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Matching with myopic and farsighted players ☆

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Abstract

We introduce the new notion of the pairwise myopic-farsighted stable set to study stable matchings under the assumption that players can be both myopic and farsighted. For the special case where all players are myopic, our concept predicts the set of matchings in the core. When all players are farsighted, we provide the characterization of pairwise myopic-farsighted stable sets: a set of matchings is a pairwise myopic-farsighted stable set if and only if it is a singleton consisting of a core element. This result confirms the result obtained by Mauleon et al. (2011) with a completely different effectivity function and provides a new special case where the farsighted stable set is absolutely maximal (Ray and Vohra, 2019) and coincides with the Strong Rational Expectations Farsighted Stable Set (Dutta and Vohra, 2017). When myopic and farsighted players interact, matchings outside the core can be stable and the most farsighted side can achieve its optimal stable matching.

JEL classification: C70; C78

Keywords: Marriage problems; Stable sets; Myopic and farsighted players

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1. Introduction

The matching literature can be divided into two streams, depending on whether players are assumed to be myopic or farsighted. In the seminal contribution by Gale and Shapley (1962), all players are assumed to be myopic and the properties of stable matchings are analyzed. In the well-known marriage model, a matching is stable if no individual player prefers to destroy an existing match and no pair of players prefers to form a match between them. Gale and Shapley (1962) have shown that the set of stable matchings of a marriage problem is non-empty, a result which implies that the core of a marriage problem is non-empty. The myopic approach is also followed in Ehlers (2007), who has characterized the von Neumann-Morgenstern (vNM) stable sets in marriage problems and has shown that the set of matchings in the core is a subset of any vNM stable set and a vNM stable set can contain matchings outside the core. Wako (2010) shows that the vNM stable set exists and is unique. Demuynck et al. (2019) introduce the myopic stable set and show it coincides with the core for matching problems.

Notions like the core, the vNM stable set, and the myopic stable set are myopic since the players do not anticipate that individual and coalitional deviations are countered by subsequent deviations. These concepts are based on the direct dominance relation and neglect the destabilizing effect of indirect dominance relations as introduced by Harsanyi (1974) and Chwe (1994). Indirect dominance captures the idea that coalitions of farsighted players can anticipate the actions of other coalitions and consider the end matching that their deviations may lead to.

Based on the concept of indirect dominance, several solution concepts assume farsighted behavior of the players in matching models. Diamantoudi and Xue (2003) have shown that in hedonic games with strict preferences core partitions are always contained in the largest consistent set due to Chwe (1994).[1] However, the largest consistent set may contain more matchings than those matchings that are in the core. Mauleon et al. (2011) characterize the vNM farsightedly stable sets as all singletons that contain a core element and show that the farsighted core, defined by Diamantoudi and Xue (2003) as the set of matchings that are not indirectly dominated by other matchings, can be empty.[2] Mauleon et al. (2014) characterize one-to-one matching problems for which indirect dominance implies direct dominance. In this kind of matching problems, any solution concept based on direct or indirect dominance (like the core or the farsighted core) will give the same predictions.

In many real-life matching problems, one may expect that players with different degrees of farsightedness interact. Also, there might be an asymmetry in the degree of farsightedness between the two sides of the market. For instance, between hospitals and physicians (or firms and workers more generally) or between schools and students, one would expect the hospitals and the schools to be more farsighted than physicians and students. Hospitals and schools have much more market experience and therefore may be better positioned to contemplate the chain of reactions that follows a deviation.

In the present paper, we study marriage problems that allow for the interaction between myopic and farsighted players and we propose the new notion of a pairwise myopic-farsighted stable set to study the matchings that are stable in this context. Central in the definition of a

[1] Other approaches to farsightedness in coalition and network formation are suggested by the work of Xue (1998), Mauleon and Vannetelbosch (2004), Page et al. (2005), Herings et al. (2004, 2009, 2019), Page and Wooders (2009), and Ray and Vohra (2015) among others.

[2] The farsighted core only exists when the core contains a unique matching and no other matching indirectly dominates the matching in the core.

pairwise myopic-farsighted stable set is the notion of a pairwise myopic-farsighted improving path. A pairwise myopic-farsighted improving path is a sequence of matchings that can emerge when farsighted players form or destroy matches based on the improvement the end matching offers relative to the current matching while myopic players form or destroy matches based on the improvement the next matching offers relative to the current matching. Each matching in the sequence differs from the previous one in that either a new match is formed or an existing one is destroyed.

A pairwise myopic-farsighted stable set is the set of matchings satisfying internal and external stability with respect to the notion of a pairwise myopic-farsighted improving path. That is, there is no pairwise myopic-farsighted improving path from any matching in the set to another matching in the set (internal stability) and there is a pairwise myopic-farsighted improving path from any matching outside the set to some matching in the set (external stability). In the special case where all players are myopic, the pairwise myopic-farsighted stable set is equivalent to the pairwise CP vNM set of Herings et al. (2017) and is equal to the core of the marriage problem.

