Epictetusian Rationality and Evolutionary Stability*

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Abstract

An economic interpretation of Epictetus's precept of 'Taking away aversion from all things not in our power' consists of extending the domain of indifference beyond its boundaries under non-ethical preferences, so as to yield indifference between outcomes differing only on things outside one's control. This paper examines the evolutionary dynamics of a population composed of Nash agents and Epictetusian agents matched randomly and interacting in the prisoner's dilemma game. It is shown that, whether or not the types of players are common knowledge, neither the Nash nor the Epictetusian type is an evolutionary stable strategy under perfectly random matching. However, if the matching process exhibits a sufficiently high degree of assortativity, the Epictetusian type is an evolutionary stable strategy, and drives the Nash type to extinction.

Keywords: ethical preferences, evolutionary stability, cooperation, prisoner's dilemma, Epictetus.

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1 Introduction

In the last decades, economists have paid increasing attention to ethical preferences. Ethical preferences include the Impartial Observer's preferences on equiprobable lotteries defined on social positions (Smith 1759, Harsanyi 1955, 1982), Kantian preferences on generalizable acts (Kant 1785, Laffont 1975, Roemer, 2010, 2019), and, also, a convex combination of Kantian and self-oriented preferences known as the *homo moralis* (Alger and Weibull, 2013, 2016). Adopting ethical preferences was shown to have key implications for how agents interact in cooperation games. For instance, Kantian agents, by focussing only on the diagonal of the normal form of the game, choose to cooperate and avoid the prisoner's dilemma (Curry and Roemer 2012). Introducing 'ethical agents' affects also, through evolutionary dynamics, the design of optimal policies.[1]

In their attempts to 'translate' philosophical insights into the concept of ethical preferences, economists have so far focused on Enlightenment Philosophies, such as Smith or Kant. However, there is *a priori* no reason to limit the study of ethical preferences to these philosophies. Philosophers have, across centuries, proposed various precepts aimed to guiding persons in their everyday life, and the history of ethics spans several millennia (MacIntyre 1971). Studying the 'translation' of these earlier precepts into the language of economics would help economists to better understand what ethical preferences are.

Ancient philosophies have remained so far largely ignored by economists studying ethical preferences. A possible reason explaining this may lie in the fact that these Ancient systems of thought are far more distant from economic thought than 18th century philosophies.[2] It should be stressed, however, that in both cases, philosophers did not formulate their ethical precepts in terms of the concept of 'preferences', so that in any case a kind of 'translation' exercise has to be done to incorporate philosophical insights into ethical preferences.

This paper proposes to contribute to the study of ethical preferences and of their consequences, by focusing on a particular kind of Ancient ethical thought: Stoicism, and, more precisely, the ethical doctrine defended by the Stoic philosopher Epictetus.[3] In the *Manual*, Epictetus argued that, in order to reach mental freedom and happiness, individuals should distinguish between things that are under their control (their judgements, desires and acts) and things that are not under their control (their body, properties, social reputation, others's acts), and

[1] For instance, standard policy instruments can, by reducing the proportion of Kantian agents with respect to Nash agents in the long-run population, worsen the situation of the Tragedy of the Commons with respect to the laissez-faire (Bezin and Ponthiere 2019).

[2] A key reason explaining this distance between Ancient philosophies and economic thought lies in the fact that Ancient philosophies define what the 'good life' is in relationship with the position of the person in the Universe or Cosmos (MacIntyre 1971, chapter 8).

[3] Epictetus (55 AC - 135 AC) is a Greek philosopher born in Phrygia. He served as a slave for many years under the Roman Empire, and kept, during the remaining of his life, some bodily stigma of bad treatments imposed by his master. While having a miserous life, Epictetus developed some practical wisdom, and gave pieces of advice to a large number of citizens. Epictetus's precepts were transcribed by his follower Arrian. This transcription is at the origin of what is known as the *Manual* or the *Handbook* (Clark 1995).

wish for nothing that is not under one's control.[4] Epictetus's precept is best summarized in this quote from the *Manual* (I, II):

> Of things some are in our power, and others are not. In our power are opinion, movement towards a thing, desire, aversion, turning from a thing; and in a word, whatever are our acts. Not in our power are the body, property, reputation, offices (magisterial power), and in a word, whatever are not our own acts. [...] Take away then aversion from all things which are not in our power, and transfer it to the things contrary to nature which are in our power.

'Taking away aversion from all things not in our power' can be interpreted in various ways. A plausible economic interpretation consists of interpreting Epictetus's precept as a requirement of *extending the symmetric factor of the preference relation beyond its boundaries under non-ethical preferences*. This interpretation amounts to regard Epictetus's Stoicism as an ethical doctrine recommending the extension of the 'domain of indifference' beyond its boundaries under non-ethical preferences.[5]

While there exist several manners to extend the 'domain of indifference', one way of formalizing Epictetus's precept is to require that individuals should be indifferent between outcomes that differ only on circumstances (circumstances being defined as things not under one's control).[6] This interpretation of Epictetus's precept does justice to the idea according to which things outside control are 'neither good nor bad', and, thus cannot make a person either better off or worse off (*Manual*, XXXII):

> For if it is any of the things which are not in our power, it is absolutely necessary that it must be neither good nor bad.

Epictetusian rationality, that is, 'wishing for nothing that is outside control', looks like an intuitive rule of behavior. But what would be the implications of a fraction of the population adopting this rule of behavior in strategic environments? Would Epictetusian players benefit from some evolutionary advantage in interacting with standard Nash players? Or, alternatively, is it the case that Nash players would lead Epictetusian players to extinction?

This paper examines the evolutionary dynamics of a population composed of Nash agents and Epictetusian agents matched randomly and interacting in the prisoner's dilemma game. We first study the outcome of a two-player prisoner's dilemma game played by Epictetusian agents and by Nash agents drawn randomly. We then examine whether the Nash behavior or the Epictetusian behavior is an evolutionary stable strategy (ESS) in the sense of Maynard-Smith

[4] On Epictetus's ethical thought, see Clark (1995), Long (2002) and Graver (2009).

[5] This economic interpretation of Epictetus's precept is developed in Ponthiere (2022).

[6] This account is not the only possible interpretation of Epictetus's precept. As discussed in Ponthiere (2022), another possible formalization of Stoicism as an extension of the 'domain of indifference' consists of a requirement of indifference between outcomes that are the best under each circumstance. The reason why the present paper focuses on the former account is that it seems closer to the Stoic ideal of 'indifference to indifferent things' (Hadot 2001).

(1982, 1989). Finally, we relax the assumption of perfectly random matching, and examine the existence of an ESS in that more general framework.

Anticipating our results, it is shown that, whether or not the types of players are common knowledge, neither the Nash type nor the Epictetusian type is an ESS under perfectly random matching. No type can drive the other type to extinction. Using the discrete time replicator's dynamics (Cressman 2003), we show that the equilibrium partition of the population is locally stable, and that the long-run proportion of Epictetusian agents is decreasing in the gain of Nash player when they defect while the other player cooperates. However, when examining a more general setting with imperfectly random matching, we show that, if the matching process exhibits a sufficiently high degree of assortativity, the Epictetusian type is an ESS and drives the Nash type to extinction.

The present paper is related to several branches of the literature. First, it is related to the literature on ethical preferences (Harsanyi, 1955, 1982, Laffont 1975, Binmore 2005, Roemer 2010, Roemer 2019, Alger and Weibull 2013, 2016).[7] While ethical preferences are (usually) defined either as preferences on equiprobable lotteries on all social positions (Harsanyi) or as preferences on generalizable acts (Laffont, Roemer), our contribution is to study another form of ethical preferences, inspired by the Stoic philosopher Epictetus, and which relies on the idea of extending the domain of indifference beyond its boundaries under non-ethical preferences. Second, this paper is also related to the previous study by Ponthiere (2022) on the formalization of Epictetusian rationality. That study examined, within a static framework, several plausible formalizations of Epictetus's precept of 'wishing for nothing that is not under one's control', and analyzed the conditions under which a preference relation on outcomes can satisfy Epictetus's precept while being complete, reflexive and transitive. Our main contributions with respect to Ponthiere (2022) are to consider Epictetusian rationality in a dynamic setting (instead of a static setting), and, also, to study interactions between Epictetusian agents with other agents (instead of focusing only on Epictetusian agents). Third, this paper is also linked to the literature on evolutionary stability under diverse kinds of preferences (Bergstrom and Stark 1993, Curry and Roemer 2012, Alger and Weibull 2013, 2016, 2019). Our contribution is here to cast light on Stoicism from an evolutionary perspective.

