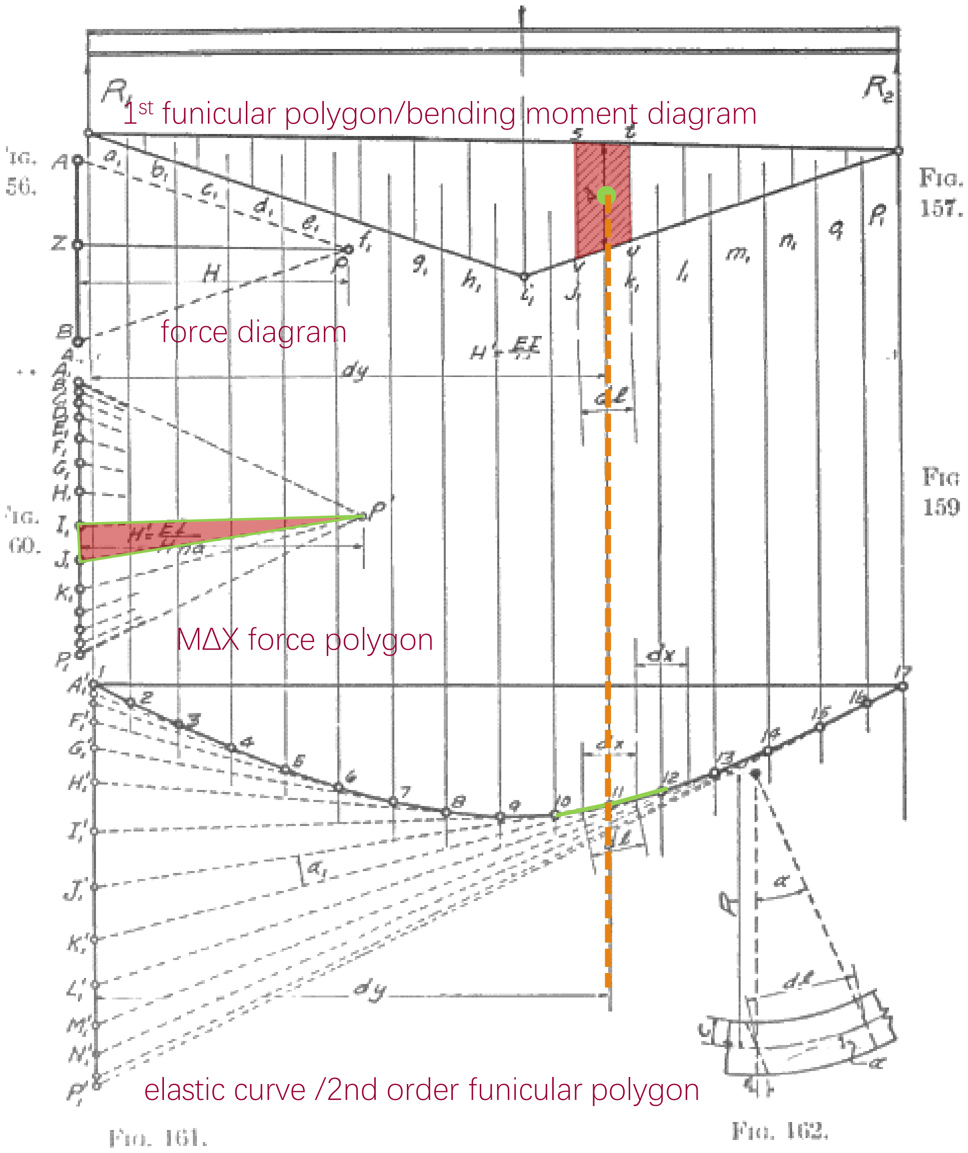
2.3 Morh’s analysis of the deflection of a statically determinate beam with graphic statics

“The deflection curve could (also) be represented by a funicular polygon analogically taking the bending moment as the “loading” instead of loads.”[Mohr, 1868]

$$M(x) = \iint q(x)\, d^2x \qquad y(x) = \frac{1}{EI}\iint M(x)\, d^2x$$

- The loads of MΔX polygon is the area of the slices of bending moment diagram
- The pole distance of MΔX polygon equals $\frac{EI}{n \cdot H}$
- The elastic curve is the second funicular polygon determined with MΔX polygon
- this second funicular polygon is “loaded” at the centroids of moment area slices



