Teaching versus Research: a Multi-Tasking Approach to Multi-Department Universities

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Abstract

The budget of a university essentially depends on the number of students it enrols. In multi-department universities resources created in one department may be redistributed to other departments. This redistribution affects the way academics share their working time between research and teaching activities. Redistribution creates free-riding on teaching efforts. In this paper, we show that by designing internal financial rules which create yardstick competition for research funds, a multi-department university may induce better teaching quality *and* research, as compared to the performance of independent departments.

Keywords: Multi-task, Incentive, University, conglomerate

JEL Classification: D21, D82, G31, I20

1 Introduction

As is widely acknowledged today, a key difference between universities and professional schools (or other actors in the higher education system) is that only the first ones count teaching *and* research as part of their core social goals. In this respect evaluating the performance of any university system calls for answering at least two questions. Do universities manage to combine high quality teaching and high quality research? And, if they do, how? Obviously, one would like to see any university to excell in both dimensions but beyond wishful thinkings, very little is known about how to effectively realize this ideal. On the one hand, evidences from UK (see Shattock, 2002) suggest that universities that perform very best in research also perform best in teaching. On the other hand, it is hard to see how things are going in those universities which are not in the Top 10. The situation is even more opaque in many continental systems where university assessement is in its very enfancy.

Fortunately enough, combining high quality teaching and high quality research is actually desirable for the universities themselves. This is in particular true in a system where universities are mostly financed on a per student basis and where students' choice depend on (1) teaching quality and (2) university's prestige (which is related to research quality). However, combining high quality teaching and high quality research is often viewed as a challenge or a source of conflicts within universities taken as an aggregate.

This is partly due to the *individual* incentives faced by academics. For them, research and teaching activities are most often substitutes. Moreover, in many education systems, academics benefit from a large discretion in the allocation of their working time. Tavernier and Wilkin (2001) show that academics do use this discretion to a large extent, resulting in very different occupation profile for academics. Accordingly the actual splitting of their time among the various tasks they are assigned to is largely a matter of taste and incentives. There are a priori many ways by which a university could reconcile the individual conflict between teaching and research faced by its academics and the vital interest of performing well in both dimensions in the aggregate. Promoting specialization, with some academics being teaching professors and others full time researchers is a possible solution. Designing incentives schemes that value both aspects simultaneously is another one. However, there are also constraints limiting what can be actually implemented. Suppose for instance that at the individual level high quality research and high quality teaching are complements, i.e. in order to be a good professor, one must also be a good researcher. Then the "specialization" route must be abandoned. Suppose instead that research and teaching quality are not equally easy to assess. Then, the actual implementation

of targeted schemes is hampered. In the context of an emergent market for academics, a similar argument could be made whenever research and teaching abilities are not equally easy to signal to alternative employers.

As a matter of fact, the tensions between teaching and research have not been widely studied in the economic literature. A few exceptions are worth being pointed out. Del Rey (2001) models competition between universities who decide on the allocation of funds between teaching and research activities. In her model, teaching achievements and research records enter the university objective function and funding is positively related to the number of students. She studies the balance between research and teaching efforts as a function of the funding rules. However, a key feature of her analysis is that academics and university authority share the same objective. De Fraja and Iossa (2002) point out that increased students' mobility favors the emergence of "elite" institutions, i.e. a limited number of high research record universities co-existing with other universities focusing on teaching activities. Beath et al. (2003) focus on the tensions between pure and applied research under binding budget constraints. However, the teaching side of the academics' job is not considered in their paper.

In this note, we start from the agency relationship that links academics to their authority (rectorate, deans,...). Specifically, we focus on the links between the multitasking nature of a professor-university relationship and the multi-unit nature of these universities. The paper is organized around two simple ideas. First, at the aggregate level, universities share key features with conglomerate firms. Indeed, universities are active in multiple fields: science, economics, law, ... Each field is organized within one department, with more or less autonomy. To a large extent, research and teaching are discipline-specific and the decisions made regarding some discipline are largely independent from those taken in other disciplines.[1] In addition, universities are headed by a central authority which in particular has the final decision on the allocation of funds. Most often, the budget is centralized and the resource constraint applies at the university level. It means that the allocation of resources is done at the university level too. Thus, universities rely on an internal financing system which is very similar to the internal capital market of a conglomerate firm (see Coupé, 2001). Such practices are widely documented and to a certain extent can be viewed as socially desirable. University completeness is sometime believed to be part of a university's mission (and this argument might be sufficient to justify a (possibly inefficient) form of redistribution). However, the ensuing solidarity between "cash-cow" departments and smaller units does not go without tensions.

The second building block of our analysis is the relationship between academics and author-

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[1] This is in particularily true if students choose first a discipline and second a university where they attend.

ities. We view it as a multitask agency problem. The university authority wants to provide incentives on the two dimensions of teaching and research and we take it a an assumption that academics must perform teaching and research. While teaching and research require some effort, we assume that research is more valuable to the academic than teaching. This might simply reflect the tastes of the academic but there are more fundamental reasons for that. In particular, the emergence of a market for academics induces more severe career concerns. As a matter of fact, while the quality of individual research output is reasonably easily assessed, teaching quality is most often evaluated at the level of a whole program, rather than at a individual level. Therefore, an academic is likely to put more effort on research than teaching because research outputs are more easily appropriable than teaching efforts.[2] All in all, inducing effort on the teaching task might be more difficult to achieve than on the research task.

Consider now the issue of funding. If a university's funding mostly depends on the number of students and if these students are (at least partially) responsive to the reputation of university programs, it is especially crucial to ensure high teaching quality. Because more funds allow for a better research environment, good teaching performance makes high research records less costly. Thus, even if academics dislike teaching, they may exert a significant effort on improving teaching quality because, by attracting students, they will obtain funds that will make high research records less costly. Think of an extreme case where each academic is totally independent: he teaches the students who choose to attend his courses and finances his research with their enrolment fees. Clearly, whatever strong her distaste for teaching might be, the academic has to teach if she wants her research to be funded.

How does the conglomerate nature of a university affect the previous argument? Obviously, the problem comes from the possibility of reallocating funds dedicated to research between the different departments. The presence of such an internal market for research funds makes it more costly to induce teaching effort. First, because of an insurance effect (even if one does not raise any fund it will benefit from research funding). Second, because the marginal value of effort is smaller (because only part of the funds raised through teaching effort will be appropriated ex post). In a multi-department university, teaching efforts can be viewed as private resources spent at contributing to the constitution of a common resource. Self-interested academics are therefore very likely to free-ride on such efforts. On the other hand, because it is a conglomerate, the university may organize yardstick competition between academics for the allocation of research

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[2]Moreover, the evaluation of research quality through publication scores or patents holdings is much easy to establish from outside the university and to transmit than the quality of teaching, which requires internal access to the institution.

funds. When such a yardstick competition is at work, high quality research by an academic is likely to induce high effort by the others. Because it induces both free-riding and yardstick competition, a mutli-unit organization for the university might a priori be thought of as a "good" with respect to research quality and a "bad" with respect to teaching efforts.

However, we show hereafter that the conflict between research and teaching may actually be resolved within the multi-unit institution. Under well-designed incentive schemes, teaching efforts and research efforts may remain complementary in the multi-unit organization, even if there are no cost synergies between the two activities. Then, the relevant issue is to compare the multi-unit university with a collection of independent departments. Does the redistribution inherent to the multi-unit form allows for a better performance in research, in teaching or both? If not, what are the trade-offs? In this respect, our key result is the following: *the conglomerate nature of universities may actually be instrumental in promoting the quality of the teaching-research bundle as compared to a collection of single departments.*

Notice that, by focusing on the tensions between teaching and research, we abstract from other important problems. For instance, we neglect the implications of the now standard distinction between pure and applied research (see Jensen and Thursby, 2001) on performance assessment. We also overlook the third and fourth basic tasks an academic is asked to perform, namely service to the society and administrative duties. (Tavernier and Wilkin (2001) show that these activities may indeed crowd out a significant share of an academic's time).

The paper is organized as follows. First, we develop our stylized model. This is done in section 2. Section 3 characterizes optimal contracts and their implications for the relation between teaching and research. Comparative statics results are also dealt with in this section. Section 4 discusses the limitations of our analysis as well as possible extensions.