This paper is organized as follows. As a preliminary step, Section 2 presents key elements of Epictetus's ethical thought. Section 3 studies the behavior of Epictetusian agents playing a prisoner's dilemma. Section 4 examines the outcomes of prisoner's dilemmas played by pairs of (either Nash or Epictetusian) players drawn randomly in the population, under two informational settings (with/without common knowledge on the opponent's type). Section 4 also studies the existence of ESS. Section 5 explores the long-run dynamics of the partition of the population into Nash and Epictetusian players under the discrete time replicator's dynamics. Section 6 examines the robustness of our results to assuming imperfectly random matching. Section 7 concludes.

[7] Other recent works on ethical preferences include Curry and Roemer (2012), Van Long (2015), Bezin and Ponthiere (2019) and De Donder et al (2021).

2 Epictetus's ethics

Unlike modern philosophers, Epictetus did not synthesize his ethical thought in a treatise, but the *Manual* - based on the transcription of Epictetus's lectures by his student Arrian - allows us to draw the contours of his ethical thought. Following the Stoic tradition, Epictetus defended a particular conception of the 'good life' known as *apathy*.[8] According to Epictetus, *a 'good life' is a life that is coherent with what the person is, that is, a life consistent with her status and her position as an infinitely small part of the Universe*.[9]

One can interpret this consistency requirement in various ways. In several influential pieces of works, the historian of thought Pierre Hadot (1978, 2001) interpreted Stoicism as a defence of a conception of the 'good life' that takes the form of a threefold discipline of the 'interior discourse': (1) the discipline of judgements; (2) the discipline of desires; (3) the discipline of acts.[10]

The discipline of judgements The Stoic discipline of judgements consists of *describing things 'as they are', in line with a purely 'physical' description of the world.* This discipline requires the person to abstract from her emotions and her imagination and, also, from added value judgements about things. The discipline of judgements includes several dimensions.

A first aspect consists of *identifying the boundaries of the self*, that is, of separating things that belong to the self (and are under control) and things that are external to the self (and are not under control).[11] This aspect of the discipline of judgements appears, for instance, in Epictetus's *Manual*, I:

> Of things some are in our power, and others are not. In our power are opinion, movement towards a thing, desire, aversion, turning from a thing; and in a word, whatever are our acts. Not in our power are the body, property, reputation, offices (magisterial power), and in a word, whatever are not our own acts. And the things in our power are by nature free, not subject to restraint or hindrance; but the things not in our power are weak, slavish, subject to restraint, in the power of others. Remember then, that if you think the things which are by nature slavish to be free, and the things which are in the power of others to be your own, you will be hindered, you will lament, you will be disturbed, you will blame both gods and men; but if you think that only which is your own to be your own, and if you think that what is another's, as it really is, belongs to

[8] See MacIntyre (1971, chapter 8).

[9] Moore (1903) and Sidgwick (1906).interpreted Stoicism as a metaphysical ethical doctrine recommending 'Living with Nature', and criticized that doctrine for its circularity. Stoic philosophers defend this circularity as a form of internal consistency.

[10] See Hadot (1978, 2001). These three disciplines are present in the works of all Stoic philosophers. However, in Epictetus's works, the last two disciplines are hard to distinguish.

[11] This task is regarded as a fundamental dogma of Stoicism. Its importance is revealed by the fact that Epictetus's *Manual* begins precisely with the distinction between things under control and things outside control (see *Manual*, I).

> another, no man will ever compel you, no man will hinder you, you will never blame any man, you will accuse no man, you will do nothing involuntarily (against your will), no man will harm you, you will have no enemy, for you will not suffer any harm.

According to Stoic philosophers, the self is limited to the 'intellect' of the person, and includes her judgements, desires/aversions and impulses to acts or acts (*Manual,* I). Other things, such as the person's body or emotions, do not belong to the self of the person, that is, to what Hadot (2001) calls, following Marcus Aurelius, the 'inner citadel' of the person.[12]

Once the self of the person is delimited, the discipline of judgements requires also that the person should take some distance with respect to her primary representations and judgements, and learn to be critical with respect to these. According to Stoicism, events do not hurt persons, but only the judgements made by persons on these events can hurt them. See Epictetus's *Manual*, V:

> Men are disturbed not by the things which happen, but by the opinions about the things; for example, death is nothing terrible, for if it were it would have seemed so to Socrates; for the opinion about death that it is terrible, is the terrible thing. When then we are impeded, or disturbed, or grieved, let us never blame others, but ourselves–that is, our opinions.

The key aspect of the discipline of judgements consists of always *valuing and judging things from the perspective of the Universe* (the 'Great Everything'). How to value these things is the object of the discipline of desires.

The discipline of desires The Stoic discipline of desires requires to *wish for nothing that is not under one's control*, that is, 'indifference to indifferent things'. Epictetus's precept 'Take away then aversion from all things which are not in our power' (*Manual*, II) is Epictetus's own reformulation of the discipline of desires. This is also formulated in the *Manual*, XIV:

> If you would have your children and your wife and your friends to live for ever, you are silly; for you would have the things which are not in your power to be in your power, and the things which belong to others to be yours. So if you would have your slave to be free from faults, you are a fool; for you would have badness not to be badness, but something else. But if you wish not to fail in your desires, you are able to do that. Practise then this which you are able to do. He is the master of every man who has the power over the things which another person wishes or does not wish, the power to confer them on him or to take them away. Whoever then wishes to be free let him neither wish for anything nor avoid anything which depends on others: if he does not observe this rule, he must be a slave.

[12] See Hadot (2001), chapter 6.

A person should wish for nothing that is not under her control (such as her body, properties, reputation and events), but, rather, should reorient her desires and aversions towards things that are under her control (her judgements and acts). As underlined by Hadot (2001, chapter 7), the discipline of desires takes its roots in the discipline of judgements, that is, the requirement of judging things from the perspective of the Universe.[13] The reasoning is the following. A person is an infinitely small part of the Universe, which is composed of myriad of things on which she has no control. Things external to the person were not 'produced' by the person, but by the Universe. Hence, the person should be indifferent to these things. Otherwise, by (dis)liking other parts of the Universe, the person would break the coherence of the Universe.

This argument relies on the idea of a coherent Universe, to which Epictetus and Marcus Aurelius adhered. It should be stressed, however, that Stoic philosophers defend also that Stoicism is the only reasonable ethical attitude even in the absence of a coherent world. Actually, if the world is driven by chaotic forces, it is still rational, by means of the Stoic discipline of desires, to preserve one's 'inner citadel' from this chaotic environment. Persons should not let happiness depend on chaotic forces.[14]

The discipline of acts The Stoic discipline of acts is strongly related to the disciplines of judgements and desires. This states that *for the things that depend on themselves, persons should act in the way that best serves the interests of the City, i.e., the Common Good.*[15] The discipline of acts requires persons to act in the way that is the most appropriate, that is, that best fits the interests and values of the City. This means that, as far as things under their control are concerned, persons should dedicate their time and their energy to activities that best contribute to serve the interests of the City. The discipline of acts requires some specialization in acts that best serve, depending on one's skills, the Common Good. See Epictetus's *Manual*, XXXVII:

> If you have assumed a character above your strength, you have both acted in this manner in an unbecoming way, and you have neglected that which you might have fulfilled.

The discipline of acts is also based on an ideal of living a life that is coherent with what the person is. Coherency is here achieved by choosing acts that, given the skills and talents received by the person, best serve the whole to

[13] For, instance, the two disciplines are presented successively in the *Manual*, III:

> In everything which pleases the soul, or supplies a want, or is loved, remember to add this to the (description, notion): What is the nature of each thing, beginning from the smallest? If you love an earthen vessel, say it is an earthen vessel which you love; for when it has been broken you will not be disturbed. If you are kissing your child or wife, say that it is a human being whom you are kissing, for when the wife or child dies you will not be disturbed.