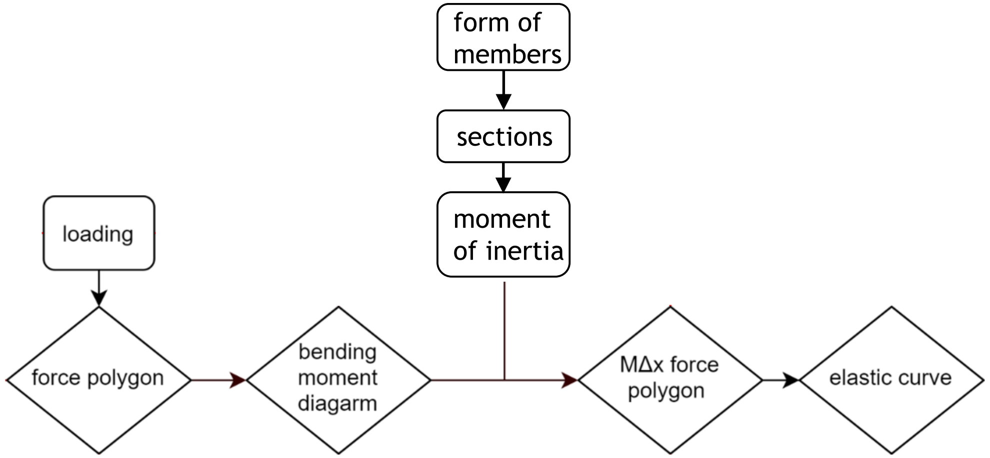


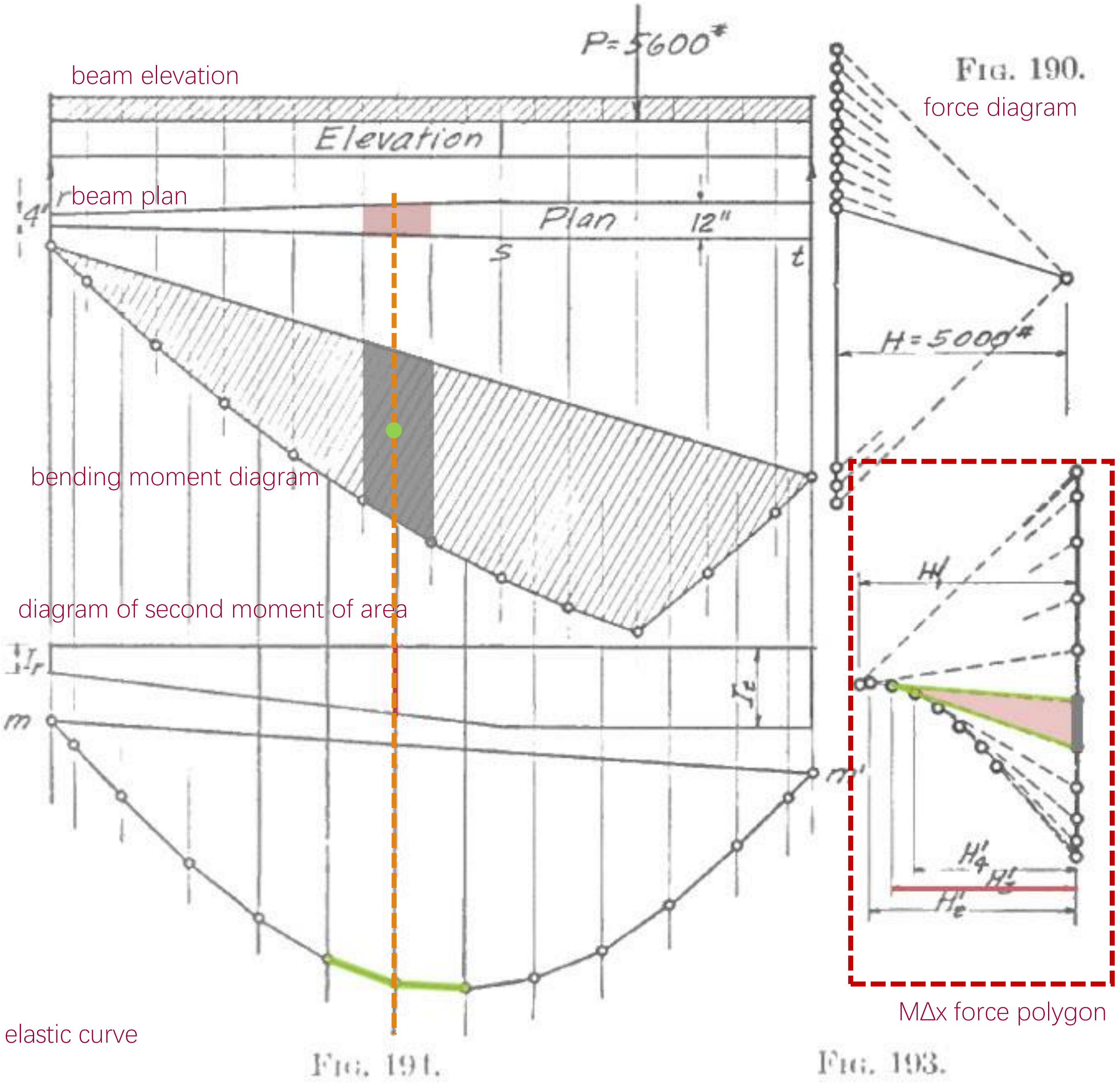
Graphic statics for bending moment and deflection of a statically determinate beam