2 The Model

We consider a university with $N \geq 2$ departments. Each department is personified by a unique professor, the so-called "academic". There are $N + 1$ players: N professors and one university dean. Thus, we only consider a two layers hierarchy: University-dean and professors.[3]

Professors allocate their time between two activities: **teaching** and **research**. The vector $a = (a_1, a_2)$ is the vector of actions where a_1 refers to teaching effort and a_2 refers to research

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[3]Obviously, reality is more complex. Universities display multiple layers hierarchy. However, the present simplification allows us to combine the mutlitasking issues faced by academics with the redistribution problem in a tractable model.

effort. Performing a level of action a costs the professor $C(a)$. The cost function is increasing and convex in both arguments. No a priori restrictions are made about the sign of the cross derivative: $\frac{\partial \partial C(a)}{\partial a_1 \partial a_2}$. A positive (negative) sign would reflect negative (positive) synergies between the two activities.

Each professor is endowed with a vector of "talent". This vector represents the professor's ability to do research and teaching. Talents are denoted by a two-dimensional vector $\eta = (\eta_1, \eta_2)$, where the first element represents the professor's teaching talent and the second, the professor's research talent.

A variable θ_i identifies a proxy for the quality of the research projects undertaken by professor i. It depends on the effort in research activity and on the professor's research talent:

$$\theta_i = f(a_{2i}, \eta_{2i})$$

with $\frac{\partial f}{\partial a_{2i}} > 0$, $\frac{\partial f}{\partial \eta_{2i}} > 0$, and research effort and talent are not perfect substitutes: $\frac{\partial \partial f}{\partial a_{2i} \partial \eta_{2i}} > 0$. A research project of quality θ_i leads to a research output only if it is combined with financial resources.

Students' choice is not explicitly modeled, however we assume that n_i, the number of students in discipline i, depends on teaching quality in field i. Quality itself depends on the combination of teaching effort and teaching talent of professor i. Hence we assume:

$$n_i = g(a_{1i}, \eta_{1i})$$

with $\frac{\partial g}{\partial a_{1i}} > 0$, $\frac{\partial g}{\partial \eta_{1i}} > 0$. Like for research, we assume that talent and effort are not perfect substitutes: $\frac{\partial \partial g}{\partial a_{1i} \partial \eta_{1i}} > 0$. Teaching efforts contribute to the constitution of the general budget of the university through enrollment fees.

Academics receive a fixed wage v from the university. For simplicity, this fixed pay is normalized to zero. In addition, there is a reward w proportional to the research output. Denoting research output by the variable r_i, we assume the utility of professor i is given by:

$$U_i = wr_i - C(a)$$

wr_i could be interpreted either as a private benefit from research or as a future job opportunity, i.e. the professor's value on the academic market. In the first interpretation, wr_i represents the private benefits an academic enjoys from his research achievement. Private benefits of research could be notoriety, job opportunities, consultancy contracts, tenure position... Clearly, in all these examples, the benefit is tied to the academic's research output. What is specific in this model is the linear specification of the private benefit.

Alternatively, wr_i can be interpreted as the academic's market value. Professorships exhibit nowadays high mobility and high turnovers[4] with the consequence that there is today a true market for academics (see Siow (1995) on the organization of the market for professors). The value of an academic on this market is largely influenced by his research performance. High research achievement signals high quality (high talent) to prospective employers. Hence, a high research output r_i translates into better job opportunities and a larger pay. Under this interpretation, our model assumes that the market for academics values research at a per unit price of w. The professor's value wr_i can be interpreted as his future reservation wage, either inside his institution or elsewhere.[5]

Notice that teaching activity does not enter positively in the professor's utility function. This rather extreme assumption is meant to capture the idea that teaching is valuable inside a given university but it has little value outside. For instance, it might be difficult to signal to the job market high teaching quality. By contrast, research has a high visibility outside university, and can be used as a signal of quality (talent) on the market. Hence, the private benefit associated to the research output and the absence of private benefit associated with teaching. Clearly, this assumption makes the worst case for teaching efforts. We will relax this assumption in section 4, and consider the case where the academic market values both the research achievement and the teaching quality.

Last, we assume that the academic talent is perfectly known inside the university but it is not outside. This is a simplifying assumption, which allow us to consider a deterministic model where uncertainty is absent. Note that all academics are identical in this model, except for talents.

Regarding *Universities*, we assume the following. The budget of the university is noted by B. The university receives a fixed transfer F from the government and a tuition fee s per student. In state owned systems, the tuition is partially paid by the government. B is then equal to:

$$B = F + s \sum_{k=1}^{N} n_k$$

In the remaining of the paper, we consider the budget B as the total amount of resources available for funding research projects. Meaning that B is the university's resources net of the academic's wages (normalized to zero) and all the other spending of a university (which could account for a large amount).

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[4]See in particular the recent contribution of Ehrenberg (2003)

[5]Note that this part of the professor's reward is delayed. It represents the future pay prospect of a professor, hence, it does not need to be paid immediately out of the university's budget. Clearly, this interpretation bears some resemblance with career concerns models. This is discussed in section 4.

The research output of professor i is denoted by r_i. It depends on (i) the project's quality θ_i and (ii) the budget y_i allocated to research in department i. Research budget includes labs, research assistants, sabbatical year,... The production function for research output is the following:

$$r_i = \theta_i v(y_i) \quad \forall \, i = 1, ..., N.$$

The function $v(.)$ is increasing and concave. The concavity of the $v(.)$ function implies that the allocation of the research budget to the departments will not be of the form "winner takes all". Moreover, we assume that the derivative v' is homogenous of degree $-h < 0$. This is equivalent to assume that $v(y) = y^{1-h}$. Note that to simplify the analysis, we consider that the production function is not department specific. This however does not mean that all academics are identical with respect to the research activity since heterogeneity could be incorporated in the academic's research talent η_{2i}. Indeed talent could be interpreted either as a specific academic talent or as a field specific talent (or both). The same is true for teaching.

This specification for the budget takes as constant other forms of funding which are to a large extent accessible to a university (typically institutional research fundings by federal or private agencies). Notice also that we assume that departments do not compete among themselves for students. In other words, students are assumed to choose first their field of studies and then their university. Teaching effort affects the choice of the university, not the field chosen.

Within this framework, we may now specify the internal financing rules of the university and the timing of the events.

1. The Academics simultaneously choose the actions a_i.

2. Students choose their university and the values of n_i and θ_i are observed.

3. Given the qualities of the research projects and the total budget, the university allocates the research budget to Academics.

Underlying this game structure, the following assumptions are made: the university cannot commit ex-ante (that is before the professors choose the actions) to a particular sharing of the university budget. Thus, the budget will be allocated ex-post (once n_i and θ_i are realized). We assume that ex-post, the university allocates funds to research projects in order to maximize the aggregate research output $\sum_{k=1}^{N} r_k$. In other words, commitment to a pre-specified distribution rule is not feasible. Actually, commitment to a particular distribution rule ex-ante would be highly demanding. Ex-ante, the university should indeed be able to specify the allocation of resources given all possible realizations of θ_k and n_k, $k = 1, ..., N$. Such a rule would be a

mapping from the N^2-dimensional space of projects quality and students' numbers to the N-dimensional space of investments. It is reasonable to assume that the costs of writing such an allocation rule are prohibitive.[6] Moreover, ex-post, the university would still have an incentive to re-negotiate such an arrangement to allocate its scare funds to the more valuable projects i.e. to maximize the aggregate research output given the budget.[7]

Lack of commitment implies that we do not need to specify the objective of the university beyond maximization the aggregate research output given the budget. The allocation of resources by the university will then be similar to the winner-picking contest of Stein (1997).[8] The analogies between our model of multi-department universities and models analyzing conglomerate -for profit- firms will be discussed further in the last section. The specificity of our analysis is to integrate the multi-tasking nature of the incentive problem, which we view as inherent to the academic job. Be it for teaching or research, the quality of the output essentially results from the academic's effort and an academic should perform both tasks.