[14] On the justification of the Stoic discipline of desires, see Hadot (2001), chapter 7.

[15] See Hadot (2001), chapter 8.

which the person belongs, that is, the City. In other words, the discipline of acts guarantees that the person, as a part of the City, does her best to promote the whole to which she belongs. Only this consistency between the whole and each of its parts can contribute to the Common Good.

The good life When a person respects the three disciplines of the 'interior discourse', that person is regarded as virtuous. According to Stoic philosophers, virtue is unique and indivisible: one cannot be partly virtuous, but must be either virtuous or not.[16] Note that Stoic philosophers associate some virtues to the three disciplines of the 'interior discourse': prudence is usually associated with the discipline of judgements, courage and temperance are associated with the discipline of desires, and justice is associated with the discipline of acts.[17]

From a Stoic perspective, a virtuous life is regarded as equivalent to a 'good life'. Stoic philosophers, including Epictetus, reduce goodness to moral goodness and badness to moral badness. This means that *the only way for the life of a person to be good is to be a virtuous life*, that is, the person should satisfy the three Stoic disciplines. There is no other way in which a life can be good.[18] See, for instance, Epictetus's *Manual*, XLVIII:

> The condition and characteristic of an uninstructed person is this: he never expects from himself profit (advantage) nor harm, but from externals. The condition and characteristic of a philosopher is this: he expects all advantage and all harm from himself.

Thus, Stoicism reduces the notion of goodness (resp. badness) to moral goodness (resp. moral badness). A person's life can only be good or bad through the acts carried out by that person. Elements external to the self (such as events) cannot make a life good or bad from a Stoic perspective.

An economic interpretation of Epictetus's thought How can one translate Epictetus's ethics into the language of modern economics? To answer that question, the first step consists of modelling the discipline of judgements. The discipline of judgements can be formalized by partitionning the world into two classes of things: on the one hand, *acts*, which are under the control of the person; on the other hand, *circumstances*, which are not under control.

Regarding the discipline of desires, Ponthiere (2022) argued that one can interpret Epictetus's precept as *requiring an extension of the symmetric factor of the preference relation beyond its boundaries under (non-ethical) preferences*. This extension of the 'domain of indifference' is an economic formalization of

[16] Note that a virtue is something that is pursued for its own sake, independently from consequences. Stoicism, like Epicureanism, belongs to the ethics of virtue, in contrast with consequentialist ethics (such as utilitarianism). See MacIntyre (1971), chapter 8.

[17] See Hadot (2001), chapter 9.

[18] Thus an event, even detrimental to a person, cannot make her life bad, because the goodness or badness of a life depends only on whether or not the person is virtuous in her life.

the Stoic discipline of desires promoting 'indifference to indifferent things'.[19] Following this interpretation, Epictetus's precept can be formalized as a requirement on the symmetric factor of a preference relation on outcomes defined as pairs (acts, circumstances). Epictetus's precept can be formalized as the requirement that *a person should be indifferent between outcomes differing only on circumstances*. This formalization of Epictetus's precept does justice to the Stoic discipline of desires: circumstances, being external to the 'inner citadel' of the person, cannot make a person better off or worse off.[20]

Once the Stoic disciplines of judgements and desires are translated into the language of economics, the study of the properties of the associated preference relation on pairs (acts, circumstances) can inform us about what persons should, from a Stoic perspective, do under each circumstance. Hence, in some sense, the discipline of acts can be 'deduced' by the disciplines of judgements and desires. The next section examines the consequences of Epictetus's precept on the discipline of acts in the particular context of simple cooperation games.

3 Epictetusian rationality in games

Epictetus's precept has important implications in game-theoretical contexts. To see this, note that, if one applies the Stoic discipline of judgements, one can see that *only the own acts of a person are within the self of that person and under her control, but not other persons's acts.*[21] Other persons's acts - being 'produced' by others - lie outside the boundaries of the self of the person. Therefore, the Stoic discipline of desires - 'Take away then aversion from all things which are not in our power' - recommends indifference with respect to these things external to the self. Hence, under our formalization of the discipline of desires, *an Epictetusian player should be indifferent between outcomes of the game that involve the same act for himself, but distinct acts for other players.*

At first glance, one may be suspicious of this result: if all players are indifferent between all outcomes of the game where they play the same strategy, this kills out all kinds of 'strategic interactions' among players, which is the main topic of game theory. But the fact that Stoicism - in particular the discipline of desires - rules out the existence of strategic interactions among persons constitutes a result on its own, which is worth being underlined. This result highlights that what game theorists call *'strategic interactions' do not consist of ahistorical phenomena*, which would exist under all kinds of representations and rationalities. Actually, persons who adopt the three Stoic disciplines will not give rise to 'strategic interactions' as usually defined by game theorists. Indeed, Epicte-

[19] That economic interpretation of Stoicism is actually supported by interpretations of contemporary philosophers considering that Stoicism is a doctrine recommending 'indifference to indifferent things'. See Hadot (2001), chapter 4.

[20] Under that formalization of Epictetus's precept, 'being indifferent to indifferent things' is translated as 'being indifferent to circumstances, *ceteris paribus*', that is, 'being indifferent between outcomes that differ only on circumstances'. See Ponthiere (2022).

[21] See Epictetus, *Manual*, II.

tusian agents reduce goodness to moral goodness.[22] From that perspective, the only way for a life to be good is to have a virtuous life, independently from what other persons do. This explains that, under Epictetusian rationality, strategic interactions cannot occur, unlike what happens with standard decision-makers.

In order to examine the consequences of adopting Epictetusian rationality in games, let us study a simple two-player simultaneous prisoner's dilemma. A two-player prisoner's dilemma (PD) takes the following form, where numbers in brackets indicate pay-offs for players A and B, respectively:[23]

	Player B	
Player A	cooperate	defect
cooperate	$(1, 1)$	(a, b)
defect	(b, a)	$(0, 0)$

Table 1. A two-person prisoner's dilemma.

A key feature of the PD game is that the outcome of the game where each player cooperates Pareto-dominates the outcome where each player defects. In Table 1, this feature is captured by the fact that (cooperate, cooperate), with pay-offs $(1, 1)$, Pareto-dominates (defect, defect), with pay-offs $(0, 0)$. Throughout this paper, we assume that *it is common knowledge to all players that the outcome where everyone cooperates Pareto-dominates the outcome where everyone defects*. That (weak) assumption is standard in analyses of PD games.[24] Moreover, we will assume in the paper that the pay-offs $(1, 1)$ and $(0, 0)$ in case of, respectively, (cooperate, cooperate) and (defect, defect), are also common knowledge to all players. This assumption of common normalization allows us to simplify notations throughout the paper, without affecting our main results.

Let us now consider how an Epictetusian agent would play a PD game. Treating the other player's acts as circumstances, an Epictetusian player should be indifferent between all outcomes of the game where he plays the same strategy. Assuming that player A is Epictetusian, this implies that the pay-off parameters a and b in Table 1 should satisfy:

$$a = 1 \text{ and } b = 0$$

The reason why Epictetusian rationality requires $a = 1$ goes as follows. From an Epictetusian perspective, things outside one's control are 'neither good nor bad'. *The only thing that matters for a person is how the person herself behaves.* In this game, the only thing that matters for player A is whether player A cooperates or defects. Therefore player A is indifferent between all outcomes where she cooperates, that is, between (cooperate, cooperate) and (cooperate, defect). Given that the pay-off for player A under (cooperate, cooperate) is equal to 1, player A's pay-off under (cooperate, defect) must also be equal to

[22] See Epictetus, *Manual*, XLVIII quoted in Section 2.

[23] See, for instance, Curry and Roemer (2012). That pay-off matrix assumes: (i) symmetric pay-offs for the two players; (ii) pay-offs in case of global cooperation normalized to 1; (iii) pay-offs in case of global defect normalized to 0.

[24] See Curry and Roemer (2012).