At the other extreme, when all players are farsighted, we show that the pairwise myopic-farsighted stable sets are characterized as the singletons consisting of a core element. To show this result, we establish first that a set of matchings is a pairwise myopic-farsighted stable set if and only if it is a vNM farsightedly stable set as defined in Mauleon et al. (2011). Next we use their characterization of the vNM farsightedly stable set as the singletons consisting of a core element. The vNM farsightedly stable set defined in Mauleon et al. (2011) results when coalitions of arbitrary size are allowed to move and improvements are required to be strict for all players involved in the move. Hence, our result improves upon and adds to the robustness of the characterization result obtained in Mauleon et al. (2011) by showing that, with a completely different effectivity function, every singleton core element is also a pairwise myopic-farsighted stable set when all players are farsighted.

We then relate our result to some recent theoretical developments regarding a conceptual drawback of the farsighted stable set: the maximality issue. Indeed, while coalitional moves improve on existing outcomes along a farsighted objection, coalitions might do even better by an alternative deviation. Also, other coalitions may not agree to a particular farsighted objection and want to impose their favored moves. Dutta and Vohra (2017) propose the rational expectations farsighted stable set (REFS) and the strong rational expectations farsighted stable set (SREFS) that restrict coalitions to hold common, history-independent expectations that incorporate maximality regarding the continuation path. They show that the REFS and the SREFS coincide with the farsighted stable set in cases where the latter consists of states with a single-payoff vector. When this is the case, the farsighted stable set also incorporates the strictest variant of maximality imposed by Ray and Vohra (2019), called absolute maximality. Absolute maximality calls for immunity to all deviations, not just by the coalition that moves or by those coalitions that intersect the one that moves, but by any coalition. We show that in case all players are farsighted, a pairwise myopic-farsighted stable set is absolutely maximal and coincides with a REFS and an SREFS.

We also study situations where the two sides of the market are heterogeneous in their degree of farsightedness. We fully analyze the typical example where the preferences of men and women are diametrically opposed and show that in case all players on one side are myopic and at least one player on the other side is farsighted, the optimal stable matching of the farsighted side constitutes the unique pairwise myopic-farsighted stable set. In all other cases, any core outcome and no other outcome is sustained by the pairwise myopic-farsighted stable set.

We present an example where all women are farsighted and all men are myopic and there exists a matching that is weakly preferred to the woman-optimal stable matching by all women and strictly preferred by some women. We show that it is possible to support this non-core matching as a singleton pairwise myopic-farsighted stable set. This illustrates that the interaction between myopic and farsighted players can lead to outcomes that cannot be supported in homogeneous cases.

Moreover, under certain conditions guaranteeing that the woman side is more farsighted than the man side, it can be shown in general that selection among core elements is possible; i.e., the most farsighted side is able to achieve its optimal stable matching. The result implies that the presence of some farsighted women is enough to guarantee that the woman-optimal stable matching can always be reached, starting from any other matching, by means of a pairwise myopic-farsighted improving path. Thus, also the myopic women benefit from the presence of farsighted women.

The paper is organized as follows. Section 2 introduces marriage problems and standard notions of stability. Section 3 defines the pairwise myopic-farsighted stable set and characterizes the implications of all possible constellations regarding farsightedness in an example where preferences of men and women are diametrically opposed. It also analyzes the special case where all players are myopic. Section 4 provides the characterization of the pairwise myopic-farsighted stable set when all players are farsighted and relates it with other results obtained by different solution concepts. Section 5 studies heterogeneous societies where myopic and farsighted players interact and shows that matchings outside the core can be stable and that the most farsighted side can achieve its optimal stable matching. Section 6 concludes.

2. Marriage problems

A marriage problem consists of a finite set of players N, partitioned into a set of men M and a set of women W. The set of non-empty subsets of N is denoted by $\mathcal{N}$. Each player $i \in N$ has a complete and transitive *preference ordering* $\succ_i$ over the players of the opposite sex and the prospect of being alone. Preferences are assumed to be strict. We write $j \sim_i j'$ if player i is indifferent between players j and j', which can only be the case if $j = j'$, and $j \succeq_i j'$ if $j \succ_i j'$ or $j \sim_i j'$. We denote the preference profile $((\succ_m)_{m \in M}, (\succ_w)_{w \in W})$ by $\succ$. A marriage problem is a triple $(M, W, \succ)$.

A *matching* is a function $\mu : N \to N$ satisfying the following properties:

(i) For every $m \in M$, $\mu(m) \in W \cup \{m\}$.
(ii) For every $w \in W$, $\mu(w) \in M \cup \{w\}$.
(iii) For every $i \in N$, $\mu(\mu(i)) = i$.

The set of all matchings is denoted by $\mathcal{M}$. Given a matching $\mu \in \mathcal{M}$, player i is said to be *single* if $\mu(i) = i$. A matching μ is individually rational if each player is acceptable to his or her partner, so for every $i \in N$ it holds that $\mu(i) \succeq_i i$. A matching μ that is not individually rational can be blocked by a player with an unacceptable partner. For a given matching μ, a pair (m, w) is said to form a *blocking pair* if m and w are not matched to one another but prefer each other to their partners at μ, i.e. $w \succ_m \mu(m)$ and $m \succ_w \mu(w)$. A matching μ is stable if it is not blocked by any single player or any pair of players.

Given a matching $\mu \in \mathcal{M}$ with man $m \in M$ matched to woman $w \in W$, so $\mu(m) = w$, the matching μ' that is identical to μ, except that the match between m and w has been destroyed by either m or w, is denoted by $\mu - (m, w)$. Given a matching $\mu \in \mathcal{M}$ such that $m \in M$ and $w \in W$ are not matched to one another, the matching μ' that is identical to μ, except that the pair (m, w) has formed at μ' and their partners at μ are now single at μ', is denoted by $\mu + (m, w)$.

