

ONE-DIMENSIONAL FINITE-VOLUME MODELING OF THE FLOW AND MORPHOLOGICAL PROCESSES DURING THE 1996 LAKE HA!HA! DYKE BREAK EVENT

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Abstract

The major flood following the Lake Ha!Ha! dyke-break in 1996 in Québec (Canada) induced important morphological changes in the valley. For instance, massive erosion was witnessed along with significant valley enlargements. Another interesting observation was the creation of an avulsion, i.e. a new river bed. Due to the very complex and irregular topography of the valley, flow simulations in that area constitute a real challenge for numerical models. Indeed, the river presents very steep slopes at some locations, or very abrupt changes in cross-section shape. The presence of rocky outcrops creates another difficulty because of its impact on the erosion process.

The focus of the research is to simulate the Lake Ha!Ha! dyke-break event using a one-dimensional finite-volume scheme solving the Saint-Venant-Exner equations in a decoupled way. First, the analysis is conducted from the pure hydrodynamical point of view allowing the characteristics of the flow to be computed. Then, these characteristics are used to determine the sediment transport and, finally, the morphological changes. Even though the model uses a one-dimensional framework, bank erosion and bank failure processes are also accounted for. Comparisons between the computed final topography and field measurements are provided, along with a discussion about the quality of the results and the key issues for future research.

Keywords: River hydraulics, sediment transport, fast transient flows, numerical modeling, morphodynamics.

1. Introduction

Even though floods are not the deadliest natural hazards, they affect a lot of people and generate heavy damages. These damages mostly come from fast flows with intense sediment transport. These flows can cause important changes in the river morphology due to both erosion and deposition of sediments. Besides, floods can be aggravated by the failure of a dam or a dyke.

The main goal of the work presented here is to simulate the fast transient flow following a dam-break event in a real, complex, river. The case chosen is the Ha!Ha! Lake dyke-break event that happened during heavy floods in Québec in 1996 for which extensive data sets exist (Capart *et al.*, 2007). This case features several challenging issues for numerical models, like steep slopes, control sections, rocky outcrops or waterfalls.

After presenting the governing equations, the numerical model is introduced. A first validation is conducted using well known experimental sets (Spinewine and Zech, 2007; Soares-Frazão *et al.* 2007). Then, the model is applied to the Ha!Ha! dyke-break event. Finally, the limitations of the current model are pointed out.

2. Governing equations

In this model, only bed load sediment transport is considered. Furthermore, the bed level evolution is considered to be much slower than the water level evolution, justifying the choice of a decoupled approach to solve the equations. In other words, at each time step, the problem is solved first considering only the hydrodynamic part and then the morphodynamic part.

2.1 Hydrodynamics

The one-dimensional Saint-Venant equations (shallow-water equations) describe the mass and momentum conservation of water. Written for cross-sections of arbitrary shape, the governing equations read:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad [1]$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} + g I_1 \right) = g A (S_0 - S_f) + g I_2 \quad [2]$$

where Q is the discharge, A the wetted area, S_0 and S_f the bed and friction slope, respectively, I_1 is the static moment used to calculate the hydrostatic pressure and I_2 accounts for the effect of non-prismaticity. With b being the cross-section width at level η , the two last terms are defined as (Figure 1):

$$I_1 = \int_0^h (h - \eta) b \, d\eta \quad [3]$$

$$I_2 = \int_0^h (h - \eta) \frac{\partial b}{\partial \eta} \, d\eta \quad [4]$$

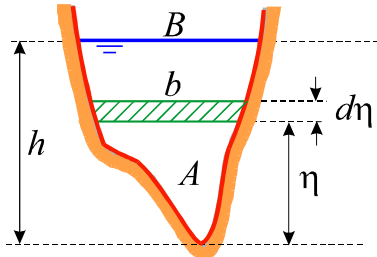


Figure 1 - Definition of the static moment

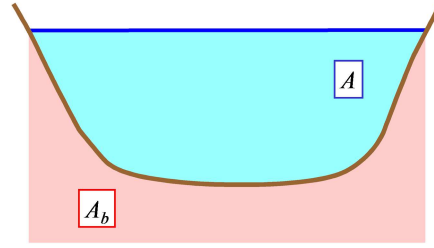


Figure 2 -Definition of the bed area

As suggested by Cunge *et al.* (1986) and developed by Petaccia *et al* (2013), the topographical source terms AS_0 and I_2 can be combined using the expression

$$\frac{\partial I_1}{\partial x} = A \frac{\partial z_w}{\partial x} - AS_0 + I_2 \quad [5]$$