1.[25] The same rationale explains why the pay-off b obtained by player A under (defect, cooperate) is, from an Epictetusian perspective, equal to the pay-off obtained by player A under (defect, defect), that is, 0. Otherwise, if $b \neq 0$, player A would not be indifferent between outcomes differing only on things not under control, and, hence, would violate the Stoic discipline of desires.

Substituting for $a = 1$ and $b = 0$ in the pay-off matrix, while supposing that both players A and B are Epictetusian, the pay-off matrix becomes:

	Player B	
Player A	cooperate	defect
cooperate	$(1, 1)$	$(1, 0)$
defect	$(0, 1)$	$(0, 0)$

Table 2. The game of Table 1 played by 2 Epictetusian players.

An immediate implication of Epictetusian rationality is that for each player, *cooperating is the dominant strategy.* Indeed, once it is acknowledged that the outcome where every player cooperates is better, for each player, than the outcome where every player defects, Epictetus's precept implies that cooperating is the dominant strategy for each Epictetusian player. This result follows from the Stoic discipline of desires, which reduces goodness to moral goodness, and badness to moral badness (see Section 2). *From a Stoic perspective, the only way to achieve a good life is through one's own acts, which have to be virtuous.* This explains why cooperate is the dominant strategy for Epictetusian agents, that is, the best strategy for them no matter what other players do. As a consequence of this, the outcome of the game is (cooperate, cooperate).

Proposition 1 *When two Epictetusian players play the game of Table 1, (cooperate, cooperate) is the dominant strategy equilibrium, and the pay-offs are* $(1, 1)$.

Proof. See above. ■

Another implication worth noticing is that the game played by Epictetusian players can no longer be regarded as a prisoner's dilemma: indeed, a prisoner's dilemma is, by definition, a game where a Nash equilibrium is Pareto-dominated by an outcome of the game that is not a Nash equilibrium. This is not the case in the game described in Table 2, where the unique Nash equilibrium, which is (cooperate, cooperate), cannot be Pareto-dominated by any other outcome of the game. Indeed, the pay-off matrix of Table 2, which was implied by our account of Epictetus's precept, rules out the possibility where an outcome of the game that is not a Nash equilibrium can Pareto-dominate a Nash equilibrium. Hence, in some sense, we can say that Epictetusian rationality, when generalized, makes the prisoner's dilemma vanish.

One way of interpreting this result is to regard it as an implication, in the present game-theoretical context, of the Stoic disciplines of judgements (requiring a separation between what is under our control and what is not under our

[25] Otherwise, if $a \neq 1$, player A would not be indifferent between outcomes of the game differing only on circumstances.

control) and of the Stoic discipline of desires (requiring indifference between all outcomes that differ only on things not under our control). This implication - cooperation is the dominant strategy - concerns the acts that should be done, and is fully compatible with the Stoic discipline of acts defined as 'for things that depend on ourselves, behave to best serve the Common Good'.

The reason why Epictetusian rationality makes the prisoner's dilemma vanish differs from the one prevailing under other ethical preferences, such as Kantian preferences (Curry and Roemer 2012). Under Kantian preferences, agents must choose the best generalizable act, that is, they must focus only on the diagonal of the game. This exclusive focus on generalizable acts makes them choose cooperation. Here, under Epictetusian rationality, the mechanism that makes agents cooperate consists of the *extension of the boundaries of the domain of indifference*: agents must regard all outcomes where they cooperate as equally good (whatever the other player does), and, also, they must regard all outcomes where they defect as equally bad (whatever the other player does).[26] These indifference relations imply, under our assumption on the pay-off matrix, that cooperation is the dominant strategy. Thus the extension of the domain of indifference provides a path out of the prisoner's dilemma, which differs from the one prevailing under Kantian preferences.

It should be stressed, however, that we focused here on the outcome of the game when all players are Epictetusian. The next section will focus on games where players are not necessarily Epictetusian, and explore the implications of the outcome of the game for the dynamics of heterogeneity in the population.

4 Natural selection

Let us consider a large population of agents who are pairwise-matched at random each period of the game. We assume, as a benchmark case, that the matching process is perfectly random. Suppose that a fraction π of agents behave in the Nash manner (i.e., choosing best reply to the other player's strategy), while a fraction $1 - \pi$ of agents are Epictetusian individuals (i.e., choosing the best strategy while being indifferent to the opponent's strategy).

As usual in evolutionary game theory, we consider that agents with a higher expected payoff will grow more quickly in the population, the payoffs being proportional to evolutionary fitness. From an evolutionary perspective, a key issue is whether or not there exists an evolutionary stable strategy (ESS) (see Maynard-Smith, 1982, 1989). An ESS is a strategy that can survive an infinitely small invasion of mutants adopting another strategy.

In our framework, the Nash behavior is an ESS if and only if the expected pay-off of Nash players is larger than the one of Epictetusian players when π is close to 1. In that case, the population is almost entirely Nash, and a mutant Epictetusian agent would do worse than Nash agents, and would be driven to extinction. Alternatively, the Epictetusian behavior is an ESS if and only if the

[26] This follows from reducing goodness to moral goodness and badness to moral badness (see Section 2).

expected pay-off of Epictetusian players is larger than the one of Nash players when π is close to 0. In that case, the population is almost entirely Epictetusian, and a mutant Nash agent would do worse than Epictetusian agents, and would be driven to extinction.

In order to examine whether the Nash or the Epictetusian behavior is an ESS, we will consider two regimes regarding the informational structure of the game. In the first regime, types of agents are common knowledge to all players, whereas, in the second one, agents's types are private information.

4.1 Natural selection (1): types are common knowledge

In order to compute expected pay-offs for Nash and Epictetusian players when pairs of players of a prisoner's dilemma are randomly selected in the population, one must first compute the pay-offs in the three possible cases: (i) the game is played by a pair of Epictetusian players; (ii) the game is played by a pair of Nash players; (iii) the game is played by a pair composed of an Epictetusian player and a Nash player. Case (i) was studied in Section 3.

Let us consider case (ii). When the game is played by two Nash players, the pay-off matrix is:

	Player B	
Player A	cooperate	defect
cooperate	$(1,1)$	(α,β)
defect	(β,α)	$(0,0)$

Table 3. The game of Table 1 played by two Nash players.

As usual in the literature, we assume that $\beta > 1$, that is, that playing defect when the other player cooperates brings an extra pay-off with respect to the outcome where all players cooperate.[27] Thus Nash players have an incentive to deviate and defect. Moreover, we also assume that $\alpha < 0$, that is, that a player who cooperates when the other player defects is worse off than under the outcome where all players defect.

Since $\beta > 1$ and $\alpha < 0$, defect is the dominant strategy for each Nash player. Hence (defect, defect) is a dominant strategy equilibrium. This is the outcome of the game played by a pair of Nash players. Note that this outcome is Pareto-dominated by the outcome where each player cooperates. But the outcome (cooperate, cooperate) is not a Nash equilibrium, since this game admits a (necessarily unique) dominant strategy equilibrium (defect, defect).

Proposition 2 *When two Nash players play the PD game, if $\beta > 1$ and $\alpha < 0$, (defect, defect) is the dominant strategy equilibrium, and the pay-offs are $(0,0)$.*

Proof. See above. ■

Consider now case (iii): an Epictetusian player A plays against a Nash player B, and each player knows the type of the opponent. The pay-off matrix is:

[27] See, for instance, Curry and Roemer (2012).

	Player B	
Player A	cooperate	defect
cooperate	$(1,1)$	$(1,\beta)$
defect	$(0,\alpha)$	$(0,0)$

Table 4. The game of Table 1 played by an Epictetusian player and a Nash player.

The Epictetusian player A knows that player B is a Nash player. But player A plays nonetheless cooperate, since this is the dominant strategy for him. Indeed, from an Epictetusian perspective, only agents's own acts can make their life good, no matter what other agents do (this is just a circumstance, to which one should be indifferent). This motivates player A choosing to cooperate, despite the knowledge that player B will defect. The Nash player B knows that player A is an Epictetusian player. Since $\beta > 1$, player B plays defect, because this is the dominant strategy for him. Thus the outcome (cooperate, defect) is the dominant strategy equilibrium. Pay-offs are $(1,\beta)$.