For every $i \in N$, we extend the preference ordering $\succ_i$ over the player's potential partners to the set of matchings $\mathcal{M}$ in the following way. We say that player i prefers the matching μ' to the matching μ if $\mu'(i) \succ_i \mu(i)$ and we write $\mu' \succ_i \mu$. We write $\mu' \sim_i \mu$ if $\mu'(i) \sim_i \mu(i)$, and $\mu' \succeq_i \mu$ if $\mu' \succ_i \mu$ or $\mu' \sim_i \mu$.

For $S \in \mathcal{N}$, $\mu(S) = \{\mu(i) \mid i \in S\}$ denotes the set of partners of players in S at μ. A coalition $S \in \mathcal{N}$ is said to block a matching $\mu \in \mathcal{M}$ if there exists a matching $\mu' \in \mathcal{M}$ such that $\mu'(S) = S$ and $\mu' \succ_S \mu$, where $\mu' \succ_S \mu$ is defined as $\mu' \succ_i \mu$ for every $i \in S$. The core of the marriage problem $(M, W, \succ)$ consists of all matchings that are not blocked by any coalition. We denote the set of matchings that belong to the core by C.

It has been shown by Gale and Shapley (1962) that the core of a marriage problem is non-empty. Also, a matching is stable if and only if it is not blocked by a coalition of size one or two if and only if it belongs to the core, see Theorem 3.3 in Roth and Sotomayor (1990). Knuth (1976) has shown that the core of a marriage problem is a distributive lattice. In particular, there is a man-optimal stable matching μ^{M} and a woman-optimal stable matching μ^{W}. For any matching μ in the core, for every $m \in M$, it holds that $\mu^{\mathrm{M}} \succeq_m \mu$. Similarly, for any matching μ in the core, for every $w \in W$, it holds that $\mu^{\mathrm{W}} \succeq_w \mu$.

3. The pairwise myopic-farsighted stable set

The literature on matching problems can be divided into two streams, depending on whether the approach taken is myopic or farsighted. Up to now, no solution concept has been proposed in order to allow for heterogeneity in the degree of farsightedness among players. In the following, we propose the notion of a pairwise myopic-farsighted stable set to study the matchings that are stable when myopic and farsighted players interact with each other.

Let $F \subset N$ denote the set of farsighted players. The set F is allowed to be empty and is also allowed to contain all players. A pairwise myopic-farsighted improving path is a sequence of matchings that can emerge when farsighted players form or destroy matches based on the improvement the end matching offers them relative to the current matching while myopic players form or destroy matches based on the improvement the next matching in the sequence offers them relative to the current one.

Definition 1. Let $(M, W, \succ)$ be a marriage problem with set of farsighted players F. A *pairwise myopic-farsighted improving path* from a matching $\mu \in \mathcal{M}$ to a matching $\mu' \in \mathcal{M} \setminus \{\mu\}$ is a finite sequence of distinct matchings $\mu_0, \ldots, \mu_L$ with $\mu_0 = \mu$ and $\mu_L = \mu'$ such that for every $\ell \in \{0, \ldots, L-1\}$ either (i) or (ii) holds:

(i) $\mu_{\ell+1} = \mu_\ell - (m, w)$ for some $(m, w) \in M \times W$ such that

$$\begin{cases} \mu_{\ell+1}(m) \succ_m \mu_\ell(m) & \text{if } m \in M \setminus F, \\ \mu_L(m) \succ_m \mu_\ell(m) & \text{if } m \in F, \end{cases}$$

or

$$\begin{cases} \mu_{\ell+1}(w) \succ_w \mu_\ell(w) & \text{if } w \in W \setminus F, \\ \mu_L(w) \succ_w \mu_\ell(w) & \text{if } w \in F. \end{cases}$$

(ii) $\mu_{\ell+1} = \mu_\ell + (m, w)$ for some $(m, w) \in M \times W$ such that

$$\begin{cases} \mu_{\ell+1}(m) \succeq_m \mu_\ell(m) & \text{if } m \in M \setminus F, \\ \mu_L(m) \succeq_m \mu_\ell(m) & \text{if } m \in F, \end{cases}$$

and

$$\begin{cases} \mu_{\ell+1}(w) \succeq_w \mu_\ell(w) & \text{if } w \in W \setminus F, \\ \mu_L(w) \succeq_w \mu_\ell(w) & \text{if } w \in F, \end{cases}$$

with at least one of these preferences being strict.

Each matching in the sequence differs from the previous one in that either an existing match in the previous matching is destroyed like in case (i) or a new match is formed between a man and a woman that are not matched to one another in the previous matching as in case (ii). Notice that in the definition of a pairwise myopic-farsighted improving path we follow the approach used by Jackson and Wolinsky (1996) when defining pairwise stable networks as well as the approach used by Jackson and Watts (2002) when defining improving paths and assume that a pair of players can form a match between them if at least one player has a strict preference to do so.