3 TheTrade-off between Research and Teaching Efforts

3.1 Optimal Efforts

At the last stage of the game, given the budget B and the value of research projects $(\theta_1, ..., \theta_i, ..., \theta_N)$, the university allocates B in order to maximize the research output. At the last stage of the game, the university then solves:

$$\max_{y_1,...,y_N} \sum_{k=1}^{N} r_k \text{ Subject to: } \sum_{k=1}^{N} y_k = B. \qquad (P1)$$

We may then specifiy the optimal allocation rule for the university budget.

PROPOSITION 3.1 *The optimal allocation of the budget is: for all $i \in N$*

$$y_i = \alpha_i B$$

where $\alpha_i = \dfrac{\theta_i^{\frac{1}{h}}}{\sum_{k=1}^{N} \theta_k^{\frac{1}{h}}}$

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[6]For references on the cost of writing contracts, see Tirole (1999).

[7]In section 4 we discuss the case where the university can fully commit ex-ante to a simple redistribution rules where a fraction γ of the budget is allocated according to the projects' relative quality and a fraction $1-\gamma$ of the budget is allocated according to the number of students. This rule may improve the global performance of the university.

[8]"Simply put, individual projects must compete for the scarce funds, and the headquarters' job is to pick the winners and the losers in this competition." Stein (1997), p111.

Proof The first order conditions for problem $(P.1)$ are:

$$\theta_i v'(y_i) = \lambda$$

where λ is the Lagrange multiplier associated with the budget constraint. Recalling that v' is homogenoeus of degree $-h$, we have $v'(y_i) = (1-h)y_i^{-h}$. Solving for y_i we obtain:

$$y_i = \frac{\lambda^{\frac{-1}{h}}}{(1-h)^{\frac{-1}{h}}} \theta_i^{\frac{1}{h}}.$$

Summing over all departments we obtain:

$$\sum_{k=1}^{N} y_k = \frac{\lambda^{\frac{-1}{h}}}{(1-h)^{\frac{-1}{h}}} \sum_{k=1}^{N} \theta_k^{\frac{1}{h}}$$

Given that $\sum_{k=1}^{N} y_k = B$, we have:

$$\frac{B}{\sum_{k=1}^{N} \theta_k^{\frac{1}{h}}} = \frac{\lambda^{\frac{-1}{h}}}{(1-h)^{\frac{-1}{h}}}$$

Replacing $\frac{\lambda^{\frac{-1}{h}}}{(1-h)^{\frac{-1}{h}}}$ by $\frac{y_i}{\theta_i^{\frac{1}{h}}}$, we have finally: $y_i = \frac{\theta_i^{\frac{1}{h}}}{\sum_{k=1}^{N} \theta_k^{\frac{1}{h}}} B.$ ∎

Our assumptions on $v(.)$ imply that the shares of the budget α are independent of the size of the budget. From proposition 3.1 the following comparative static results are immediate:

Corollary 3.1 *The optimal allocation of the budget satisfies the following:*

$$\frac{\partial \alpha_i}{\partial \theta_i} > 0 \qquad (3.1)$$

$$\frac{\partial \alpha_i}{\partial \theta_j} < 0 \qquad (3.2)$$

In other words, the resource allocation process is based on the *relative quality* of research projects, an effect which is typical in winner pricing contest.[9] This means that in a Multi-Department University (hereafter MDU), quality of project i alone cannot explain the budget allocated to professor i. In a MDU, budget allocation depends on the quality of all projects. As we will see, this allocation scheme, which is specific to a MDU, creates yardstick competition between academics and as such, is an important part of the incentive package.

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[9]"Specifically, the extent to which any given project gets funded in an internal capital market will depend not only on that project's own *absolute* merits, but also on its merits *relative* to other projects in the company's overall portfolio." Stein (1997), page 112.

University behaviour in the last stage is perfectly anticipated by academics. Using Proposition 3.1, we may now analyze the first stage of the game. Integrating the optimal budget allocation scheme, the professor i's utility function is:

$$U_i = w[\theta_i v(\alpha_i B)] - C(a)$$

Let us denote,

- $C_{a_{li}}$ is the partial derivative of $C(a_i)$ with respect to a_{li}, $l = 1, 2$,

- α_i' is the partial derivative of α_i with respect to θ_i.

where indices i referring to professor i. First order conditions read as follows:

$$C_{a_{1i}} = w[\theta_i v'(\alpha_i B)\alpha_i] \frac{\partial B}{\partial a_{1i}} \tag{3.3}$$

$$C_{a_{2i}} = w[v(\alpha_i B) + \theta_i v'(\alpha_i B)\alpha_i' B] \frac{\partial \theta_i}{\partial a_{2i}} \tag{3.4}$$

The right hand sides of (3.3) and (3.4) are respectively the marginal benefit of teaching effort and research effort. We assume that there exists a unique solution to this system.[10] We denote this solution a_{1i}^* and a_{2i}^*. Obviously, an increase in the marginal benefit of task l leads to an increase of a_{li}^*, $l = 1, 2$. We now state:

LEMMA 3.1 *Ignoring potential (positive or negative) synergies on cost, research and teaching are complementary activities:* $\frac{\partial a_{1i}}{\partial a_{2i}} > 0$

Proof: Take the derivative of the LHS of (3.3) with respect to a_{2i}. The sign of the expression is given by:

$$v'(\alpha_i B)\alpha_i + \theta_i v'(\alpha_i B)\alpha_i' + \theta_i v''(\alpha_i B)\alpha_i \alpha_i' B \tag{3.5}$$

Only the last term in this expression is negative (since $v(.)$ is concave and $\alpha_i' > 0$). A sufficient condition for a positive sign of (3.5) is: $v'(\alpha_i B) > -v''(\alpha_i B)\alpha_i B$, which is always true given the assumption on $v(.)$. Note that we could reach the same result by taking the derivative of (3.3) with respect to a_{1i}; the sign of this expression is obviously given by (3.5). ∎

The previous Lemma shows that the multi-unit structure of the university preserves the complementary nature of teaching and research efforts.[11]. Obviously, a necessary condition for

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[10]Since $v(.)$ is strictly concave and $C(.)$ is convex, a sufficient condition for the program to be globally concave is that efforts are weak substitutes.

[11]This result is established for the case where there are no synergies on cost. However, it is clear that the result is preserved even if there exist negative synergies between teaching and research activities

this result is that some link is preserved, through financing rules, between efforts in the first stage and the scope for rewards in the second stage. We are now in a position to address our key issue, i.e. assess the efficiency of a conglomerate form for the university. Since under our assumptions, there exists no synergies between departments outside those resulting from the budget sharing, any difference between a SDU (single department university) and a MDU (multi department university) is explained by the organizational form of the university.

3.2   SDU and MDU

In a single department university (SDU), there is no budget allocation scheme, since there is only one research project available: $y = B$. The professor's utility is then:

$$U = w[\theta v(B)] - C(a)$$

Take the first order condition to derive the optimal teaching and research levels:

$$C_{a_1} = w[\theta v'(B)]\frac{\partial B}{\partial a_1} \tag{3.6}$$

$$C_{a_2} = w[v(B)]\frac{\partial \theta}{\partial a_2} \tag{3.7}$$

Compare first the marginal benefit of teaching in a multi vs a single department university (equations (3.3) and (3.6)). Everything else being equal (the research budget B in a single department university equals $\alpha_i B$ in a multi department university; and the value of the research project), the marginal benefit of teaching in a multi department university is $\alpha_i < 1$ times the marginal benefit in a single department. Hence, a multi department university (MDU) provides the teachers with less incentives to do teaching.

The reason is that the budget is a pure private good in a SDU while it tends to be a common resource in a MDU. If in a SDU any additional resource created by attracting more students is invested in the department's research project, in a MDU, any additional resource goes to the common pool of resources from which department i gets only a fraction $\alpha_i < 1$. Hence an academic can only appropriate a fraction α_i of the additional budget.

An insurance effect is also at play in our framework. Indeed, the availability of funds within the university affects the marginal benefit of teaching through the term $v'(\alpha_i B)$. By concavity of $v(.)$, the larger the budget, the lower the marginal benefit of teaching. An academic would have lower incentives to teach if its university is endowed with a large budget B and conversely a low budget stimulates teaching effort. Lemma 3.2 will more clearly illustrate this effect.