This results in the following form of the momentum equation (2), where the derivative of I_1 is calculated for a constant water level $\bar{z}_w$.

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} + gI_1 \right) = -gAS_f + g \frac{\partial I_1}{\partial x} \Big|_{\bar{z}_w} \quad [6]$$

The friction slope is determined using Manning equation, with R the hydraulic radius and n the Manning friction coefficient

$$S_f = \frac{n^2 Q^2}{A^2 R^{4/3}} \quad [7]$$

2.2 Morphodynamics

To take into account erosion and deposition, the Exner equation which describes the mass conservation of sediments in the bed is added to the system:

$$\frac{\partial A_b}{\partial t} + \frac{1}{1 - \epsilon_0} \frac{\partial Q_s}{\partial x} = 0 \quad [8]$$

where (Figure 2) A_b is the bed area, ϵ_0 the porosity of the bed material, and Q_s is the sediment discharge. Non-equilibrium transport is considered using the Daubert-Lebreton equation (1967), where Q_s^* is the sediment discharge at equilibrium and L is the adaptation length.

$$\frac{\partial Q_s}{\partial x} = \frac{Q_s^* - Q_s}{L} \quad [9]$$

The sediment transport at equilibrium is calculated here using the Meyer-Peter and Müller formula, where B is the cross-section width, s the relative density, d the grain diameter and τ_c the critical shear stress.

$$Q_s^* = 8B\sqrt{(s-1)gd^3} \left(\frac{R S_f}{(s-1)d} - \tau_c \right)^{1.5} \quad [10]$$

3. Numerical scheme

3.1 Hydrodynamic equations

Equations (1) and (6) are discretized using first-order finite volumes with a LHLL scheme for the flux calculations (Fraccarollo *et al.* 2003, Goutière *et al.* 2008).

$$U_i^{n+1} = U_i^n + \frac{\Delta t}{\Delta x} (F_{i+0.5}^* - F_{i-0.5}^*) + S_i^* \Delta t \quad [11]$$

The LHLL approach consists in considering at each cell interface a discontinuity between two constant states, creating a Riemann problem. It is assumed that an intermediate state denoted U^* is delimited by two waves issued from the initial discontinuity and travelling at speeds $S_L = \min(V - c)$ and $S_R = \max(V + c)$.

Within this framework, the mass flux is written as

$$Q^* = \frac{S_R Q_L - S_L Q_R + S_L S_R (z_R - z_L)}{S_R - S_L} \quad [12]$$

where $z_R - z_L$ is used instead of $h_R - h_L$ to ensure equilibrium when water is at rest on a sloping bed (Nujic, 1995).

For the momentum flux, the LHLL scheme introduces a difference between the right and the left flux, by accounting for the topographical source terms $(\partial I_1 / \partial x)|_{\bar{z}}$ in the expression:

$$F_L^* = \frac{S_R \sigma_L - S_L \sigma_R + S_R S_L (Q_R - Q_L) - (S_R - S_L) g I_{1L}}{S_R - S_L} \quad [13]$$

$$F_R^* = \frac{S_R \sigma_L - S_L \sigma_R + S_R S_L (Q_R - Q_L) - (S_R - S_L) g I_{1R}}{S_R - S_L} \quad [14]$$

For the friction source term, a semi-implicit method is used. First, equation (11) is solved without considering friction but only the various fluxes, resulting in an intermediate state. Then, the friction is accounted for by using the Manning formula (7).