If there exists a pairwise myopic-farsighted improving path from a matching μ to a matching μ', then we write $\mu \to \mu'$. The set of all matchings that can be reached from a matching $\mu \in \mathcal{M}$ by a pairwise myopic-farsighted improving path is denoted by $h(\mu)$, so

$$h(\mu) = \{\mu' \in \mathcal{M} \mid \mu \to \mu'\}.$$

The pairwise myopic-farsighted stable set results when we replace the conditions of internal and external stability in the vNM stable set as based on direct dominance by the conditions as based on the pairwise myopic-farsighted improving paths.

Definition 2. Let $(M, W, \succ)$ be a marriage problem with set of farsighted players F. A set of matchings $V \subset \mathcal{M}$ is a *pairwise myopic-farsighted stable set* if it satisfies:

(i) Internal stability: For every $\mu, \mu' \in V$, it holds that $\mu' \notin h(\mu)$.
(ii) External stability: For every $\mu \in \mathcal{M} \setminus V$, it holds that $h(\mu) \cap V \neq \emptyset$.

Condition (i) of Definition 2 corresponds to internal stability. For any two matchings μ and μ' in the pairwise myopic-farsighted stable set V it does not hold that $\mu \to \mu'$. Condition (ii) of Definition 2 expresses external stability. For every matching μ outside the pairwise myopic-farsighted stable set V it holds that there is $\mu' \in V$ such that $\mu \to \mu'$.

The next result shows that all matchings in a pairwise myopic-farsighted stable set are individually rational.

Theorem 1. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players F. Let V be a pairwise myopic-farsighted stable set and let $\mu \in V$. Then μ is individually rational.*

The proof of Theorem 1, as well as the other proofs, can be found in the Appendix. The proof is by contradiction. Suppose some not individually rational μ belongs to V. Then one can show that $\mu \notin h(\mu - (m, w))$, where (m, w) is a match in μ involving an unacceptable partner, irrespective of whether m and w are myopic or farsighted. At the same time it holds that

$\mu - (m, w) \in h(\mu)$, so by internal stability it holds that $\mu - (m, w) \notin V$, whereas by external stability there is $\mu' \in V$ such that $\mu' \in h(\mu - (m, w))$. We argue next that then also $\mu' \in h(\mu)$, leading to a violation of internal stability of V, a contradiction.

Consider the case where all players are myopic, so $F = \emptyset$. Definition 2 then boils down to the pairwise CP vNM set as defined in Herings et al. (2017). This set differs from the standard notion of a vNM set in three important ways. The standard definition, as used for instance in Ehlers (2007) and Wako (2010), violates the assumption of coalitional sovereignty, the property that an objecting coalition cannot determine the organization of players outside the coalition.[3] On the importance of respecting coalitional sovereignty, we also refer to Ray and Vohra (2015). Second, the standard definition of the vNM set is such that it does not take into account that a deviation by a coalition can be followed by further deviations. The pairwise CP vNM set follows the approach by van Deemen (1991) and Page and Wooders (2009), which takes into account that if a matching is blocked by some coalition and the resulting matching is not in the stable set itself, then further deviations will take place. This observation leads van Deemen (1991) to define the generalized stable set for abstract systems and Page and Wooders (2009) to define the stable set with respect to path dominance. Third, we restrict ourselves to deviations by single players and pairs. It follows from the results in Herings et al. (2017) that identical results are obtained when coalitions of arbitrary size are allowed to move.

The following result is stated as Theorem 1 in Herings et al. (2017). The proof is based on the fact that, at a core element, myopic pairwise improvements are impossible and on the result by Roth and Vande Vate (1990) claiming that, from any matching that does not belong to the core, a core element can be reached by a finite sequence of myopic pairwise improvements.

Theorem 2. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F = \emptyset$. A set of matchings is a pairwise myopic-farsighted stable set if and only if it is equal to the core.*

The pairwise myopic-farsighted stable set coincides with the core when all players are myopic. But, in general, under the presence of some farsighted players, does the pairwise myopic-farsighted stable set coincide with the core?

In Example 1, we consider the most basic situation of conflict between the objectives of men and women, where the most preferred woman of each man ranks him as the worst possible marriage partner.

Example 1. Consider the marriage problem $(M, W, \succ)$ with two men, $M = \{m_1, m_2\}$, and two women, $W = \{w_1, w_2\}$. Assume $F = W$, so the women are farsighted and the men are myopic. The preferences of men and women are diametrically opposed to each other:

m_1	m_2	w_1	w_2
w_1	w_2	m_2	m_1
w_2	w_1	m_1	m_2.

There are seven possible matchings, illustrated in Fig. 1. The man-optimal stable matching is equal to μ_6 and the woman-optimal stable matching to μ_7. Table 1 presents the matchings that can be reached from a given initial matching by means of a pairwise myopic-farsighted improving path. Notice that $\mu^{\mathrm{W}} \in h(\mu^{\mathrm{M}})$ because the farsighted woman w_1 first leaves m_1 to become

[3] Echenique and Oviedo (2006) and Konishi and Ünver (2006) study solution concepts for many-to-many matching problems that respect coalitional sovereignty.

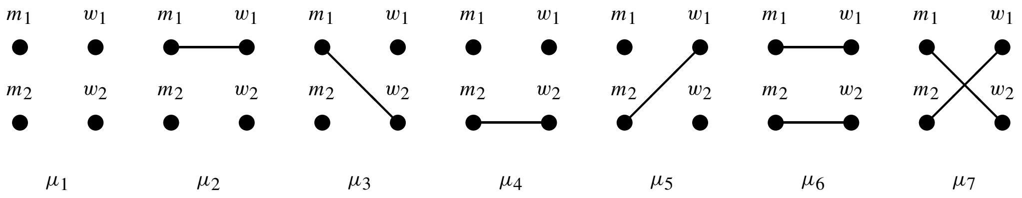

Fig. 1. All possible matchings in Example 1, where $\mu_6 = \mu^{\mathrm{M}}$ and $\mu_7 = \mu^{\mathrm{W}}$.