Turning to the comparison of marginal benefits of research in a MDU and a SDU (equations (3.4) and (3.7)), we note the additional term $\theta_i v'(\alpha_i B)\alpha_i \prime B \frac{\partial \theta_i}{\partial a_{2i}}$ in (3.4). This term is positive.

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Hence, everything else being equal (the research budget B in a single department university equals $\alpha_i B$ in a multi department university), the marginal benefit of a research effort is larger in a MDU than in a SDU.

The additional term in (3.4) measures the competitive effect of having a MDU. In a MDU, professors compete for the research budget. Given that the budget allocation scheme is based on the relative quality of projects, the professors have to produce higher quality research to grab the resources from the university. The competition for research funding, as induced by the MDU structure, leads to an increase of research quality.

Summing up, integrating departments in a MDU creates free riding on teaching and yardstick competition on research. We have shown that MDU provides more incentives to do research and less incentives for teaching than SDU. Does it mean that MDU have a better research and a lower quality teaching than SDU. The answer is no. There are two reasons for the negative answer: first, lower quality teaching reduces the university's budget and hence, it reduces the research quality. It also reduces the incentives to do research (lemma 3.1). Second, again following lemma 3.1, the increase in incentives to do research increases the incentives to do teaching.

Hence, is it possible that a MDU offers a better quality teaching and performs better research than a SDU. The reverse could also be true. Last, it is also possible that MDU performs better on one dimension only. However, the next proposition shows that a MDU cannot perform better than a SDU on the teaching side only. To establish this result, we compare a MDU with a collection of SDU replicating the MDU's departments.

PROPOSITION 3.2 *If for all $i = 1, ..., N$, $a_{1i}^{MDU} > a_1^{SDU}$ then $a_{2i}^{MDU} > a_2^{SDU}$*

PROOF: Each academic i performs more teaching effort in a MDU than in a SDU if:

$$\theta_i^{MDU} v'(\alpha_i B^{MDU})\alpha_i > \theta_i^{SDU} v'(B^{SDU}) \tag{3.8}$$

Replacing $v'(y)$ by $(1-h)y^{-h}$, we have:

$$\theta_i^{MDU} \alpha_i^{1-h}(B^{MDU})^{-h} > \theta_i^{SDU}(B^{SDU})^{-h} \tag{3.9}$$

Since $\alpha_i < 1$ and $B^{MDU} > B^{SDU}$ as we posit that all academic are doing more teaching effort, a necessary condition for (3.9) to hold is that $\theta_i^{MDU} > \theta_i^{SDU}$. But this precisely means that for all i, $a_{2i}^{MDU} > a_2^{SDU}$. And it completes the proof. ∎

Proposition 3.2 shows that a necessary condition for having more teaching effort in all departments is to have more research in all departments. However, depending on the distribution of talents, it is possible that in some department, a MDU has a better performance while in

other department a MDU does worse. Unfortunately, we have not been able to solve the model for a general characterization of the occurence of the various cases. We propose hereafter an example which allows to discuss the basic intuitions underlying the general trade-off between teaching and research incentives.

3.3   An Example

Let us assume that each academic is characterized by a vector $\eta_i = (\eta_{1i}, \eta_{2i})$ of talent. The research output in department i is $r_i(\theta_i, y_i) = \theta_i y_i^{1-h}$, with $h < 1$ where $\theta_i = a_{2i}\eta_{2i}$. We assume that all the academics are identical with respect to talent. This means that $\eta_i = (\eta_1, \eta_2)$, $\forall i$.

The university is financed exclusively by a per-student fee s. The number of students in department i is $n_i(a_{1i}, \eta_{1i}) = a_{1i}\eta_1$. Hence the total budget is $B = s\eta_1 \sum_{k=1}^{N} a_{1k}$.

We assume that the costs of teaching and research efforts are separable. Specifically, we assume $C(a_i) = \frac{a_{1i}^2}{2} + \frac{a_{2i}2}{2}$.

In a Single Division University, the optimal behaviour of the academic is obtained by solving the following program:

$$\max_{a_1, a_2} w[\theta B^{1-h}] - C(a)$$

The first order conditions are:

$$w[\theta(1-h)(s\eta_1 a_1)^{-h}](s\eta_1) = a_1 \tag{3.10}$$

$$w[(s\eta_1 a_1)^{1-h}]\eta_2 = a_2. \tag{3.11}$$

Solving them for the efforts we obtain:

$$a_1^{SDU} = (1-h)^{\frac{1}{2h}} w^{\frac{1}{h}} \eta_2^{\frac{1}{h}} (s\eta_1)^{\frac{1-h}{h}} \tag{3.12}$$

$$a_2^{SDU} = (1-h)^{\frac{1-h}{2h}} w^{\frac{1}{h}} \eta_2^{\frac{1}{h}} (s\eta_1)^{\frac{1-h}{h}} \tag{3.13}$$

In a Multi department university each academic i solves:

$$\max_{a_{1i}, a_{2i}} w[\theta_i(\alpha_i B)^{1-h}] - C(a_i)$$

where $B = s\eta_1 \sum_{k=1}^{N} a_{1k}$, α_i is given in proposition 3.1: $\alpha_i = \dfrac{\theta_i^{\frac{1}{h}}}{\sum_{k=1}^{N} \theta_k^{\frac{1}{h}}}$ and α_i' is given by:

$$\alpha_i' = \frac{\partial \alpha_i}{\partial \theta_i} = \frac{1}{h}\theta_i^{\frac{1}{h}}\theta_i^{-1}\frac{\sum_{k=1,k\neq i}^{N} \theta_k^{\frac{1}{h}}}{(\sum_{k=1}^{N} \theta_k^{\frac{1}{h}})^2} = \frac{1}{h\theta_i}\alpha_i \sum_{k=1,k\neq i}^{N} \alpha_k > 0 \tag{3.14}$$

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The first order conditions are:

$$w[\theta_i(1-h)(s\eta_1\alpha_i\sum_{k=1}^{N}a_{1k})^{-h}](s\eta_1\alpha_i) = a_{1i} \qquad (3.15)$$

$$w[(s\eta_1\alpha_i\sum_{k=1}^{N}a_{1k})^{1-h} + \theta_i(1-h)(s\eta_1\alpha_i\sum_{k=1}^{N}a_{1k})^{-h}(s\eta_1\sum_{k=1}^{N}a_{1k})\alpha_i']\eta_2 = a_{2i} \qquad (3.16)$$

Using the facts that all academics are identical, we can replace $\sum_{k=1}^{N}a_{1k}$ by Na_{1i}, α_i by $\frac{1}{N}$ and α_i' by $\frac{1}{h\theta_i}\alpha_i(1-\alpha_i) = \frac{1}{h\theta_i}\frac{N-1}{N^2}$.

Simplifying and solving the system for the efforts a_{1i} and a_{2i} we obtain an explicit relation between optimal values in the single division university and a multi division one:

$$a_{1i}^{MDU} = (1-h)^{\frac{1}{2h}}w^{\frac{1}{h}}\eta_2^{\frac{1}{h}}(s\eta_1)^{\frac{1-h}{h}}\frac{1}{N^{\frac{1}{h}}}(\frac{N+h-1}{h})^{\frac{1}{2h}} = a_1^{SDU}g_1(N) \qquad (3.17)$$

$$a_2^{MDU} = (1-h)^{\frac{1-h}{2h}}w^{\frac{1}{h}}\eta_2^{\frac{1}{h}}(s\eta_1)^{\frac{1-h}{h}}\frac{1}{N^{\frac{1}{h}}}(\frac{N+h-1}{h})^{\frac{1+h}{2h}} = a_2^{SDU}g_2(N) \qquad (3.18)$$

It follows that

$$a_{1i}^{MDU} > a_1^{SDU} \Leftrightarrow g_1(N) > 1$$

$$a_{2i}^{MDU} > a_2^{SDU} \Leftrightarrow g_2(N) > 1.$$

with $g_1(N) = \frac{1}{N^{\frac{1}{h}}}(\frac{N+h-1}{h})^{\frac{1}{2h}}$ and $g_2(N) = \frac{1}{N^{\frac{1}{h}}}(\frac{N+h-1}{h})^{\frac{1+h}{2h}}$. Moreover, direct computations yield

$$r_i^{MDU} > r^{SDU} \Leftrightarrow g_2(N)g_1(N)^{1-h} > 1$$

Proposition 3.3 summarizes our findings:

PROPOSITION 3.3 *For all $h < 1$, it exists $N^1 < N^r < N^2$, with $N^r > 1$ and $N^1 \geq 1$ if $h \leq \frac{1}{2}$ such that*

- $\forall N \in [1, Max[1, N^1]]$, we have $a_{1i}^{MDU} \geq a_1^{SDU}$, $a_{2i}^{MDU} \geq a_2^{SDU}$, and $r_i^{MDU} \geq r^{SDU}$

- $\forall N \in [Max[1, N^1], N^r]$, we have $a_{1i}^{MDU} \leq a_1^{SDU}$, $a_{2i}^{MDU} \geq a_2^{SDU}$, and $r_i^{MDU} \geq r^{SDU}$

- $\forall N \in [N^r, N^2]$, we have $a_{1i}^{MDU} \leq a_1^{SDU}$, $a_{2i}^{MDU} \geq a_2^{SDU}$, and $r_i^{MDU} \leq r^{SDU}$

- $\forall N \geq N^2$, we have $a_{1i}^{MDU} \leq a_1^{SDU}$, $a_{2i}^{MDU} \leq a_2^{SDU}$, and $r_i^{MDU} \leq r^{SDU}$

PROOF: The equation $g_1(N) = 1$ is equivalent to $-hN^2 + N + h - 1 = 0$. The roots of this second degree equations are: $N = 1$ and $N = N^1 = \frac{1-h}{h}$. $g_1(N)$ is larger than one when

N lies in between the two roots. When $h > \frac{1}{2}$, $N^1 < 1$, hence for all $N > 1$, $g_1(N) < 1$ and $a_{1i}^{MDU} < a_1^{SDU}$. When $h \leq \frac{1}{2}$, $N^1 \geq 1$, hence for all $N \in [1, N^1]$, $g_1(N) \geq 1$ and $a_{1i}^{MDU} \geq a_1^{SDU}$, and for all $N > N^1$, $g_1(N) < 1$ and $a_{1i}^{MDU} < a_1^{SDU}$.

The equation $g_2(N) = 1$ is equivalent to $-hN^{\frac{2}{1+h}} + N + h - 1 = 0$. We first show that if $h < 1$, this equation has two roots in the interval $[1, +\infty[$: $N = 1$ and $N^2 > 1$. First, $N = 1$ is clearly a root of this equation. Second, $-hN^{\frac{2}{1+h}} + N + h$ is increasing up to $N = (\frac{1+h}{2h})^{\frac{1-h}{1+h}} > 1$ and decreasing after. Hence, there is another root $N^2 > 1$. $g_2(N)$ is larger than one when N lies in between the two roots 1 and N^2. Hence for all $N \in [1, N^2]$, $g_2(N) \geq 1$ and $a_{2i}^{MDU} \geq a_2^{SDU}$, and for all $N > N^2$, $g_2(N) < 1$ and $a_{2i}^{MDU} < a_2^{SDU}$.

The root N^2 is larger than N^1. Indeed, $g_2(N) = g_1(N)(\frac{N+h-1}{h})^{\frac{1}{2}}$. Clearly $g_2(N) > g_1(N)$ for all $N > 1$. Hence, when $g_1(N^1) = 1$, $g_2(N^1)$ is still larger than one.

The equation $g_1(N)^{1-h}g_2(N) = 1$ is equivalent to $-hN^{2-h} + N + h - 1 = 0$. For $h < 1$, this equation has two roots in the interval $[1, +\infty[$: $N = 1$ and $N^r > 1$. The analysis is similar to the equation $g_2(N) = 1$: $-hN^{2-h} + N + h - 1 = 0$ increases up to $N = (\frac{1}{h(2-h)})^{\frac{1}{1-h}} > 1$ and decreases after. Hence for all $N \in [1, N^r]$, $r^{MDU} \geq r^{SDU}$, and for all $N > N^r$ $r^{MDU} < r^{SDU}$

Last, the root N^r lies in between N^1 and N^2. Indeed, when $N = N^1$, $g_1(N^1)^{1-h}g_2(N^1)$ is larger than one since the first term equals one and the second is > 1. When $N = N^2$, $g_1(N^2)^{1-h}g_2(N^2)$ is lower than one since the second term equals one and the first is < 1. ∎

Figures 1 and 2 illustrates proposition 3.3 for $h = \frac{1}{4}$ and $h = \frac{1}{2}$.
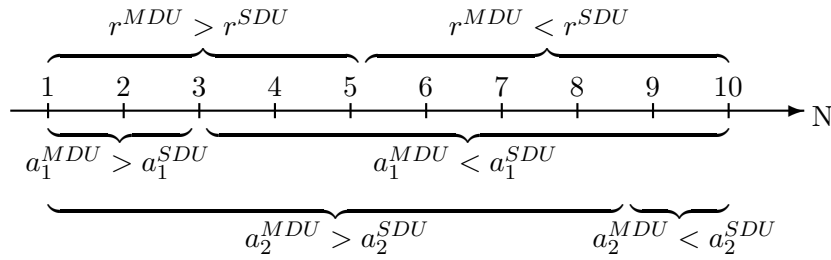

Figure 1: $h = \frac{1}{4}$

N^1 then defines the critical "size" beyond which the free-riding effect of the MDU more then compensates the initial positive effect. N^2 defines the critical level beyond which the MDU structure leads to less research efforts. Too many departments makes competition too fierce, which induces less effort. As predicted by proposition 3.2, we observe that $N^1 < N^2$. This ordering is explained by the following argument: when the number of department increases,
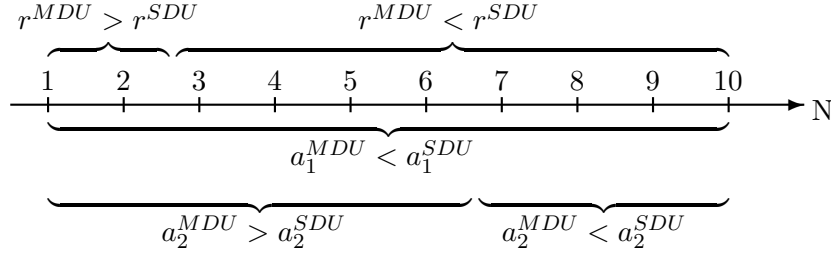
Figure 2: $h = \frac{1}{2}$

the free-riding effects comes into play as well as the competition effect. However, when the number is small enough the introduction of competition for research funding increases efforts in the research dimension which increases effort in the teaching dimension as well, because of the complementarity effect. In other words, because teaching and research activities remain complements, a bit of competition for research fundings overcomes the free-riding effect.

The numerical approximations for our critical values for N are $\{N^1, N^r, N^2\} = \{3, 5.15, 8.66\}$ for $h = \frac{1}{4}$ and $\{N^1, N^r, N^2\} = \{1, 2.62, 6.22\}$ for $h = \frac{1}{2}$. Although we have not been able to prove it formally, our computations indicate that all of these treshold values are decreasing in h. Recall that in this example a larger h means that $v(.)$ is more concave, so that the marginal contribution of funds to research output is decreasing quickly. By contrast, when $h = 1$ the marginal contribution is constant. In other words, a lower h means that redistribution possibilities become more valuable. It is therefore not surprising that a MDU remains more efficient than a collection of SDU for a larger number of divisions.

The following corrolary is an immediate consequence of Proposition 3.3.