Source: Wolfe 1921

Beams with varying section/second moment of area

- the pole distance for each segment varies in proportional to its corresponding second moment of area/moment of inertia and equals $\frac{EI}{n \cdot H}$,
- Each ray intersects the previous one or its extension on the pole.

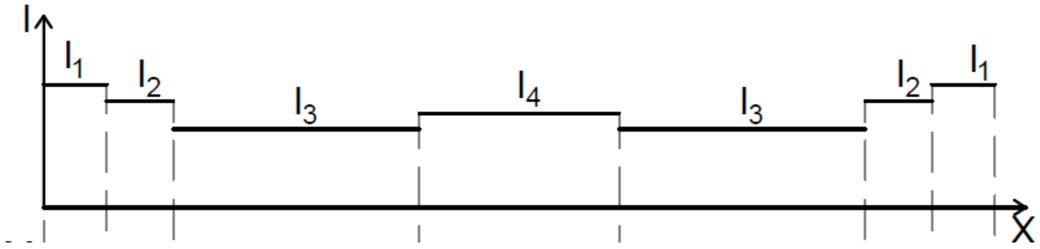




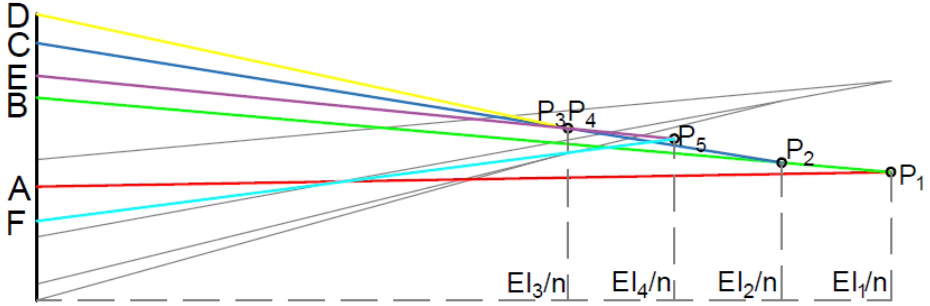
 Source: Wolfe 1921

Beams with shifting moment signs and varying section/second moment of area

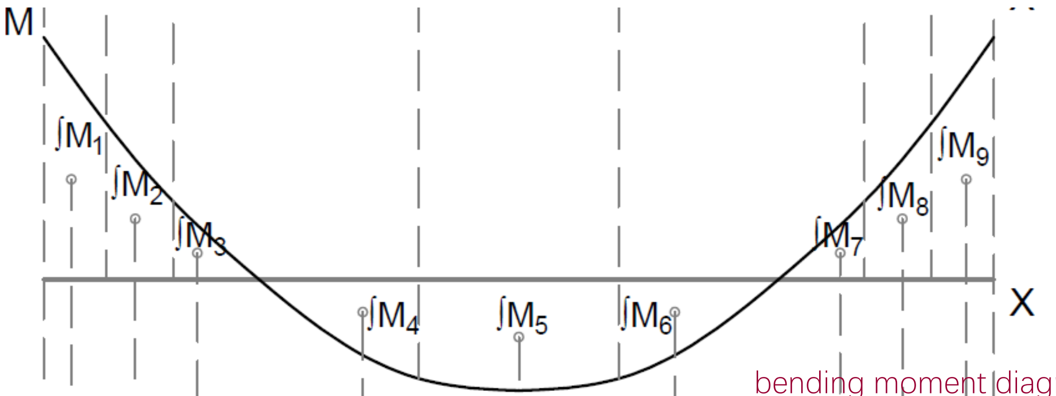
- All the fans in the MΔx polygon can be scaled to share the same pole distance
- The positive and negative moment “loads” are laid in the opposite direction on the “load line” of MΔx polygon