Proposition 3 *When an Epictetusian player A and a Nash player B play the PD game, their type being common knowledge, if $\beta > 1$ and $\alpha < 0$, the dominant strategy equilibrium is (cooperate, defect), and the pay-offs are $(1,\beta)$.*

Proof. See above. ■

Having computed the outcome of the game and the associated pay-off in cases (i), (ii) and (iii), we can now compute the expected pay-offs for the Epictetusian agents and the Nash agents of playing two-person games with a randomly assigned opponent.

An Epictetusian agent has a probability $1-\pi$ of being matched with another Epictetusian agent, and a probability π of being matched with a Nash agent. In any case, an Epictetusian agent will cooperate, and enjoy the same pay-off, equal to 1 (since for the Epictetusian agent pay-offs are invariant to the other player's strategy). Hence the expected pay-off for an Epictetusian agent is:

$$E(\text{payoffs})_{\text{Epictetusian}} = (1-\pi)1 + \pi 1 = 1 \tag{1}$$

For a Nash agent, the pay-off is β when the agent is matched with an Epictetusian player (which occurs with probability $1-\pi$), and the pay-off is 0 when the agent is matched with another Nash player (which occurs with a probability π). Thus the expected pay-off for a Nash agent is:

$$E(\text{payoffs})_{\text{Nash}} = (1-\pi)\beta + \pi 0 = (1-\pi)\beta \tag{2}$$

Interestingly, the higher the proportion of Epictetusian players in the population $1-\pi$, the higher the expected pay-off of Nash players is. Indeed, playing their dominant strategy (defect) yields a higher pay-off for Nash players when they play against an Epictetusian player in comparison to when they play against another Nash player. Proposition 4 summarizes our results.

Proposition 4 *Suppose that the types of players are common knowledge.*

- *If* $1 > (1-\pi)\beta$*, the expected pay-off is larger for the Epictetusian player than for the Nash player.*
- *If* $1 = (1-\pi)\beta$*, the expected pay-offs are equal for the Epictetusian player and for the Nash player.*
- *If* $1 < (1-\pi)\beta$*, the expected pay-off is larger for the Nash player than for the Epictetusian player.*
- *Neither the Nash type nor the Epictetusian type is an ESS. No type can drive the other type to extinction.*

Proof. The first part of the proposition follows from the calculus of expected pay-offs for the two types of players.

Regarding whether or not the Nash behavior is an ESS, consider the case where π is close to 1. In that case, the expected pay-off of the Nash player is:

$$\lim_{\pi \to 1} (1-\pi)\beta = 0 < 1$$

which is lower than the expected pay-off of Epictetusians, which is 1. Hence a population composed (almost) entirely of Nash players cannot resist an infinitely small invasion of Epictetusian players. Thus Nash behavior is not an ESS.

Consider now the case where π is close to 0. In that case, the expected pay-off of the Nash player is:

$$\lim_{\pi \to 0} (1-\pi)\beta = \beta > 1$$

which is higher than the expected pay-off of the Epictetusian player, which is 1. Hence a population composed (almost) entirely of Epictetusian players cannot resist an infinitely small invasion of Nash players. Thus Epictetusian behavior is not an ESS. ■

Proposition 4 states that neither Nash behavior nor Epictetusian behavior is an ESS. No type can drive the other type to extinction. The intuition behind that result is the following. When the population is composed almost entirely of Nash players, expected pay-offs of Nash players are low, because they are trapped in the coordination failure and play defect against each others. The expected pay-offs of Nash players are then lower than the expected pay-offs of the few Epictetusian players invading the population. Thus Nash players cannot resist that invasion, and thus cannot push the Epictetusian to extinction. Alternatively, when the population is composed almost entirely of Epictetusian players, expected pay-offs for Epictetusian players is 1, which is smaller than the expected pay-offs of Nash players, because, under this composition of the population, the small population of invading Nash players benefits a lot from playing defect (since they play mainly against Epictetusian players). Enjoying a higher expected pay-off, Nash players tend then to invade the population. Thus Epictetusian players cannot resist an invasion of Nash players. This explains why neither the Nash nor the Epictetusian types are ESS in our framework.

4.2 Natural selection (2): types are private information

Let us now consider the alternative informational regime, where players do not know the type of their opponent.

Note first that, for an Epictetusian player, knowing or not knowing the type of the opponent playing the game does not affect the best way to behave. Indeed, from an Epictetusian perspective, the pay-offs are invariant to the other player's strategy. Hence, from that perspective, whether the other player is also an Epictetusian player, or a Nash player does not make any difference. In any case, Epictetusian players will play cooperate, as they do under the common knowledge assumption, and they will obtain a pay-off equal to 1, no matter the type of the other player and the strategy chosen by the other player.

As for Nash players, a Nash player knows that the other player cooperates if he is an Epictetusian player, which happens with a probability $1-\pi$. Moreover, a Nash player knows that the other player defects if he is a Nash player, which happens with a probability π. These pieces of information can be used to compute the expected pay-offs of cooperating and defecting for a Nash player in the absence of knowledge about the opponent's type.

Hence, for a Nash player, in the absence of information about the other player's type, the expected payoff of playing defect is:

$$\pi 0 + (1-\pi)\beta = (1-\pi)\beta \tag{3}$$

whereas the expected pay-off of playing cooperate is:

$$\pi\alpha + (1-\pi)1 = \pi\alpha + 1 - \pi \tag{4}$$

Note that, since $\beta > 1$ and $\alpha < 0$, we have:

$$(1-\pi)\beta > 1 - \pi > \pi\alpha + 1 - \pi \tag{5}$$

so that the expected payoff of defect is higher than the expected pay-off of cooperation, for all levels of π.

As a consequence of this, a Nash player ignoring the type of his opponent will choose to play defect, because this strategy yields a higher expected pay-off for him. The expected pay-off of the Nash player is thus $(1-\pi)\beta$. Proposition 5 summarizes our results.

Proposition 5 *Suppose that the type of each player is private knowledge.*

- *If $1 > (1-\pi)\beta$, the expected pay-off is larger for the Epictetusian player than for the Nash player.*
- *If $1 = (1-\pi)\beta$, the expected pay-offs are equal for the Epictetusian player and for the Nash player.*
- *If $1 < (1-\pi)\beta$, the expected pay-off is larger for the Nash player than for the Epictetusian player.*

- *Neither the Nash type nor the Epictetusian type is an ESS: no type can drive the other type to extinction.*

Proof. The first part of Proposition 5 follows from the above calculations.

The second part of Proposition 5 can be proved by showing that the expected pay-off of Nash players is smaller than the expected pay-off of Epictetusian players when π tends to 1, whereas the expected pay-off of Epictetusian players is smaller than the expected pay-off of Nash players when π tends to 0. This is similar to the proof of Proposition 4. ■

Proposition 5 suggests that the previous results, obtained under the assumption that the type of each player is common knowledge, are robust to relaxing that informational assumption, and to supposing that players do not know the type of their opponent. Neither the Nash behavior nor the Epictetusian behavior is an ESS in this game: no type - neither the Nash, nor the Epictetusian - can drive the other type to extinction, and this result holds whether players' types are common knowledge or not.

5 The dynamics of heterogeneity

Up to now, we showed that, under perfectly random matching, neither the Nash type nor the Epictetusian type can drive the other type to extinction. In the light of these results, one may still wonder what the long-run dynamics of heterogeneity looks like. In order to examine that issue, this section studies the dynamics of the partition of the population, by assuming an explicit dynamics of the variable π, the fraction of Nash players in the population.

For that purpose, let us assume that the population partition follows the discrete time replicator dynamics (see Cressman 2003):

$$\pi_{t+1} - \pi_t = \left(\frac{(1-\pi_t)\beta - [\pi_t(1-\pi_t)\beta + (1-\pi_t)\,1]}{\pi_t(1-\pi_t)\beta + (1-\pi_t)\,1} \right) \tag{6}$$

The left-hand-side (LHS) of the equation is the variation in the proportion of Nash in the population between two successive time-periods. The RHS is the relative gap in expected pay-offs between Nash players and the entire population (relative to the expected pay-off of the population as a whole).