Corollary 3.2 *There exists a unique $N^* > 1$ such that the aggregate research output is maximal.*

PROOF: N^* is the maximum of $g_1(N)^{1-h}g_2(N)$. The derivative of $g_1(N)^{1-h}g_2(N)$ with respect to N is (after simplification):

$$(\frac{N+h-1}{h})^{\frac{1}{h}}\frac{1}{hN^{\frac{2}{h}}}(\frac{1}{N(N+h-1)} - (2-h))$$

N^* is the solution of $\frac{1}{N(N+h-1)} = (2-h)$. Solving the equation, if $h < 1$, there is a unique positive root $N^* = \frac{1}{2}[(1-h) + \sqrt{(1-h)^2 + \frac{4}{2-h}}]$. Clearly $N^* > 1$. ∎

This last result shows that in order to maximize the research output, the university should have some level of diversification. Notice however that N^* is actually less than 2, whatever the

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value of h. Accordingly, when we take the restriction $N \geq 2$ into account, the desirability of a multiunit university cannot be evaluated through N^*. The relevant comparison is between N^r and 2. Condition $N^r > 2$ is not satisfied for all $h \in [0, 1]$ but our numerical computations show that N^r is decreasing in h. Actually, unless h is large, there always exists a feasible MDU structure which exhibits a better research output than the corresponding collection of single-unit divisions. The fact that h cannot be too large is intuitive: Suppose h is arbitrarily close to 1, then research output (almost) does not depend on research funding. In this case, very few is to be gained through redistribution opportunities while free-riding already undermines teaching efforts. On the other hand, is h is smaller, the competition for funds is fiercer. Accordingly, its positive effects overcome the negative free-riding effect for a larger number of divisions.

3.4 Comparative static analysis

In the previous subsection, we have compared the performance of a MDU university to that of a collection of single departments. We have shown that MDU induced a trade-off between teaching and research efforts reflecting the "free-riding" vs "yardstick competition" sides of the MDU coin. In this section, we establish some comparative statics results regarding the departments' interdependence that results from the MDU structure. More precisely, what are the consequences on a_{1i}^* and a_{2i}^* of a change in teaching ability (η_{1j}) and research ability (η_{2j}) of professor $j \neq i$. Recall that we have assumed that ability and effort are not perfect substitute in order to have: $\frac{\partial a_{1j}}{\partial \eta_{1j}} > 0$ and $\frac{\partial a_{2j}}{\partial \eta_{2j}} > 0$. Then the analysis of the conditions (3.3) and (3.4) leads to the following Lemma.

LEMMA 3.2 *In a MDU, teaching efforts a_{1i} and a_{1j} are <u>strategic substitutes</u>:*

$$\frac{\partial a_{1i}^*}{\partial a_{1j}} < 0$$

PROOF: An increase in η_{1j} leads to an increase in a_{1j} and then, the university budget B increases. To measure the effect of a change in η_{1j} on a_{1i}, take the derivative of (3.3) with respect to a_{1j}:

$$w[\theta_i v''(\alpha_i B)\alpha_i^2]\frac{\partial B}{\partial a_{1i}}\frac{\partial B}{\partial a_{1j}} < 0 \tag{3.19}$$

Clearly, the marginal benefit of teaching for professor i decreases when the budget increases i.e. when a_{1j} increases. ∎

Thus teaching efforts are unambiguously strategic substitutes. This illustrates the insurance effect we mentioned. When there are more resources, the individual incentives to create additional resources diminishes.

On the other hand, the nature of the strategic interaction in research efforts is less clear-cut. We would like research efforts to be strategic complements. Indeed, in this case, more effort in one department induces more efforts at the other departments. The benefits of yardstick competition are clearly dependent on this virtuous circle. Unfortunately, in our MDU, research efforts a_{2i} and a_{2j} could be either strategic substitutes ($\frac{\partial a_{2i}^*}{\partial a_{2j}} < 0$) or complements ($\frac{\partial a_{2i}^*}{\partial a_{2j}} > 0$).

An increase in η_{2j} leads to an increase in a_{2j} and then an increase in θ_j. To measure the effect of a change in η_{2j} on a_{2i}, take the derivative of (3.4) with respect to a_{2j}:

$$w[v''(\alpha_i B)\frac{\partial a_i}{\partial \theta_j}B + \theta_i v'(\alpha_i B)\alpha_i' B\frac{\partial \alpha_i}{\partial \theta_j}B + \theta_i v(\alpha_i B)\frac{\partial \alpha_i'}{\partial \theta_j}B]\frac{\partial \theta_i}{\partial a_{2i}}\frac{\partial \theta_2}{\partial a_{2j}} \qquad (3.20)$$

In order to simplify (3.20) we use the following: $\alpha_i = \frac{\theta_i^{\frac{1}{h}}}{\sum_{k=1}^{N}\theta_k^{\frac{1}{h}}}$ and $\alpha_i' = \frac{1}{h\theta_i}\alpha_i\sum_{k=1,k\neq i}^{N}\alpha_k$.

Using the fact that $\sum_{k=1}^{N}\alpha_k = 1$, we have

$$\alpha_i' = \frac{1}{h\theta_i}\alpha_i(1 - \alpha_i) > 0 \qquad (3.21)$$

$$\frac{\partial \alpha_i}{\partial \theta_j} = -\frac{1}{h\theta_i}\alpha_i^2 < 0 \qquad (3.22)$$

$$\frac{\partial \alpha_i'}{\partial \theta_j} = \frac{1}{h\theta_i}\frac{\partial \alpha_i}{\partial \theta_j}(1 - 2\alpha_i) \qquad (3.23)$$

Using (3.21), (3.22) and (3.23), we can rewrite (3.20) as follows:

$$w\frac{\partial \alpha_i}{\partial \theta_j}B[v''(\alpha_i B) + v'(\alpha_i B)\frac{1}{h}\alpha_i(1 - \alpha_i)B + v(\alpha_i B)\frac{1}{h}(1 - 2\alpha_i)]\frac{\partial \theta_i}{\partial a_{2i}}\frac{\partial \theta_2}{\partial a_{2j}} \qquad (3.24)$$

The sign of (3.20) will be given by the sign of

$$v''(\alpha_i B) + v'(\alpha_i B)\frac{1}{h}\alpha_i(1 - \alpha_i)B + v(\alpha_i B)\frac{1}{h}(1 - 2\alpha_i) \qquad (3.25)$$

(3.20) is positive (resp. negative) if (3.25) is negative (resp. positive).

After simplification (3.25) is:

$$-h^2(1 - h) + (\alpha_i B)^2[(2 - h) - (3 + h)\alpha_i] \qquad (3.26)$$

The sign of this condition is hard to establish since it depends on all the parameters of the model. If we return to our previous example with symmetric departments, a sufficient condition for strategic complementarity is:

$$(2 - h) - (3 + h)\alpha_i \leq 0 \Rightarrow N \leq \frac{3 + h}{2 - h}$$

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This condition holds for example for $h = \frac{1}{2}$ and $N = 2$. If $N \geq \frac{3+h}{2-h}$, effort are strategic substitutes only if the budget is low enough. For low values of B, (3.25) will be negative even if $N \geq \frac{3+h}{2-h}$. It illustrates the fact that competition for resources is more severe when resources are relatively scarce. When the university is endowed with more resources, research efforts are strategic substitute i.e. the intensity of competition for resources diminishes.

Thus, we have strict complementarity if budget is low and the α_k not too dissimilar, i.e. an increase in η_{2j} leads to an increase in the research quality of *all* departments. This conclusion is no longer valid when large asymmetries prevail.

4 Comments

4.1 Robustness

Section 3 has been devoted to disentangling the nature of the trade-offs between teaching and research efforts in a multi-department university. Roughly speaking, our analysis suggests that a mutli-department university may actually acheive some form of redistribution in research funding among departments without sacrifying efficiency, i.e. while inducing more efforts on teaching quality and research quality. This is especially true in cases where research output are heavily dependent on funding levels (h small). Since our model is quite specific, we now question its robustness to alternative assumptions?

• *Mixed Research Funding* So far, we assumed that the allocation of the university's budget to research projects is a pure winner-picking contest, meaning that the financing of an academic's research only depends on the relative quality of its project. This allocation rule has a positive effect on research incentives due to yardstick competition but a negative effect on teaching incentives due to free riding. In fact, the absence of reward for the teaching effort is the most important problem associated with winner-picking, especially when the number of academics is large.

Obviously, the university could alleviate this problem by departing from winner picking and allocate its resources not only according to the relative quality of research projects but also depending on the teaching's quality, measured for example by the number of students in field i.[12] In fact, in most of the universities, the number of students per departments matters for deciding on research resources allocation.

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[12]Notice that such a rule is likely to conflict with equity considerations since it introduces a bias in favour of those disciplines which are lucky enough to attract large cohorts of students simply because of labour market conditions.