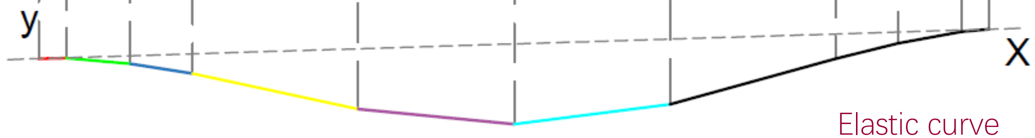
second moment of area diagram



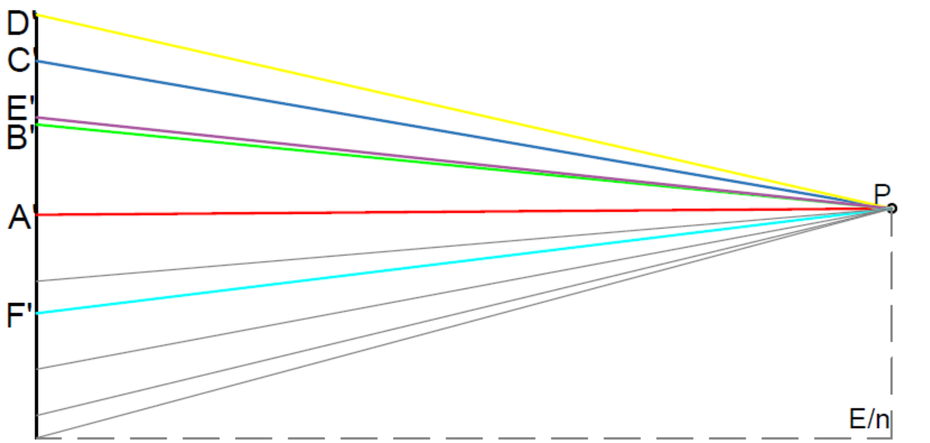
MΔx force polygon with varing pole distance