3.2 Morphodynamics

A four-point Preissmann scheme is used to discretize the Exner and Daubert-Lebreton equations (Figure 3), with the following general expressions where m and i represent time and space, respectively, and θ and ψ are weights:

$$f = \theta\psi f_{i+1}^{m+1} + \theta(1-\psi)f_i^{m+1} + \psi(1-\theta)f_{i+1}^m + (1-\theta)(1-\psi)f_i^m \quad [20]$$

$$\frac{\partial f}{\partial t} = \psi \frac{f_{i+1}^{m+1} - f_{i+1}^m}{\Delta t} + (1-\psi) \frac{f_i^{m+1} - f_i^m}{\Delta t} \quad [21]$$

$$\frac{\partial f}{\partial x} = \theta \frac{f_{i+1}^{m+1} - f_i^{m+1}}{\Delta x} + (1-\theta) \frac{f_{i+1}^m - f_i^m}{\Delta x} \quad [22]$$

Using $\theta = 1$ results in a fully implicit scheme, that is unconditionally stable. This allows the use of a large time step and so enables the use of the same or a larger time step than the hydrodynamics one.

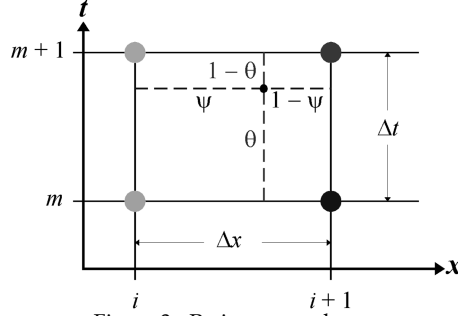


Figure 3 - Preissmann scheme.

With the discretization illustrated in Figure 4, in order to change the bed section following erosion or deposition, it is mandatory to translate the area variation into an elevation variation for each point ($\Delta z_{b,j}$). To do that, two different assumptions can be made. The erosion can be either considered as uniform over the whole cross-section, yielding

$$\Delta z_{b,j} = \frac{\Delta A_b}{B} \quad [23]$$

Or, the erosion can be considered proportional to the water depth, and a linear relation is used

$$\Delta z_{b,j} = \frac{h_i}{A} \Delta A_b \quad [24]$$

The presence of non-erodible bed rock is considered by setting up a lower limit for the bed elevation, and adapting the sediment transport accordingly.

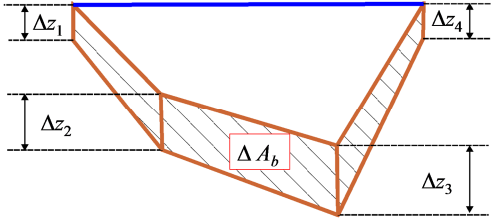


Figure 4 - Discretization of the cross-sections.

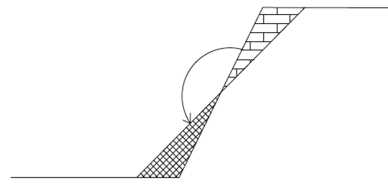


Figure 5 - Bank failure mechanism.

4. Bank failure operator

During the erosion process, portions of the banks located under the water surface are eroded. This process can lead to an excessive steepness of the bank, beyond the stability limits of the material. A bank failure operator is introduced, based on tilting of the unstable portions of the bank (Swartenbroekx *et al*, 2012). This results in moving sediments from the top to the toe of the bank (Figure 5).

To determine the slope limit, strong assumptions are required. First, sediments are assumed to be solely composed of non-cohesive particles. Secondly, the influence of groundwater is not accounted for. And, finally, the last assumption is the strongest of them all, the bank is considered to be infinite. These allow the use of the following relation between the limit (β) and the frictional angle (ϕ) through the density ratio (s).

$$\tan \beta = \frac{s - 1}{s} \tan \phi \quad [25]$$

The same mechanism is used to represent the failure of the bed slope in the longitudinal direction. As regards the frictional angles, one value is provided for humid material and one for dry material.