Table 1
The set of matchings that can be reached by a pairwise myopic-farsighted improving path in Example 1 when $F = \{w_1, w_2\}$. It holds that $\mu_6 = \mu^{\mathrm{M}}$ and $\mu_7 = \mu^{\mathrm{W}}$.

μ	$h(\mu)$					
μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7
μ_2				μ_5	μ_6	μ_7
μ_3	μ_2			μ_5	μ_6	μ_7
μ_4		μ_3			μ_6	μ_7
μ_5		μ_3	μ_4		μ_6	μ_7
μ_6						μ_7
μ_7		μ_3		μ_5		

single at the matching μ_4. Next, the farsighted woman w_2 marries m_1 to arrive at the matching μ_3. Next, w_1 marries m_2 to reach μ^{W}.

We argue that the set $V = \{\mu^{\mathrm{W}}\}$ is a pairwise myopic-farsighted stable set. The condition of internal stability in Definition 2 is satisfied since the set V is a singleton. Since for every $\mu \in \mathcal{M} \setminus \{\mu^{\mathrm{W}}\}$, it holds that $\mu^{\mathrm{W}} \in h(\mu)$, the condition of external stability in Definition 2 is satisfied as well. We have shown that $V = \{\mu^{\mathrm{W}}\}$ is a pairwise myopic-farsighted stable set.

It is not hard to demonstrate that there are no other pairwise myopic-farsighted stable sets. Let V be a pairwise myopic-farsighted stable set not equal to $\{\mu^{\mathrm{W}}\}$. The condition of internal stability in Definition 2 together with the fact that $\mu^{\mathrm{W}} \in h(\mu)$ for every $\mu \in \mathcal{M} \setminus \{\mu^{\mathrm{W}}\}$ implies that

$$\mu^{\mathrm{W}} \notin V.$$

To satisfy the condition of external stability in Definition 2, it should therefore hold that

$$\mu_3 \in V \text{ or } \mu_5 \in V.$$

Since $h(\mu_6) = \{\mu_7\}$ and $\mu_7 \notin V$, external stability implies that

$$\mu_6 \in V.$$

Since $\mu_6 \in h(\mu_3)$ and $\mu_6 \in h(\mu_5)$, we obtain a contradiction with internal stability.

We next analyze the case in which exactly one player is farsighted, say woman w_1. Table 1 remains almost unchanged, except that it is no longer the case that μ_3 belongs to $h(\mu_7)$. The argument that $V = \{\mu^{\mathrm{W}}\}$ is a pairwise myopic-farsighted stable set remains unaffected. The argument that there is no other pairwise myopic-farsighted stable set proceeds along the same lines as before and becomes slightly easier. △

Table 2
The set of matchings that can be reached by a pairwise myopic-farsighted improving path in Example 2 when $F = N$. It holds that $\mu_6 = \mu^{\mathrm{M}}$ and $\mu_7 = \mu^{\mathrm{W}}$.

μ	$h(\mu)$					
μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7
μ_2				μ_5	μ_6	μ_7
μ_3	μ_2				μ_6	μ_7
μ_4		μ_3			μ_6	μ_7
μ_5			μ_4		μ_6	μ_7
μ_6						μ_7
μ_7					μ_6	

Example 1 shows that in the most basic situation of conflict between the objectives of men and women, farsighted women are able to obtain their most preferred solution. Moreover, this is the unique prediction as made by the concept of the pairwise myopic-farsighted stable set. In fact, it is not even needed that both women are farsighted. Even if only one of them is farsighted, the woman-optimal stable matching results as the unique prediction. These findings are generalized in the Theorems 6 and 7 in the Appendix.

Example 2. We take the same primitives as in Example 1, but now vary the assumptions with respect to farsightedness.

The case where nobody is farsighted leads to a concept that is equivalent to the pairwise CP vNM set of Herings et al. (2017). It follows from Theorem 2 that the core is the unique pairwise myopic-farsighted stable set, so $V = \{\mu^{\mathrm{M}}, \mu^{\mathrm{W}}\}$.

We now turn to the case where all players are farsighted, $F = N$. Table 2 presents the matchings that can be reached from a given initial matching by means of a pairwise myopic-farsighted improving path. Since both μ_6 and μ_7 can be reached from any other matching, it follows that both $\{\mu_6\}$ and $\{\mu_7\}$ are pairwise myopic-farsighted stable sets. It is easily verified that there are no other pairwise myopic-farsighted stable sets. As in the case of completely myopic players, we obtain all core elements as the unique prediction, be it that these elements are predicted as singletons in the farsighted case. In Section 4, we generalize this finding to arbitrary matching problems where all players are farsighted.

The case where all players are farsighted with the exception of one player, say $F = \{m_1, w_1, w_2\}$, is very close to the situation where everyone is farsighted. Compared to Table 2, the only change is that $\mu_3 \in h(\mu_5)$ and $\mu_3 \in h(\mu_7)$. This will not affect the analysis and the conclusion that the woman-optimal stable matching and the man-optimal stable matching can be both sustained as singleton pairwise myopic-farsighted stable sets and there are no other pairwise myopic-farsighted stable sets remains.