bending moment diagram



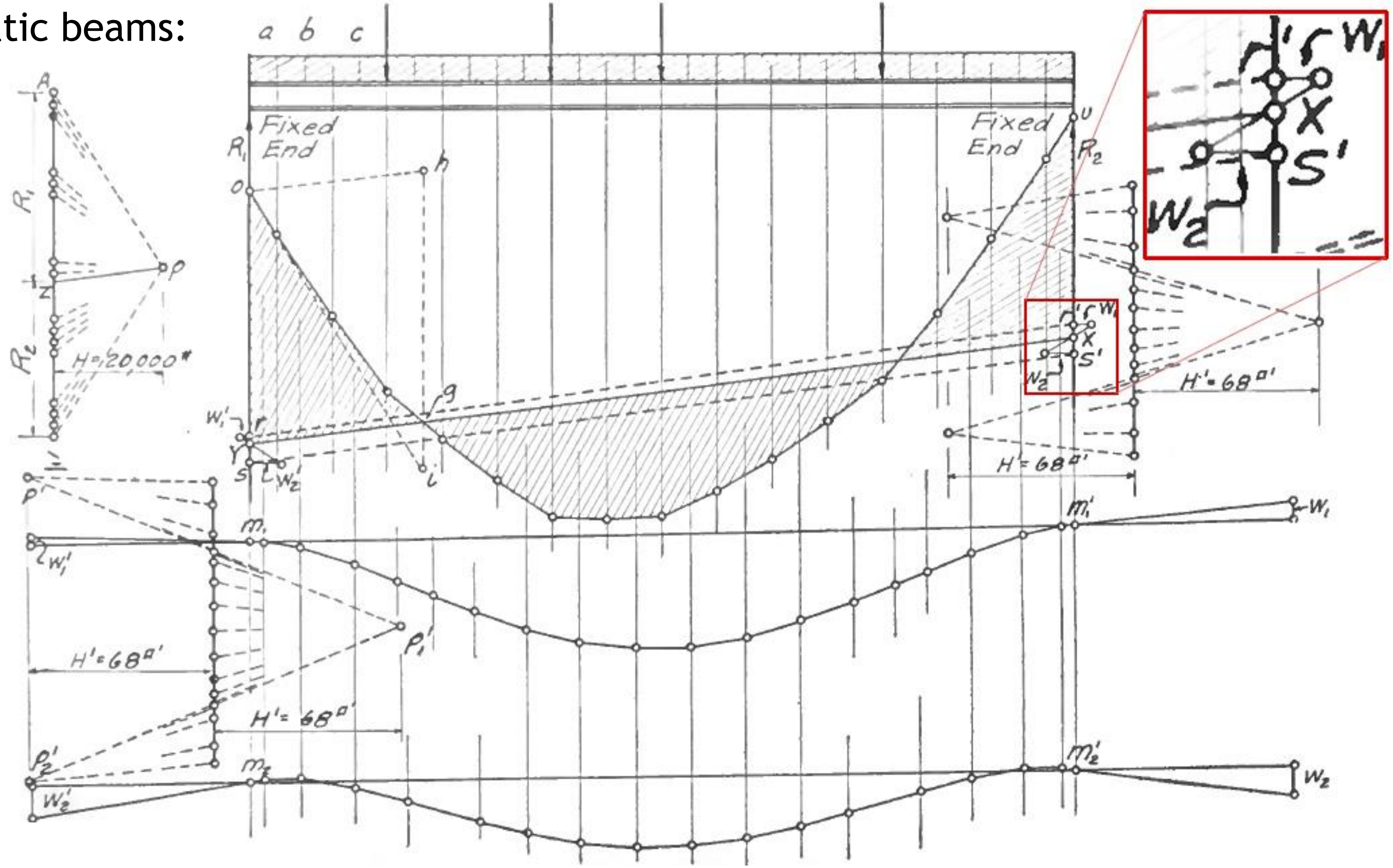
Elastic curve

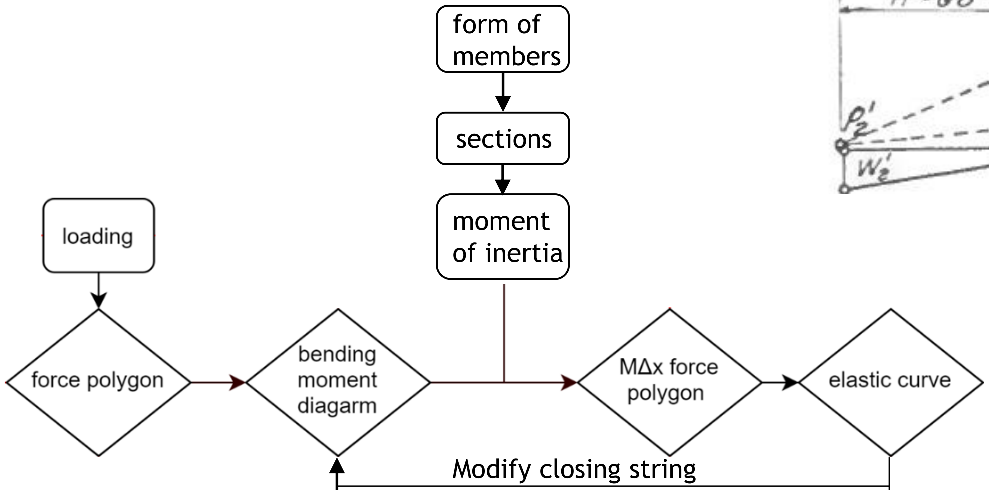


MΔx force polygon with scaled constant pole distance and moment line

2.4 Wolfe’s techniques for hyperstatic beams:

- Using trial bending moment diagrams for statically indeterminacy
- Using elastic curves to check and adjust
- Use a pair of triangles to check the elastic curve
- The positive and negative moment “loads” are laid in the same direction on the “load line” of MΔx polygon, but their rays are on the opposite sides