5. Comparison with experimental results

Before using the model in a complex geometry, it had to be validated on simplified cases. Two experiments were chosen as test cases. The first one is a dam break experiment over movable bed (Spinewine and Zech, 2007). As this test case is purely one-dimensional with a rectangular cross-section, it is used to validate the model with uniform erosion and without bank failure. The second one is a dam break experiment in a trapezoidal channel (Soares-Frazão *et al*, 2007). It will be used to validate the model with non-uniform erosion and bank failure operator.

5.1 Dam-break on rectangular sections

5.1.1 Experimental setup

The experiments were conducted in a 6 meter long flume with 0.25 meter width that was initially filled with sediments. Two kinds of grains were used: PVC and sand. Experiments were conducted on flat bed but also with a step located at the dam (Figure 6).

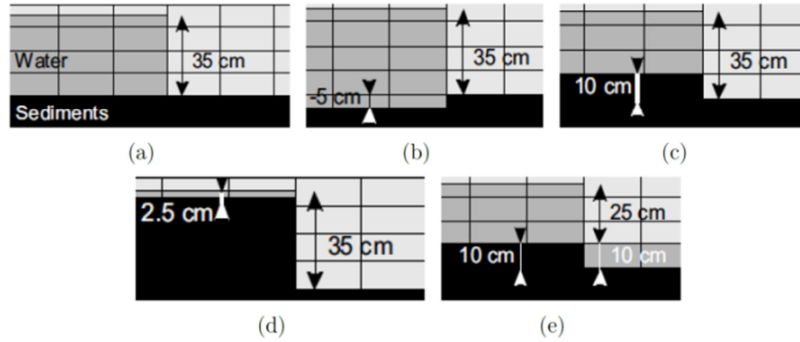


Figure 6 - Experimental setup (from Spinewine and Zech, 2007): various configurations used. Sediments are either composed of sand or PVC.

5.1.2 Numerical simulations and results

All the configurations illustrated in Figure 6 were simulated, using both types of sediments. In the simulations, sediment transport is considered at equilibrium, erosion / deposition are distributed uniformly over the width and bank failure is not considered. The flume was divided into cells of 5 centimeters. The Preissmann weights for the space ψ and time θ were 0.5 and 0.8 respectively.

It can be seen in Figure 7 (left) that the results are quite good with a pretty close prediction of the final bed and the water level, for case (c) of Figure 6. All other cases were also successfully reproduced, except for case (d) with a huge step (Figure 7, right). A sensitivity analysis was conducted (Franzini, 2013) showing that the equilibrium assumption and the Manning coefficient both have a significant impact on the solution (Figure 8 and 9). Inversely, the size of grains did not have any influence. It can be explained by the fact that the bed shear stresses occurring in this case are a lot bigger than the critical shear stress.

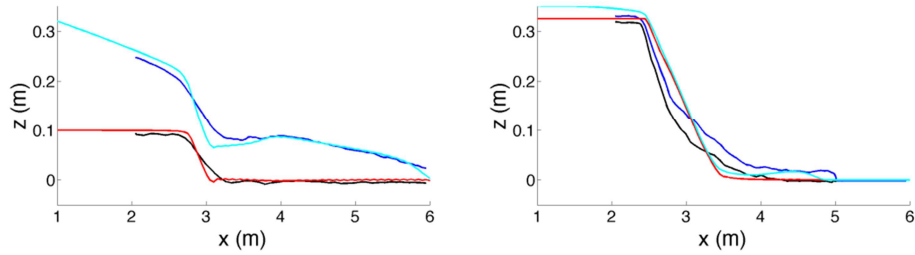


Figure 7 - Comparison between experiment (bed: black, water: blue) and model (bed: red, water: light blue) for the use of sand in two different step size. Left: case c (sand), Right: case d (sand).

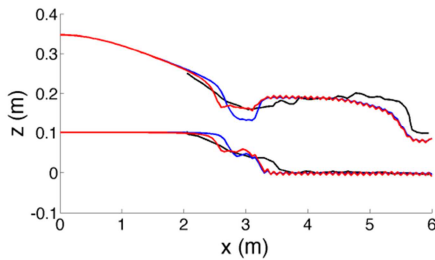


Figure 8 - Impact of Manning coefficient change from 0.0188 (blue) to 0.025 $\text{m}^{1/3}/\text{s}$ (red) for case (e) in Figure 6.