The final case is where one player on each side is farsighted, say $F = \{m_1, w_1\}$. Table 3 presents the matchings that can be reached from a given initial matching by means of a pairwise myopic-farsighted improving path. Since both μ_6 and μ_7 can be reached from any other matching, it follows that both $\{\mu_6\}$ and $\{\mu_7\}$ are pairwise myopic-farsighted stable sets. It is easily verified that there are no other pairwise myopic-farsighted stable sets. The predictions are therefore identical to the case where all players are farsighted. △

Table 3
The set of matchings that can be reached by a pairwise myopic-farsighted improving path in Example 2 when $F = \{m_1, w_1\}$. It holds that $\mu_6 = \mu^{\mathrm{M}}$ and $\mu_7 = \mu^{\mathrm{W}}$.

μ	$h(\mu)$					
μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7
μ_2				μ_5	μ_6	μ_7
μ_3	μ_2				μ_6	μ_7
μ_4	μ_2	μ_3			μ_6	μ_7
μ_5	μ_2	μ_3	μ_4		μ_6	μ_7
μ_6						μ_7
μ_7	μ_2				μ_6	

Example 2 illustrates that any core element can be sustained in some pairwise myopic-farsighted stable set in case both sides of the markets are similar in terms of their degree of farsightedness as well as in the case where all players are farsighted except one.

In Examples 1 and 2, the number of farsighted women is at least as large as the number of farsighted men. The analysis for all other cases follows by symmetry arguments. For instance, if the only farsighted player is a man, then symmetry dictates that the unique pairwise myopic-farsighted stable set is the singleton containing the man-optimal stable matching.

4. The farsighted society

In this section we consider the case where all players are farsighted. Contrary to the case where all players are myopic, the myopic-farsighted stable set does not reduce to a concept that has been studied in the literature before. Still, it turns out to be helpful to give the definition of the vNM farsightedly stable set of Mauleon et al. (2011), which results when coalitions of arbitrary size are allowed to move and improvements are required to be strict for all players involved in a move.

Definition 3. Given a matching $\mu \in \mathcal{M}$, a coalition $S \in \mathcal{N}$ is able to *enforce* the matching $\mu' \in \mathcal{M}$ if the following conditions hold: (i) $\mu'(i) \notin \{\mu(i), i\}$ implies $\{i, \mu'(i)\} \subseteq S$ and (ii) $\mu'(i) = i \neq \mu(i)$ implies $\{i, \mu(i)\} \cap S \neq \emptyset$.

If, given some matching μ, coalition S can enforce matching μ', then any match in μ' that does not exist in μ should be between players in S. Moreover, if a match in μ is destroyed, then at least one of the two players involved in that match should belong to S.

Definition 4. Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F = N$. A *coalitional farsighted strictly improving path* from a matching $\mu \in \mathcal{M}$ to a matching $\mu' \in \mathcal{M} \setminus \{\mu\}$ is a finite sequence of distinct matchings $\mu_0, \ldots, \mu_L$ with $\mu_0 = \mu$ and $\mu_L = \mu'$ such that for every $\ell \in \{0, \ldots, L-1\}$ there is a coalition $S_\ell \in \mathcal{N}$ that can enforce $\mu_{\ell+1}$ from μ_ℓ and, for every $i \in S_\ell$, $\mu_L(i) \succ_i \mu_\ell(i)$.

Let some $\mu \in \mathcal{M}$ be given. The set of matchings $\mu' \in \mathcal{M}$ such that there is a coalitional farsighted strictly improving path from μ to μ' is denoted by $g(\mu)$.

Definition 5. Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F = N$. A set of matchings $V \subset \mathcal{M}$ is a *vNM farsightedly stable set* if it satisfies:

(i) Internal stability: For every $\mu, \mu' \in V$, it holds that $\mu' \notin g(\mu)$.
(ii) External stability: For every $\mu \in \mathcal{M} \setminus V$, it holds that $g(\mu) \cap V \neq \emptyset$.

Mauleon et al. (2011) show that the vNM farsightedly stable sets are characterized as all singletons that consist of a core element. The next result confirms that the same characterization applies to the pairwise myopic-farsighted stable set when all players are farsighted.

Theorem 3. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F = N$. A set of matchings is a pairwise myopic-farsighted stable set if and only if it is a singleton consisting of a core element.*

For the proof of Theorem 3, we show that in case $F = N$ a set of matchings V is a pairwise myopic-farsighted stable set if and only if it is a vNM farsightedly stable set. In other words, in case $F = N$ it does not make a difference whether arbitrary coalitions are allowed to move while requiring strict improvements or only singleton and pairs while requiring weak improvements with at least one player improving strictly. This result significantly improves upon the characterization result obtained in Mauleon et al. (2011) by showing that, with a completely different effectivity function, every singleton core element is also a pairwise myopic-farsighted stable set when all players are farsighted.