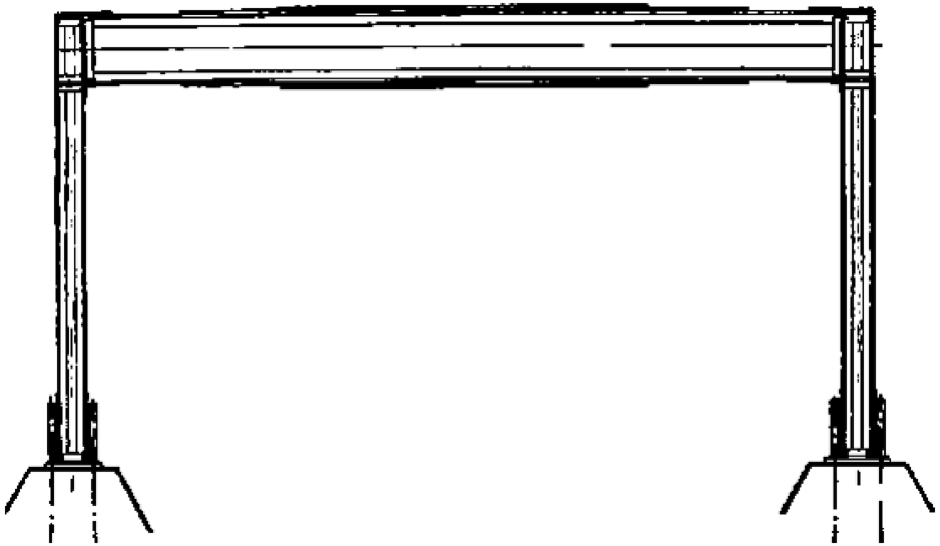




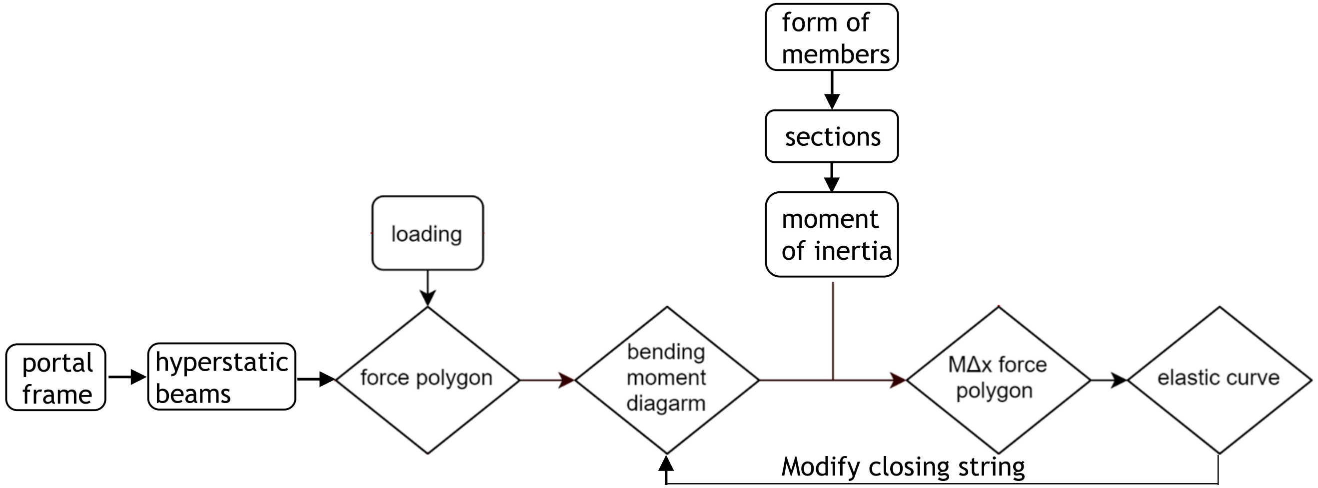
Source: Wolfe 1921

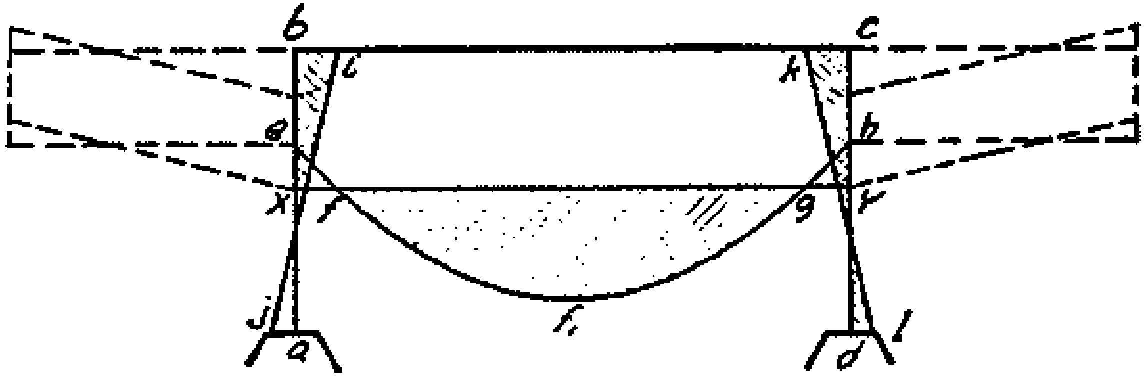
3 Wolfe’s techniques for hyperstatic portal frames:

- Frames are rotated into statically indeterminate beams
- Using trial bending moment diagrams for static indeterminacy
- Using elastic curves and triangle pairs to check and adjust the bending moment diagram

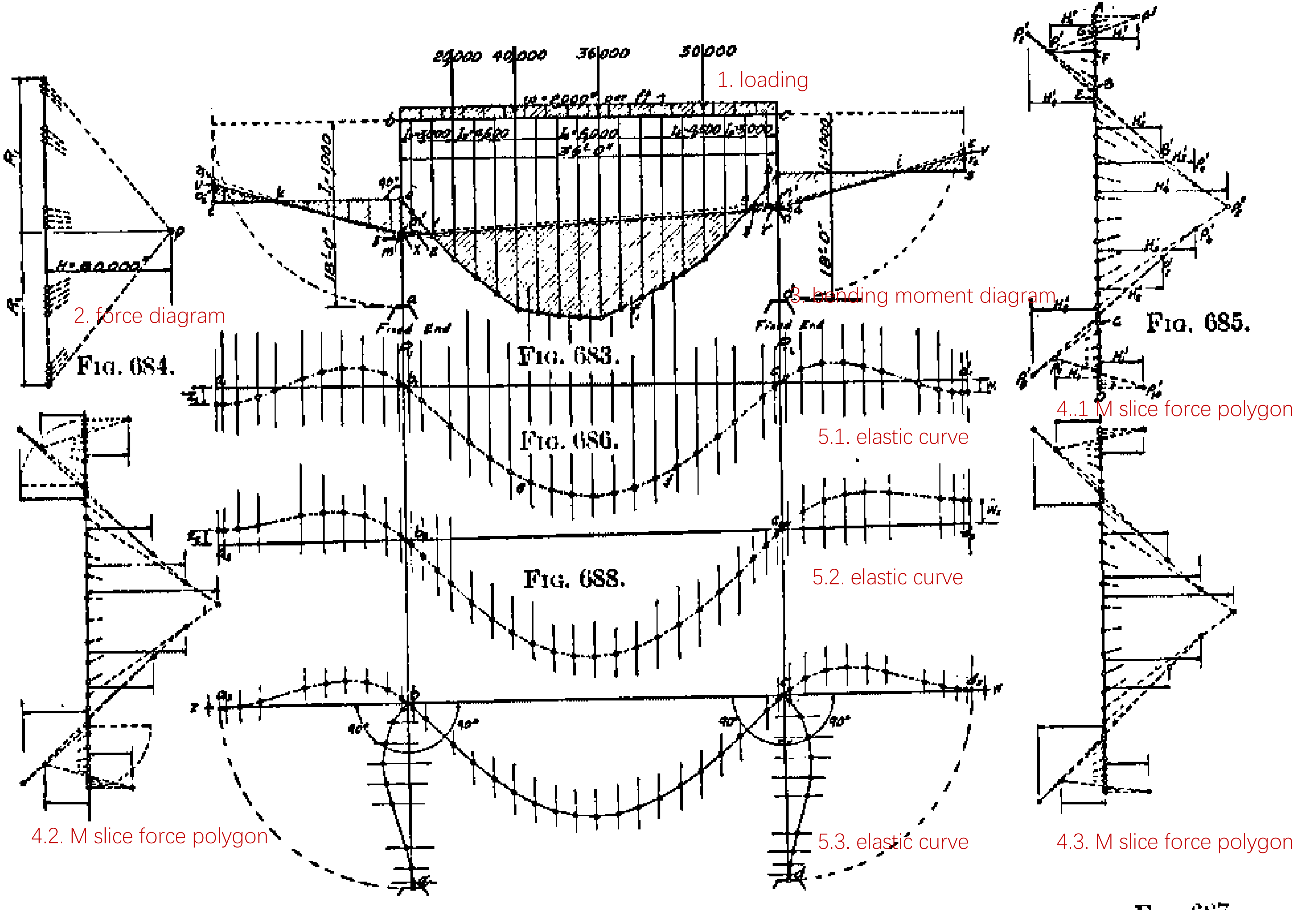


a portal frame fixed at both ends





convert the portal frame into a statically indeterminate beam



- Frames are rotated into statically indeterminate beams
- Using trial bending moment diagrams for static indeterminacy
- Using elastic curves and triangle pairs to check and adjust the bending moment diagram