That equation simplifies to:

$$\pi_{t+1} = \pi_t + \left(\frac{\beta(1-\pi_t) - 1}{\pi_t\beta + 1} \right) \equiv \varphi(\pi_t) \tag{7}$$

This expression shows that the proportion of Nash players in the population at time $t+1$ exceeds the proportion of Nash players in the population at time t when the expected pay-off of Nash players exceeds the expected pay-off of Epictetusian players, that is, when $\beta(1-\pi_t) > 1$. Alternatively, when $\beta(1-\pi_t) < 1$, that is, when the expected pay-off of Nash players is smaller than the expected pay-off of Epictetusian players, the proportion of Nash players in the population at period $t+1$ is smaller than the proportion of Nash players at time t.

Proposition 6 summarizes our results.

Proposition 6 *Assume that the dynamics of π_t follows the discrete time replicator's dynamics. The equilibrium proportion of the Nash type in the population is $\pi^* = 1 - \frac{1}{\beta}$. The equilibrium proportion of Epictetusian type in the population is $1 - \pi^* = \frac{1}{\beta}$. This equilibrium partition is locally stable.*

Proof. The equilibrium partition of the population can be found by setting $\pi_{t+1} = \pi_t$ in the dynamic equation. We then have:

$$\left(\frac{\beta(1-\pi_t)-1}{\pi_t\beta+1}\right) = 0 \iff \pi_t = 1 - \frac{1}{\beta} = \pi^* \tag{8}$$

Thus, at the equilibrium, there is a fraction $1-\frac{1}{\beta}$ of Nash players, and a fraction $\frac{1}{\beta}$ of Epictetusian players.

We have:

$$\frac{\partial \pi_{t+1}}{\partial \pi_t} = 1 + \left(\frac{-\beta\left[\pi_t\beta\right] - \beta\beta(1-\pi_t)}{\left(\pi_t\beta+1\right)^2}\right) = 1 + \left(\frac{-\beta^2}{\left(\pi_t\beta+1\right)^2}\right) \tag{9}$$

The local stability of the equilibrium requires:

$$\begin{aligned}
\left|\frac{\partial \pi_{t+1}}{\partial \pi_t}\right|_{\pi^*=1-\frac{1}{\beta}} < 1 \iff & \quad \left|1 + \left(\frac{-\beta^2}{\left(\left(1-\frac{1}{\beta}\right)\beta+1\right)^2}\right)\right| < 1 \\
\iff & \quad \left|1 + \left(\frac{-\beta^2}{\left(\beta-1+1\right)^2}\right)\right| < 1 \\
\iff & \quad |1+(-1)| < 1 \\
\iff & \quad |0| < 1
\end{aligned}$$

Thus the equilibrium $\pi^* = 1 - \frac{1}{\beta}$ is stable. ■

Proposition 6 states that, in the long-run, the population is composed of a fraction $1 - \frac{1}{\beta}$ of Nash agents and of a fraction $\frac{1}{\beta}$ of Epictetusian agents, and that this equilibrium partition is locally stable.

As an illustration, Figure 1 shows the form of the function $\varphi(\pi_t)$ for distinct values of the parameter β, together with the 45° line. We can see that, as β increases, the equilibrium proportion of Nash agents in the population tends to grow. We can also see that the slope of $\varphi(\pi_t)$ at the intersection with the 45° line is always less than 1, so that the equilibrium partition is locally stable.

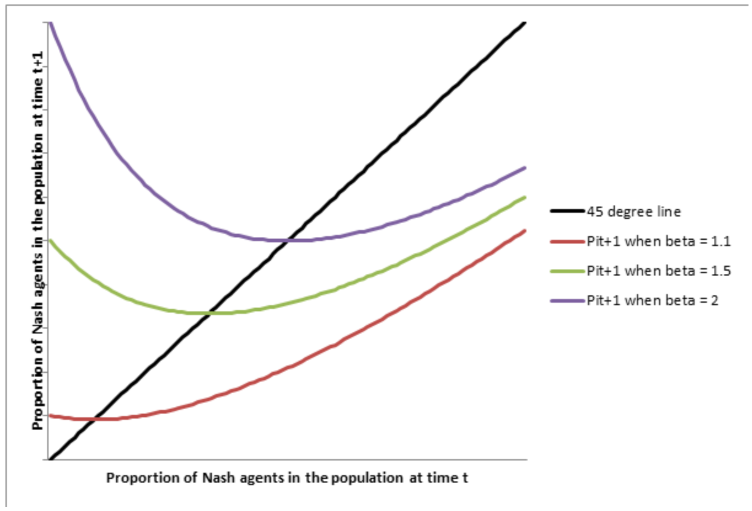

Figure 1. Transition function $\varphi(\pi_t)$ for various levels of β.

In sum, this section shows that, under the discrete time replicator's dynamics, the long-run partition of the population into Nash and Epictetusian agents is locally stable, and includes a proportion of Epictetusian agents that is decreasing with the pay-off gain of defecting for Nash agents when their opponents do not defect.

6 Imperfectly random matching

While the above sections assumed that Nash and Epictetusian agents were matched perfectly randomly to play the prisoner's dilemma, one may wonder whether our results are robust to allowing for some degree of assortativity in the matching. This section examines that issue, and revisits our results under a more general matching function allowing for some degree of assortativity.

For that purpose, let us first define by $\Pr(N|N,\pi)$ the probability of being matched with a Nash type N conditionally on being of Nash type N when the proportion of Nash agents is π. Similarly, let us define by $\Pr(N|E,\pi)$ the probability of being matched with a Nash type N conditionally on being of Epictetusian type E when the proportion of Nash agents is π.

Following Alger and Weibull (2013), we can define by $\phi(\pi)$ the difference between the conditional probabilities for an agent to be matched with an agent with type Nash, given that the agent himself either also has the Nash type N, or, alternatively, the Epictetusian type E:

$$\phi(\pi) = \Pr(N|N,\pi) - \Pr(N|E,\pi) \tag{10}$$

The difference $\phi(\pi) : (0,1) \to [-1,1]$ defines the assortment function.

Using the following necessary balancing condition, which states that the number of Nash agents matched with Epictetusian agents must be strictly equal with the number of Epictetusian agents matched with Nash agents:

$$\pi \left[1 - \Pr\left(N|N,\pi\right)\right] = (1-\pi)\Pr\left(N|E,\pi\right) \tag{11}$$

one can rewrite the conditional probability $\Pr\left(N|N,\pi\right)$ as a function of π and $\phi(\pi)$:

$$\begin{aligned} \Pr\left(N|N,\pi\right) &= \phi\left(\pi\right) + \Pr\left(N|E,\pi\right) \\ &= \phi\left(\pi\right) + \frac{\pi}{1-\pi}\left[1 - \Pr\left(N|N,\pi\right)\right] \\ &= (1-\pi)\,\phi\left(\pi\right) + \pi \end{aligned} \tag{12}$$

Note that the conditional probability of being matched with an agent of type N collapses to π when the assortment function $\phi(\pi)$ equals 0, which is the case when the matching is perfectly random.

Moreover, the conditional probability $\Pr\left(N|E,\pi\right)$ can be rewritten as:

$$\begin{aligned} \Pr\left(N|E,\pi\right) &= \Pr\left(N|N,\pi\right) - \phi\left(\pi\right) \\ &= (1-\pi)\,\phi\left(\pi\right) + \pi - \phi\left(\pi\right) \\ &= \pi(1-\phi\left(\pi\right)) \end{aligned} \tag{13}$$

Note also that $\Pr\left(N|E,\pi\right)$ equals π when the assortment function $\phi(\pi)$ equals 0, which is the case when the matching is perfectly random.