In a recent paper, Dutta and Vohra (2017) criticize the farsighted stable set introduced in Ray and Vohra (2015) because it does not fully capture the idea of optimal behavior embodied in backward induction. Coalitions involved in a farsighted objection are not required to make the, in a Pareto sense, most profitable moves that may be available to them. Consequently, when there are multiple continuation paths, the farsighted stable set can yield unreasonable predictions. Dutta and Vohra (2017) restrict coalitions to hold common, history-independent expectations that incorporate maximality regarding the continuation path and propose two related solution concepts: the rational expectations farsighted stable set (REFS) and the strong rational expectations farsighted stable set (SREFS).[4] Let us define both concepts formally for matching problems.

In order to deal simultaneously with the pairwise approach of the pairwise myopic-farsighted stable set and the coalitional approach of the vNM farsightedly stable set, we introduce the effectivity correspondence $E : \mathcal{M} \times \mathcal{M} \to 2^N$, where, for $\mu, \mu' \in \mathcal{M}$, the collection $E(\mu, \mu')$ consists of all the coalitions that can replace μ with μ'. The empty coalition is assumed to be able to replace μ with μ, i.e., to stay at μ. Under the pairwise approach, singletons can destroy their link, so in case $\mu' = \mu - (m, w)$ then both $\{m\}$ and $\{w\}$ are part of $E(\mu, \mu')$. Pairs can form a link, while simultaneously destroying the links with their existing partners, so in case $\mu' = \mu + (m, w)$ then $\{m, w\}$ belongs to $E(\mu, \mu')$. Under the coalitional approach, the conditions under which $S \in E(\mu, \mu')$ are given in Definition 3.

Dutta and Vohra (2017) modify Jordan's (2006) approach to farsightedness by interpreting an

[4] Dutta and Vartiainen (2020) have extended the rational expectation stable set of Dutta and Vohra (2017) by allowing coalitions to condition their behavior on the history of states.

expectation as describing the expected transition from one state to the next.[5] More formally, for a matching $\mu \in \mathcal{M}$, we define $\Gamma(\mu) = (\gamma(\mu), T(\mu))$, where $\gamma(\mu)$ is the matching that players expect to result after matching μ by a move of coalition $T(\mu) \in E(\mu, \gamma(\mu))$. If $\gamma(\mu) = \mu$, then $T(\mu) = \emptyset$, meaning that no coalition is expected to change μ. Such a matching is said to be *stationary.* An *expectation* is a function $\Gamma = (\gamma, T) : \mathcal{M} \to \mathcal{M} \times 2^N$. The set of stationary matchings is denoted by $\Sigma(\Gamma)$.

Given an expectation function $\Gamma = (\gamma, T)$, let γ^k denote the k-fold composition of γ. We define the function $\Gamma^k : \mathcal{M} \to \mathcal{M} \times 2^N$ by defining $\Gamma^k(\mu) = \Gamma(\gamma^{k-1}(\mu))$.

An expectation Γ is *absorbing* if for every $\mu \in \mathcal{M}$, there exists $k \in \mathbb{N}$ such that $\gamma^k(\mu)$ is stationary. For an absorbing expectation Γ, the function $\gamma^* : \mathcal{M} \to \mathcal{M}$ is obtained by defining $\gamma^*(\mu) = \gamma^k(\mu)$, where $k \in \mathbb{N}$ is such that $\gamma^k(\mu)$ is stationary.

Definition 6. An absorbing expectation Γ is *rational* if it has the following three properties:

(I) If $\gamma(\mu) = \mu$, then, for every $\mu' \in \mathcal{M}$, for every $S \in E(\mu, \mu')$, it holds for some $i \in S$ that $\mu \succeq_i \gamma^*(\mu')$.

(E) If $\gamma(\mu) \neq \mu$, then $(\mu, \gamma(\mu), \ldots, \gamma^L(\mu) = \gamma^*(\mu))$ is such that, for every $\ell \in \{0, \ldots, L - 1\}$, for every $i \in T(\gamma^\ell(\mu))$, $\gamma^L(\mu) \succ_i \gamma^\ell(\mu)$.

(M) If $\gamma(\mu) \neq \mu$, then for every $\mu' \in \mathcal{M}$ such that $T(\mu) \in E(\mu, \mu')$, it holds for some $i \in T(\mu)$ that $\gamma^*(\mu) \succeq_i \gamma^*(\mu')$.

The set of stationary matchings $\Sigma(\Gamma)$ of a rational absorbing expectation Γ is a *rational expectations farsighted stable set* (REFS).

Condition (I) requires that if μ is stationary, then from μ no coalition is effective in making a profitable move consistent with γ. Condition (I) is weaker than farsighted internal stability since it requires internal stability only with respect to those farsighted objections that are consistent with the common expectation Γ. Condition (E) requires that if μ is a non-stationary state, then there is a farsighted strictly improving path from μ to a stationary matching $\gamma^L(\mu)$ that is consistent with Γ. It follows from the definition in (E) that $\gamma^{L-1}(\mu) \neq \gamma^L(\mu)$, so L is the first period that a stationary matching is reached along the path. Condition (E) is stronger than farsighted external stability since it requires that the path is consistent with the common expectation Γ. We can simplify (E) to the equivalent requirement that if $\gamma(\mu) \neq \mu$, then, for every $i \in T(\mu)$, $\gamma^*(\mu) \succ_i \mu$. Condition (M) is the *maximality* condition. It requires that if $T(\mu)$ is expected to move from μ to $\gamma(\mu)$, then the members of $T(\mu)$ cannot strictly improve in the long run by making another move.