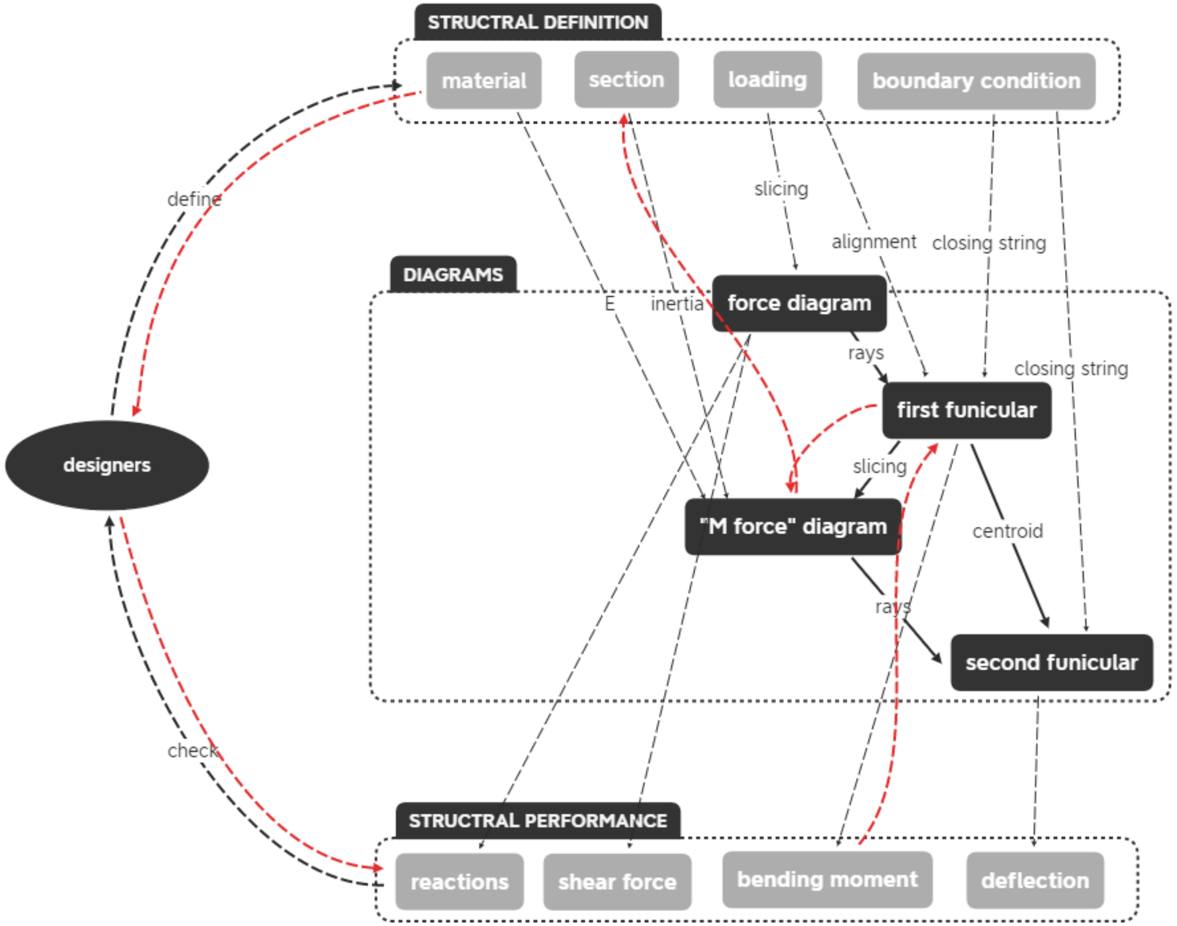
4. The values and drawbacks of the historical geometric-based method for rigid frames

Advantages:

- Advantageous for problems involving compound loading or varying second area moment.
- Less demanding for mathematics knowledge and skills
- Chronology-free can operate on both directions
- Very insightful into the relation between form and forces

Drawbacks

- Slow and cumbersome with hand-drawing
- Can only be applied to symmetric frames under symmetric vertical loading
- This method has not been well developed for more complex frames



Chronology-free procedure of this geometric-based approach
(gray line for analysis procedure red lines for a possible modification)

5. Summary and future work

Summary

- The history of the geometric-based method for rigid frames -(section II)
- Technique details explanation -(section III and IV)
- A discussion on the advantage and drawbacks of the methods- (section V)

Future work：

- Other historical geometic-based methods for rigid frames-fixed point method

- Theoretical development and parameterization of this approach from the perspective of design

THANK YOU !