The other conditional probabilities $\Pr\left(E|N,\pi\right)$ and $\Pr\left(E|E,\pi\right)$ can be computed as:

$$\Pr\left(E|N,\pi\right) = 1 - \Pr\left(N|N,\pi\right) = (1-\pi)\left(1-\phi\left(\pi\right)\right) \tag{14}$$

$$\Pr\left(E|E,\pi\right) = 1 - \Pr\left(N|E,\pi\right) = 1 - \pi\left(1-\phi\left(\pi\right)\right) \tag{15}$$

Following Alger and Weibull (2013), we assume that $\phi(\pi)$ is continuous and that $\phi(\pi)$ converges to some number as π tends to 1:

$$\lim_{\pi\to 1}\phi(\pi) = \sigma \tag{16}$$

for some real number σ. σ is the index of assortativity of the matching process. It follows from the formula of $\Pr\left(N|E,\pi\right)$ that $\sigma \in [0,1]$. The extreme case where $\sigma = 0$ corresponds to the case of perfectly random matching. The opposite extreme case, where $\sigma = 1$, corresponds to the case of perfectly correlated matching (each type is matched only with agents of the same type).[28]

[28] To see why this latter case corresponds to perfectly correlated matching, note that, when $\sigma = 1$, we have:

$$\lim_{\pi\to 1}\Pr\left(N|E,\pi\right) = 0$$

that is, even when the population is almost entirely composed of Nash agents, an Epictetusian agent is matched with a Nash agent with a probability equal to 0.

Having defined the assortment function $\phi(\pi)$, let us now reexamine the existence of ESS under imperfectly random matching. For that purpose, let us first consider the case where the types of agents are common knowledge.

Introducing imperfectly random matching has no effect on the expected pay-offs of Epictetusian agents in comparison with the perfectly random matching case. The reason is that, for Epictetusian agents, their pay-offs do not depend on the other agent's strategy, and, hence, do not depend on the type of agent with whom they are matched. More formally, we have:

$$\begin{aligned} E(\text{payoffs})_{\text{Epictetusian}} &= \Pr(E|E,\pi)\,1 + \Pr(N|E,\pi)\,1 \\ &= [1-\pi(1-\phi(\pi))]\,1 + [\pi(1-\phi(\pi))]\,1 = 1 \end{aligned} \quad (17)$$

Thus the precise form of the assortment function does not affect the expected pay-offs of Epictetusian agents.

For a Nash agent, the pay-off is β when the agent is matched with an Epictetusian player (which occurs with probability $\Pr(E|N,\pi)$), and the pay-off is 0 when the agent is matched with another Nash player (which occurs with a probability $\Pr(N|N,\pi)$). Thus the expected pay-off for a Nash agent is:

$$\begin{aligned} E(\text{payoffs})_{\text{Nash}} &= \Pr(E|N,\pi))\beta + \Pr(N|N,\pi)\,0 \\ &= (1-\pi)(1-\phi(\pi))\,\beta \end{aligned} \quad (18)$$

The expected pay-off of the Nash type depends on the form of the assortment function. Actually, it is only in the special case where $\phi(\pi) = 0$ (perfectly random matching) that the expected pay-off equals $(1-\pi)\beta$, as in the baseline model. When $\phi(\pi) > 0$ (some positive degree of assortativity), the expected pay-off for the Nash type is reduced in comparison with the perfectly random matching case. On the contrary, when $\phi(\pi) < 0$ (some negative degree of assortativity), the expected pay-off for the Nash type is increased in comparison with the perfectly random matching case.

Given that the expected pay-off of the Nash type depends on the assortment function, whereas the expected pay-off of the Epictetusian type does not, we can expect that the form of the assortment function affects also the issue of whether any of these two types can be an ESS. This issue can be examined by comparing the expected pay-offs of the two types.

Proposition 7 *Suppose that the types of players are common knowledge and that the matching process is imperfectly random with the assortment function* $\phi(\pi)$.

- *If* $1 > (1-\pi)(1-\phi(\pi))\,\beta$*, the expected pay-off is larger for the Epictetusian player than for the Nash player.*
- *If* $1 = (1-\pi)(1-\phi(\pi))\,\beta$*, the expected pay-offs are equal for the Epictetusian player and for the Nash player.*
- *If* $1 < (1-\pi)(1-\phi(\pi))\,\beta$*, the expected pay-off is larger for the Nash player than for the Epictetusian player.*

- *The Nash type is not an ESS. When $\lim_{\pi \to 0} \phi(\pi) \leq \frac{\beta - 1}{\beta}$, the Epictetusian type is not an ESS. When $\lim_{\pi \to 0} \phi(\pi) > \frac{\beta - 1}{\beta}$, the Epictetusian type is an ESS.*

Proof. The first part of the proposition follows from the calculus of expected pay-offs for the two types of players.

Regarding the second part, consider first the case where π is close to 1. In that case, the expected pay-off of the Nash player is:

$$\begin{aligned} & \lim_{\pi \to 1} \left[(1-\pi)(1-\phi(\pi))\beta\right] \\ = & \beta \lim_{\pi \to 1} \left[(1-\pi)(1-\phi(\pi))\right] \\ = & \beta (1-\sigma) 0 = 0 < 1 \end{aligned}$$

which, given that $\sigma \in [0,1]$, is lower than the expected pay-off of Epictetusians, which is 1. Hence a population composed (almost) entirely of Nash players cannot resist an infinitely small invasion of Epictetusian players. Thus Nash behavior is not an ESS.

Consider now the case where π is close to 0. In that case, the expected pay-off of the Nash player is:

$$\begin{aligned} & \lim_{\pi \to 0} \left[(1-\pi)(1-\phi(\pi))\beta\right] \\ = & \beta \lim_{\pi \to 0} \left[(1-\phi(\pi))\right] \\ = & \beta \left[1 - \lim_{\pi \to 0} \phi(\pi)\right] \end{aligned}$$

When $\beta \left[1 - \lim_{\pi \to 0} \phi(\pi)\right] \geq 1$, the expected pay-off of Nash is higher than or equal to the expected pay-off of the Epictetusian player, which is 1. Hence the Epictetusian type is not an ESS. However, when $\beta \left[1 - \lim_{\pi \to 0} \phi(\pi)\right] < 1$, the expected pay-off of Nash is lower than the expected pay-off of the Epictetusian player, which is 1. Hence the Epictetusian type is an ESS. This condition can be rewritten as: $\lim_{\pi \to 0} \phi(\pi) > \frac{\beta - 1}{\beta}$. ■

Whatever the form of the assortment function is, the Nash type is never an ESS. When the population is almost entirely composed of Nash agents, it is impossible for them to drive the Epictetusian type to extinction. The intuition goes as follows. When the degree of assortativity σ is low, the expected pay-off of the Nash is lower than the one of Epictetusian agents, because of the same reason as in the baseline model ($\sigma = 0$).[29] However, when the degree of assortativity σ is increased, the situation becomes even worse for the Nash, since in that case the few Epictetusian agents are increasingly matched among themselves, which guarantees them an even higher expected pay-off than Nash agents (in comparison to $\sigma = 0$). Thus increasing assortativity of the matching does not allow the Nash type to drive the Epictetusian type to extinction.

[29] The reason is that most Nash agents are matched with Nash agents (because π is close to 1), and are thus trapped in the prisoner's dilemma. Hence the Nash agents have here a lower expected pay-off than the few Epictetusian agents.

If the assortment function exhibits a sufficiently high positive assortment when π is close to zero, the Epictetusian type is an ESS. In other words, if the degree of assortativity in the matching is sufficiently high when the Nash type agents are not numerous, the Epictetusian type can drive the Nash to extinction. The intuition is that, when the degree of assortativity is sufficiently high when π is close to 0, most Epictetusian agents are matched with Epictetusian agents, which guarantees them higher expected pay-offs than the few Nash being matched, for most of them, with Nash agents. Hence, a higher degree of assortativity of the matching prevents the few invading Nash agents from realizing higher pay-offs than the Epictetusian types, by preventing most of interactions between Nash and Epictetusian agents (which are the only way for Nash agents to have higher pay-offs than Epictetusian agents). Thus, whereas the Epictetusian type could not drive the Nash type to extinction under perfectly random matching, it is possible, when the degree of assortativity of the matching is sufficiently high, that the Epictetusian type can drive the Nash to extinction.

Finally, let us conclude this section by considering, under a general assortment function with imperfect random matching, the case where the type of the opponent is private information.