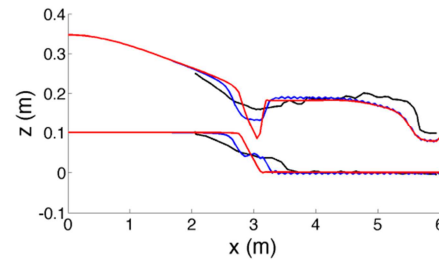


Figure 9 - Comparison between equilibrium (blue) and non-equilibrium assumptions (red) for case (e) in Figure 6.

5.2 Dam-break on trapezoidal sections

5.2.1 Experimental setup

This experiment was conducted in a 6 meter long flume with semi-trapezoidal cross-sections (Figure 10). The bed upstream from the gate is not erodible.

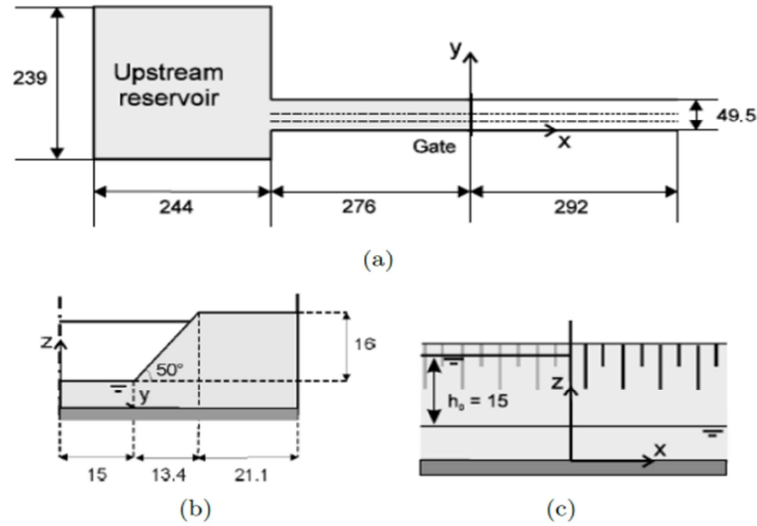


Figure 10 - Experimental setup (from Soares-Frazão *et al.*, 2007). a) Top view. b) Section. c) Initial conditions.

5.2.2 Numerical simulations and results

For this experiment, it was chosen to use a non-equilibrium sediment transport with non-uniform erosion, and to activate the bank failure. The flume was divided into cells of 5 centimeters. The Preissmann weights for the space ψ and time θ were both 0.8.

It can be seen in Figure 11 that the results are pretty close to what was expected with a good estimation of the final cross-section shape, even with the strong assumptions made for the bank failure operator. Similar results were obtained for the other cross-sections.

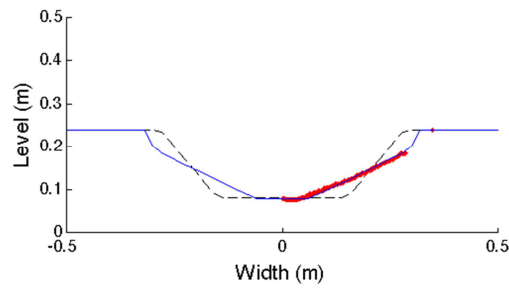


Figure 11 - Comparison between the experiment (red dots) and the model (blue lines). The initial section is also represented (black dashed lines).

6. Lake Ha!Ha! dyke break event

6.1 Model set-up

The test case is described in Capart *et al.* (2007) who provide the initial and final topography, the location of bed-rock and the boundary conditions (upstream discharge and downstream water level). Figure 12 shows some typical cross-sections in the river that experienced severe morphological changes.



Figure 12 - Helicopter photos of the Ha!Ha! river after the 1996 flood (Capart *et al.*, 2007).