Limiting the scope of winner picking and integrating the number of students, together with the project's relative quality, as a determinant of the budget sharing rule would have positive effect on teaching incentives since it would make the teaching effort more approriable by the academic. However, this kind of sharing rule proves hard to use because the university should ex-ante, that is before the academics choose the efforts, design (and commit to) a sophisticated sharing rule. As discussed in section 2, full commitment would imply that the university decides ex-ante the way the budget will be allocated given all possible realizations of n_k and θ_k, $k = 1, ..., N$. Commitment to such a rule would require that both the number of students and the projects' quality are observable and verifiable.[13] If it does not seem to be a problem for the student's numbers, it is much more demanding in term of information for the projects' quality.[14] Outsiders, such as a court would need a lot of information and a great expertise to verify the qualities of the research projects.

With non verifiable θ, the university can implement ex-ante two kind of sharing rules: winner picking and rules based on the number of students only. Despite the non verifiability of θ, winner picking is implementable since it corresponds to the optimal allocation of resources ex-post (once they are created). Hence, to implement winner picking, the university simply decide to postpone the definition of the sharing rule till the budget is known. Rules based on the number of students create strong incentive for teaching because it reduces the free-riding, but it also reduces the yardstick competition effect. It is particularly clear in the rule that replicates the stand alone university: $y_i = sn_i$, where there is no free riding but no yardstick competition. Rules based on the number of students (or more generally on teaching quality) reduces the benefits of the conglomerate structure of a university.

To see how things are going with full commitment and a verifiable project's quality, let us consider a particular rule that mixes winner picking with an allocation based on the number of students. Suppose that the university commits to split the budget B in two parts. a fraction γ of the budget B will be allocated according to the relative quality of the projects (winner picking); a fraction $1 - \gamma$ will be allocated according to the relative number of students enrolled in the departments. With this rule, the research budget of academic i is:

$$y_i = \gamma \alpha_i B + (1 - \gamma) \beta_i B \qquad (4.27)$$

where α_i is given by proposition 3.1 and $\beta_i = \frac{n_i}{\sum_{k=1}^{N} n_k}$.

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[13]Recall indeed that only observable and verifiable informations could be included in a contract.

[14]Remember that the project quality θ_i is only a part of the final research record r_i, the latest being perfectly observable and verifiable.

If the university applies the sharing rule given by (4.27), funds could be inefficiently allocated ex-post: once the n_k and θ_k are realized, there is room for a redistribution that increases the aggregate research output. Strong commitment by the university is then necessary to apply this rule.

When the research budget is given by (4.27), the first order conditions read as follows:

$$C_{a_{1i}} = w[\theta_i v'(y_i)][(\gamma \alpha_i + (1 - \gamma)\beta_i)\frac{\partial B}{\partial a_{1i}} + (1 - \gamma)B\frac{\partial \beta_i}{\partial a_{1i}}] \tag{4.28}$$

$$C_{a_{2i}} = w[v(y_i) + \theta_i v'(y_i)\gamma \alpha'_i B]\frac{\partial \theta_i}{\partial a_{2i}} \tag{4.29}$$

Consider first the incentive to perform research effort. Yardstick competition is still present but its incentive effect is reduced because the academics compete only for a fraction γ of the budget. Moreover, for those academics who benefit from a large research financing because a lot of students attend their field, the benefit of competing for the university budget is lower. Consider next the teaching effort. There is still free riding because only a fraction of the incremental budget created by academic i will be invested in his research project, but there is more incentives to teach because the share of the budget for project i is increasing with teaching effort by academic i. Hence, the new sharing rule increases the incentives for teaching and decreases the incentives for research. But because of the complementarity between the two tasks, the global effect is ambiguous.

To have a more clear picture of the changes induced by $\gamma < 1$, let us return to the example of section 3.3. Given that all academics are identical, in the first order conditions (4.28) and (4.29), α_i and β_i can both be replaced by $\frac{1}{N}$. Solving for the effort levels we have:

$$a_{1i}^{MDU} = a_1^{SDU}g_3(N,\gamma) \tag{4.30}$$

$$a_{2i}^{MDU} = a_2^{SDU}g_4(N,\gamma) \tag{4.31}$$

with $g_4(N,\gamma) = \frac{1}{N^{\frac{1}{h}}}\left(\frac{hN+(1-h)(n-1)\gamma}{h}\right)^{\frac{1+h}{2h}}(1 + (1-\gamma)(N-1))^{\frac{1-h}{2h}}$ and $g_3(N,\gamma) = [g_4(N,\gamma)(1 + (1-\gamma)(N-1))]^{\frac{1}{1+h}}$. Notice that $g_3(N,1) = g_1(N)$ and $g_4(N,1) = g_2(N)$.

The aggregate research output is:

$$\sum_{k=1}^{N} r_k = Na_{2i}\eta_2(sa_{1i}\eta_1)^{1-h} = N\eta_2 a_2^{SDU}(s\eta_1)^{1-h}(a_1^{SDU})^{1-h}g_3(N,\gamma)^{1-h}g_4(N,\gamma)$$

If the university sets γ in order to maximize the aggregate research output, we have:

PROPOSITION 4.4 *If $N \geq 2$, it is efficient to set $\gamma = \gamma^* = \frac{(1-h)N}{(N-1)(2-h)}$ with $0 < \gamma^* < 1$.*

PROOF: $\gamma^* \equiv \max_\gamma g_3(N, \gamma)^{1-h} g_4(N, \gamma)$. ∎

With identical academics, there is no distortion in the allocation of resources to the academics since $\alpha_i = \beta_i$. Hence, there is no loss due to a misallocation of resources ex-post. This would no longer be true if the academics were different. Selecting γ^* just reflects the balance between incentives to teach and to do research. The fact that the university optimally sets $\gamma^* < 1$ means that it achieves a larger aggregate research output with the redistribution rule (4.27). Hence, the benefits of conglomerate organization for a university increases when it can makes research budgets contingent on both the research's and the teaching's quality.

- *Pay related to performance scheme*

In the analysis, we assumed that there is no direct pay related to performance for the academics and, in particular, there is no reward to teaching. Indeed, for academics, a higher teaching quality is valuable only because it increases the total research budget. This assumption can be justified by the fact that teaching quality, unlike research quality, is difficult to assess. Moreover, if research quality is comparable across academics in the same field, measures of teaching quality are often institution specific, hence less comparable. Suppose however that the university designs a measure of teaching quality. Typically this measure could result from students' assessments. Denote by q_i a proxy for the teaching quality by professor i. The relevant measure of teaching quality should be correlated with teaching effort and teaching talent: $q_i(a_{1i}, \eta_{1i})$ with $\frac{\partial q_{i1}}{\partial a_{i1}} > 0$ and $\frac{\partial q_{i1}}{\partial \eta_{i1}} > 0$.

With observable teaching quality q_i, the market value of a professor i is now: $w[r_i + \delta q_i]$ where δ is the weight given to teaching. Integrating this pay structure in the academic's utility function, the first order condition of the optimization problem are:

$$C_{a_{1i}} = w[\theta_i v'(\alpha_i B)\alpha_i]\frac{\partial B}{\partial a_{1i}} + w\delta\frac{\partial q_i}{\partial a_{1i}} \tag{4.32}$$

and (3.4) which remains unchanged.

Obviously, the effect on optimal teaching effort is posisitve. Moreover, the more precise is the quality measure, the more positive the impact. Given lemma 3.1 we may also conclude that research efforts will increase as well.

- *Alternative Financing Sources*

A key feature of our model is that departments rely exclusively on the University central budget to finance research. Obviously, realife departments have also access to alternative source of funding. We will not address here the possible funding related to consultancy, and more generally private funding related to appplied research. Obviously, the incentives to rely on such sources are larger when basic research efforts are less appropriable. Aside from consultancy, academics

may also rely on institutional funds to finance their basic research. Obviously, academics also face yardstick competition for these funds. Moreover, their competitors are in general working in a similar discipline, so that competition is likely to be tougher. It therefore seems reasonable to consider that research effort is likely to be larger there. However, the key implication of these fundings is that they are unrelated to the result of teaching efforts. Accordingly, wider access to these funds breaks the complementarity between teaching and research efforts. Notice here that the argument equally applies to SDUs and MDUs. However, because the MDU already faces a free-riding problem in the teaching activitiy, the negative impact of these external fundings on teaching efforts could actually be stronger.