As under perfectly random matching, the fact that an Epictetusian player knows the type of his opponent or not does not affect the best way to behave for him, because, from an Epictetusian perspective, the pay-offs are invariant to the other player's strategy (see above). In any case, Epictetusian players will play cooperate, as they do under the common knowledge assumption, and they will obtain a pay-off equal to 1, no matter the type of the other player and the strategy chosen by the other player.

As for Nash players, a Nash player knows that the opponent cooperates if the opponent is an Epictetusian player, which happens with a probability $\Pr(E|N,\pi))$. Moreover, a Nash player knows that the opponent defects if he is a Nash player, which happens with a probability $\Pr(N|N,\pi))$. Hence, for a Nash player, in the absence of information about the other player's type, the expected payoff of playing defect is:

$$\begin{aligned}
& \Pr(N|N,\pi))0 + \Pr(E|N,\pi))\beta \\
= & \Pr(E|N,\pi))\beta \\
= & \beta(1-\pi)(1-\phi(\pi))
\end{aligned} \tag{19}$$

whereas the expected pay-off of playing cooperate is:

$$\begin{aligned}
& \Pr(N|N,\pi))\alpha + \Pr(E|N,\pi))1 \\
= & \Pr(N|N,\pi))\alpha + \Pr(E|N,\pi)) \\
= & \alpha[(1-\pi)\phi(\pi)+\pi] + (1-\pi)(1-\phi(\pi))
\end{aligned} \tag{20}$$

Note that, since $\beta > 1$ and $\alpha < 0$, we have:

$$\beta(1-\pi)(1-\phi(\pi)) > (1-\pi)(1-\phi(\pi)) > \alpha[(1-\pi)\phi(\pi)+\pi] + (1-\pi)(1-\phi(\pi)) \tag{21}$$

so that the expected payoff of defect is higher than the expected pay-off of cooperation, for all levels of π.

Hence, a Nash player ignoring the type of his opponent will choose to play defect, because this strategy yields a higher expected pay-off for him. The expected pay-off of the Nash player is thus $\beta(1-\pi)(1-\phi(\pi))$. Proposition 8 summarizes our results.

Proposition 8 *Suppose that the type of each player is private knowledge and that the matching process is imperfectly random with the assortment function $\phi(\pi)$.*

- *If $1 > (1-\pi)(1-\phi(\pi))\beta$, the expected pay-off is larger for the Epictetusian player than for the Nash player.*
- *If $1 = (1-\pi)(1-\phi(\pi))\beta$, the expected pay-offs are equal for the Epictetusian player and for the Nash player.*
- *If $1 < (1-\pi)(1-\phi(\pi))\beta$, the expected pay-off is larger for the Nash player than for the Epictetusian player.*
- *The Nash type is not an ESS. When $\lim_{\pi\to 0}\phi(\pi) \leq \frac{\beta-1}{\beta}$, the Epictetusian type is not an ESS. When $\lim_{\pi\to 0}\phi(\pi) > \frac{\beta-1}{\beta}$, the Epictetusian type is an ESS.*

Proof. The first part of Proposition 8 follows from the above calculations.

The second part of Proposition 8 can be proved by showing that the expected pay-off of Nash players is smaller than the expected pay-off of Epictetusian players when π tends to 1, whereas the expected pay-off of Epictetusian players is smaller than the expected pay-off of Nash players when π tends to 0 when $\lim_{\pi\to 0}\phi(\pi) \leq \frac{\beta-1}{\beta}$, but larger than the expected pay-off of Nash players when π tends to 0 when $\lim_{\pi\to 0}\phi(\pi) > \frac{\beta-1}{\beta}$. This is similar to the proof of Proposition 7. ■

In sum, this section suggests that our previous results are only partly robust to the assumptions made concerning the randomness of the matching process. True, whether or not the matching of players is perfectly random, the Nash type can never drive the Epictetusian type to extinction. However, whereas the Epictetusian type cannot drive the Nash type to extinction under perfectly random matching, it is possible that the Epictetusian type can drive the Nash type to extinction when the matching exhibits a sufficiently high degree of assortativity. The intuition is that, when the matching is sufficiently assortative, the few invading Nash agents will play with other Nash agents, and the large majority of Epictetusian agents will play among themselves, which yields higher expected pay-offs for the Epictetusian than for the few Nash. Thus, when the matching process is sufficiently assortative, the Epictetusian agents can resist an invasion of a few Nash agents, something that is not true under perfectly random matching. This explains that a sufficiently high degree of assortativity of the matching process makes Stoicism an ESS.

These findings can be related with recent results in the literature about ethical preferences. For instance, Alger and Weibull (2019) emphasized, when studying game-theoretical interactions of Nash agents with other agents having (partly) ethical preferences (of the Kantian type), that more frequent interactions of ethical agents with each other (that is, a higher degree of assortment of the matching process) contributed to make evolution more favourable to ethical agents, in comparison to situations where the degree of assortment of the matching process is lower. The present paper, by focusing on interactions Nash/Epictetusian agents (instead of Nash/Kantian agents), is complementary to Alger and Weibull (2019). In line with that paper, we corroborate the theoretical result: assortativity in the matching process 'protects' ethical preferences.

7 Conclusions

This paper proposes to reexamine Epictetus's precept of 'Taking away aversion from all things not in our power' in an evolutionary light. For that purpose, we considered a population composed of Nash agents and of Epictetusian agents, the latter being indifferent between outcomes that differ only on things not under one's control (in line with the Stoic discipline of desires). Pairs of members of that population are randomly drawn and play a prisoner's dilemma.

When playing a prisoner's dilemma, Epictetusian agents are indifferent between outcomes of the game that differ only regarding the opponent's strategy, because they reduce goodness to moral goodness. Hence, if it is common knowledge that the outcome where everyone cooperates Pareto-dominates the outcome where everyone defects, the extension of the domain of indifference to all outcomes of the game where the player plays the same strategy implies that cooperating is the dominant strategy for Epictetusian agents. Epictetusian rationality, by encouraging cooperation, provides a way out of the prisoner's dilemma. This path out of the prisoner's dilemma differs from the one prevailing under Kantianism (focusing only on the diagonal of the game).

Our analyses show that, under perfectly random matching, neither Nash behavior nor Epictetusian behavior is an ESS: no type leads the other type to extinction. These results are robust to the informational setting, that is, to whether or not the types of agents are common knowledge. Moreover, assuming the discrete time replicator dynamics, we showed that the (locally stable) equilibrium proportion of Nash players is equal to $1-\frac{1}{\beta}$, while the one of Epictetusian agents is equal to $\frac{1}{\beta}$. Thus the long-run prevalence of Stoicism depends on how large the pay-offs of Nash agents are in case of defection.

However, when considering a more general framework with imperfectly random matching, we showed that, provided the matching process is sufficiently assortative, the Epictetusian type is an ESS and can drive the Nash type to extinction. The underlying intuition is that, when the matching process is sufficiently assortative, the few Nash agents invading a population composed almost entirely of Epictetusian agents will play among Nash agents (for most of them), whereas most of Epictetusian agents will play among Epictetusian agents. This guarantees that only very few Nash agents will be able to defect in presence of Epictetusian agents. As a consequence, the Epictetusian type exhibits a higher expected pay-off than the Nash type even when the population is almost entirely Epictetusian, something that is not possible under perfectly random matching. Thus assortativity of the matching, if sufficiently high, preserves most Epictetusian agents from interactions with Nash agents, which makes the Epictetusian type drive the Nash type to extinction.

These results have some important implications. First, at the level of history, our analyses allow us to explain the survival of attitudes based on Ancient moral philosophies such as Stoicism, despite the passage of long periods of time. The fact that Nash behavior is not an ESS in our framework - neither under perfectly random matching, nor under imperfectly random matching - provides an explanation for the survival of these ethical attitudes defended more than two millennia ago. Second, also at the level of history, the fact that the Epictetusian type could not drive the Nash type to extinction reveals that the degree of assortativity in the matching of interactions was not sufficiently high to make the Epictetusian type an ESS. Third, at the policy level, our results suggest that, in order to deal with climate change and other global disorders due to coordination failures, a possible path is to encourage the prevalence of ethical preferences (such as Epictetusian rationality) by reducing, through transfer policies, the gains obtained from defection.

8 References

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