The 35 kilometer long river was divided into computational cells of approximately 100 meter each. Simulations were run using a two-step method. First, the initial water level was determined by letting the water flow without allowing any erosion. After stabilization, a non-equilibrium transport was activated with non-uniform erosion and bank failure operator. The Preissmann weights for the space ψ and time θ were both 0.8.

6.2 Results and discussion

Figure 13 shows the initial and final bed elevations, following the field survey and the computations. Some features are reasonably well reproduced, but overall, the results did not meet the expectations. Major differences occurred especially near the waterfalls (around kilometer 7, 8, 12 and 22 in Figure 13) or at steeper slopes (from kilometer 30 in Figure 13).

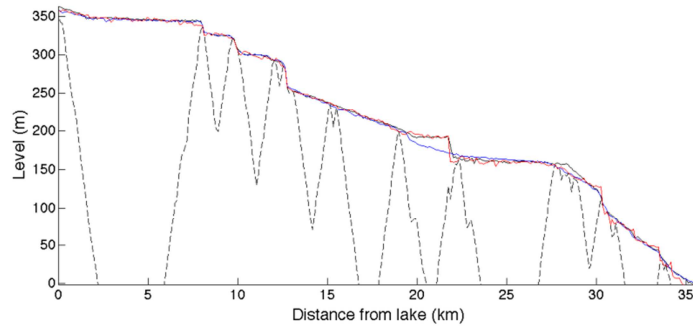


Figure 13 - Comparison between real data (initial: black, final: blue) and the model (red). The bed rock level is also represented (dash).

However, looking only at the first reach until kilometer 7, before the first waterfall, the cross-section evolution is quite closed to the observation (Figure 14 (left)).

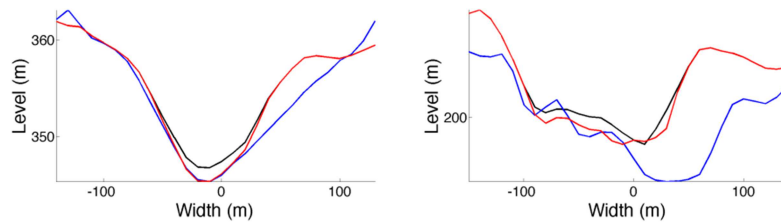


Figure 14 - Example of a well-represented (left) and badly-represented (right) section with the model (red) and data (initial: black, final: blue).

From these results, it can be deduced that the model still has serious limitations when used on real test cases, although it performed quite well on simplified laboratory cases. Firstly, it is based on a 1D approach, whereas there are some clear 2D features in the Ha!Ha! case, such as the avulsion for example that cannot be reproduced by a 1D model, or secondary flows in meanders that will enhance the erosion at the outer bank (Figure 14, right). Secondly, the model uses several parameters that are considered as being constant all over the river, that must be calibrated and that can have a significant effect on the result, e.g. the Manning friction parameter, the grain size, the soil porosity, the adaptation length in the non-equilibrium model, the angle of stability of the bank material. Thirdly, it does not take into account lateral input of water and sediment from runoff or tributaries. Furthermore, it is also based on a clear water assumption where sediment is transported only along the bed and therefore, the model cannot consider the impact of a high concentration of sediment. Lastly, it appears that the flux calculation scheme does not handle well the presence of waterfalls along the course of the river.

7. Conclusions

A one-dimensional model for solving shallow water and Exner equations in complex geometry is presented. The problem is solved in an uncoupled way. First, the hydrodynamical data are determined using a finite volume scheme with a LHLL-type solver for the fluxes. Then, the morphological evolution is computed using a four-point Preissmann scheme allowing non equilibrium sediment transport. A basic bank failure operator is also included in the present work.

Three examples of applications are presented: two experimental cases and one real case. The model predicted well the bed changes in the idealized experiments. Unfortunately, results in the real case did not meet the expectation.

In future works, the use of various models could be investigated such as two phases (Greco *et al.*, 2012) or mixture layer (Cao *et al.*, 2004; Wu and Wang, 2008). More emphasis will also be put on handling complex geometry with a focus on waterfalls fluxes and erosion on steep slope.

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