4.2 Related Literature

- *Career Concerns*

There is no direct incentive or performance related to pay in our model. Rather, the model relies on implicit incentives where current actions influence future opportunities. To put it differently, implicit incentives mean that the university cannot control the per-unit reward w.

This incentive structure shares features of the career concern model of Holmström (1982) and Dewatripont *et. al.* (1999a, 1999b). We should stress however that our framework differs from career concern ones in a fundamental respect. In the career concern model, the agent's pay reflects the market's expectation of the unknown agent's talent given the observables. Applied to our framework, this means that wr_i is the expected wage of a professor on the academic market, given observables i.e. given research records. In this case, the market would pay the academics according to their perceived talent. Clearly, the market will use the research output to assess the talent of an academic, and the academic's pay will increase with the research output. But market assessment of research talent would also take into account the amount available for research of professor i. And to make correct assessment about the research budget y_i, the market needs to infer the value of the total budget B and the redistribution rules that apply within the university. If it is common knowledge that the university allocates its budget in order to maximize the aggregate research output (an assumption we make in this model) the market expectation of the budget y_i will depend on the expectation of the unknown talents η_{2j} of all professors $j = 1, ..., N$ within the same university since redistribution will be based not only on the project's quality θ_i but also on the qualities of the other projects θ_j as all professors compete for the same budget. Moreover, the market should evaluate the total budget size, which depends on the expectation of the unknown talents $\eta_{1j}, \ j = 1, ..., N$. Hence, the market value of professor i depends on his observed research output but also on the observed

research output of his colleague in other fields. This would make the model extremely difficult to solve. We then take another route and assume the benefit of research is simply proportional to the professor's research output. Accordingly, our model cannot be viewed as a career concern model. The market values identically a research record resulting from a high effort and/or high talent than a research record resulting from a poor talent but a large research budget. Hence, we consider a myopic market, where the professor's value depends only on his observable: his research achievement r_i.

- *Conglomerates*

Our analysis can be viewed as an application of the more general literature on conglomerate, for profit, firms. The literature on conglomerate firms identifies two important features of conglomerates which are also present in the framework we adopt for universities.

(1) Brusco and Panuzzi (2002) and Gautier and Heider (2002) show that the fund raising activities at the division level are strategic substitutes when the conglomerate's headquarter redistributes the resources ex-post across the divisions. Winner-picking (Stein, 1997) has innate agency costs: when the scare resources are redistributed, managerial incentives to create these resources are weaker. Redistribution (a) reduces the investment cash flow sensitivity and (b) insures the divisional manager against failure at the fund raising stage. Thereby, it reduces the managerial incentives to produce resource. The existence of these agency costs is independent of the fact that the conglomerate redistribute the resources efficiently or not (independent of the fact that the fraction α_i of fund allocated to division i is computed or not to maximize the total firm value) but it arises when the budget constraint applies at the conglomerate level. In this respect, our results adds two insights: first the value of the conglomerate depends on the number of divisions and second, the weak side of the conglomerate (lack of incentives for raising funds) can be overcome. In our model, the multi-dimensional structure may actually induce more efforts on the "raising funds" activity.

(2) Inderst and Laux (2002) show that developments of projects' quality are strategic complements activities accross divisions when the divisions are not too dissimilar ex-ante. In their model, the conglomerate resources are given but not the division's future value. Resources are allocated by the corporate headquarter to the most valuable projects. If the project's quality depends on a managerial effort, agglomerating divisions in a single firm increases the incentives to produce valuable projects. Integrating activities mean that the resource allocation process is based on the relative qualities of the projects. Competition for producing valuable projects increases the managerial incentives. Notice that the strategic complementarity property arises only if the conglomerate allocates the resources efficiently i.e. only if the conglomerate's head-

quarter picks the winners.

In the present paper, we combine the two dimensions in a mutlitasking framework. This is in contrast with the literature on conglomerate considers only single task problems: either the managers should receive incentive to produce resources, the future value of projects being given (1) or managers should receive incentives to develop projects, the resources being given (2). This paper considers a multi-task problem, where the resources and the projects' value are endogenous and result from the performance of a manager (professor). In this respect, the present paper suggests that the benefits of a conglomerate structure essentially depend on the number of divisions, i.e. the value of diversification has to be linked with the level (or degree) of diversification.

5   Final Remarks

Universities are most often organized as multi-unit departments headed by a single central authority. They also obtain a very significant share of their funds from enrollment fees and/or per students subsidies. Universities are asked to perform well in teaching and research activities, In this respect, a multi-unit organization allows for a redistribution of research funds which may be relatively independent of the origin of funds. However, it is often argued that such a redistribution weakens academics' incentives to perform well in their teaching duties. This is especially intuitive if the bulk of an academic pay and prestige depends on research records rather than teaching performance.

In this paper, we show that the multi-unit organization of universities is not incompatible with improved performance in both teaching and research. In other words, the lack of direct incentives towards teaching activities can be overcome. Central to this result is the organization of yardstick competition between departments which combined with the strategic complementarity between teaching and research may promote teaching and research efforts. However, the number of departments cannot be too large.

References

[1] Beath J., R. Owen, J. Payago-Theotoky, D. Ulph (2003), Optimal Incentives for IncomeGeneration in Universities: the rule of thumb for the Compton Tax, International Journal of Industrial Organization, 21, 1301-1322

[2] Brusco S. and Panunzi F. (2002), Reallocation of Corporate Resources and Managerial Incentives in Internal Capital Markets, forthcoming European Economic Review.

[3] Coupé T. (2001), Should University be Conglomerate, in "The Strategic Analysis of Universities: Microeconomics and Management Perspectives", Dewatripont, Thys-Clément and Wilkin eds, Université Libre de Bruxelles

[4] De Fraja G. and Iossa E. (2002), Competition among Universities and the Emergence of the Elite Institution, Bulletin of Economic Research, 54, 275-294

[5] Del Rey E. (2001), Teaching versus Research: a Model of State University Competition, Journal of Urban Economics, 49, 356-373

[6] Dewatripont M, Jewitt M. and J. Tirole (1999a), The Economics of Career Concerns, Part I: Comparing Information Structures, Review of Economic Studies, 66, 183-198

[7] Dewatripont M, Jewitt M. and J. Tirole (1999b), The Economics of Career Concerns, Part II: Applications to Missions and Accountability of Governmental Agencies, Review of Economic Studies, 66, 199-217

[8] Erhenberg R. (2003), Studying Ourselves: the Academic Labour Market, Journal of Labor Economics, 21-2, 267-287

[9] Gautier A. and F. Heider (2002), The Benefits and Costs of Winner-Picking: Redistribution vs Incentives, cahier du CEREC 2002/5

[10] Holmström, B. (1982), Managerial Incentive Problems: A dynamic Perspective, in essays in Economics and Management in Honor of Lars Wahlbeck, Helsinki: Swedish School of Economics.

[11] Inderst R. and Ch. Laux (2002), Incentives in Internal Capital Markets: Capital Constraints, Competition and Investment opportunities

[12] Jensen R. and M. Thursby (2001), Objectives, charecteirsitcs and Outcomes of University Licensing: a survey of Major US Universities, Journal of Technology Transfer, 26, 59-72

[13] Shattock M. (2002), Evaluating University Success, in "European University Change or Convergence" in Dewatripont, Thys-Clément and Wilkin eds, Université Libre de Bruxelles

[14] Siow A. (1995), The Organization of the Market for Professors, University of Toronto WP UT-ECIPA 95-01

[15] Stein, J. (1997). Internal Capital Markets and Competition for Corporate Resources, Journal of Finance, 52: 111-133.

[16] Tavernier M. and L. Wilkin (2001), le Budget Temps des Professeurs d'Université: une enquête exploratoire, in "The Strategic Analysis of Universities: Microeconomics and Management Perspectives", Dewatripont, Thys-Clément and Wilkin eds, Université Libre de Bruxelles.

[17] Tirole, J. (1999). Incomplete Contracts: Where do we stand? Econometrica, 67, 741-781.