A stricter version of maximality is *strong maximality*. Strong maximality rules out deviations by any coalition that intersects the coalition stipulated to move.

(M') If $\gamma(\mu) \neq \mu$, then, for every $\mu' \in \mathcal{M}$, for every $S \in E(\mu, \mu')$ such that $S \cap T(\mu) \neq \emptyset$, it holds for some $i \in S$ that $\gamma^*(\mu) \succeq_i \gamma^*(\mu')$.

An expectation Γ satisfying (I), (E), and (M') is a *strong rational expectation*. The set of stationary matchings $\Sigma(\Gamma)$ of a strong rational absorbing expectation Γ is said to be a *strong rational expectations farsighted stable set* (SREFS).

[5] Jordan (2006) expresses farsightedness in terms of commonly held consistent expectations regarding the final outcome from any state. A stationary state of ϕ is $\mu \in \mathcal{M}$ such that $\phi(\mu) = \mu$. For matching problems, an expectation is then a function $\phi : \mathcal{M} \to \mathcal{M}$ such that for every $\mu \in \mathcal{M}$, $\phi(\phi(\mu)) = \phi(\mu)$.

Dutta and Vohra (2017) show that REFS and SREFS can be very different from farsighted stable sets as defined in Ray and Vohra (2015). However, one interesting case in which both solution concepts coincide with a farsighted stable set is when all elements of the farsighted stable set lead to the same payoff vector, see Theorem 1 of Dutta and Vohra (2017).[6]

We use the terminology pairwise REFS and pairwise SREFS when the effectivity function only allows for moves by singletons and pairs and we refer to coalitional REFS and coalitional SREFS when coalitions of arbitrary size can be effective. It is easily verified that the proof of Theorem 3 remains valid when we replace weak improvements by strict improvements. Theorem 1 of Dutta and Vohra (2017) applies and we obtain the following result.

Theorem 4. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F = N$. A set of matchings is a pairwise myopic-farsighted stable set if and only if it is a pairwise (coalitional) REFS if and only if it is a pairwise (coalitional) SREFS.*

Recently, Ray and Vohra (2019) have incorporated the strictest variant of maximality, called absolute maximality. Absolute maximality asks for immunity to all deviations, not just by the coalition that moves or by those coalitions that intersect the one that moves. Apart from the strengthening of maximality, Ray and Vohra (2019) also allow for history-dependent expectations. They refer to a history-dependent expectation as a negotiation process. Let us define this concept formally for matching problems.

A *farsighted strictly improving path* from a matching $\mu \in \mathcal{M}$ to a matching $\mu' \in \mathcal{M} \setminus \{\mu\}$ is a finite sequence of distinct matchings $\mu_0, \ldots, \mu_L$ with $\mu_0 = \mu$ and $\mu_L = \mu'$ such that for every $\ell \in \{0, \ldots, L-1\}$ there is a coalition $S_\ell \in E(\mu_\ell, \mu_{\ell+1})$ such that, for every $i \in S_\ell$, $\mu_L(i) \succ_i \mu_\ell(i)$. The set of matchings $\mu' \in \mathcal{M}$ such that there is a farsighted strictly improving path from μ to μ' is denoted by $f(\mu)$. A set of matchings $V \subset \mathcal{M}$ is a *farsighted stable set* if it satisfies internal and external stability with respect to f.

A history α is a finite sequence of matchings along with coalitions that are effective in making the transitions. If there is no change of matching, the empty coalition is recorded. An initial history is just a single matching. Let $\mu(\alpha)$ be the last matching in history α. A negotiation process Γ is such that for each history α, $\Gamma(\alpha) = (\gamma(\alpha), T(\alpha))$, where $T(\alpha) \in E(\mu(\alpha), \gamma(\alpha))$. Moreover, if $\gamma(\alpha) = \mu(\alpha)$, then it holds that $T(\alpha) = \emptyset$. Given any history α, Γ induces a continuation chain. A matching μ is *stationary* if, at any α with $\mu(\alpha) = \mu$ it holds that $\gamma(\alpha) = \mu$. That is, once at μ, the continuation chain displays μ forever. The set of stationary matchings is denoted by $\Sigma(\Gamma)$.

The negotiation process Γ is *absorbing* if after every history the continuation chain ends in a stationary matching. For an absorbing negotiation process Γ, for every history α, let $\gamma^*(\alpha)$ denote the stationary matching reached from α.

The next definition is based on Section 4 of Ray and Vohra (2019), where farsighted dominance of the negotiation process is required at every conceivable history.

Definition 7. A farsighted stable set V is *absolutely maximal* if $V = \Sigma(\Gamma)$, where Γ is an absorbing negotiation process such that:

(I) If $\gamma^*(\alpha) = \mu(\alpha)$, then, for every $\mu' \in \mathcal{M}$, for every $S \in E(\mu(\alpha), \mu')$, it holds for some $i \in S$ that $\mu(\alpha) \succeq_i \gamma^*(\alpha, S, \mu')$.

[6] Bloch and van den Nouweland (2020) analyze farsighted stable sets in abstract systems when agents have heterogeneous expectations over the dominance paths. They show that any singleton farsighted stable set with common expectations is a farsighted stable set with heterogeneous expectations.

(E) If $\gamma^*(\alpha) \neq \mu(\alpha)$, then, for every $i \in T(\alpha)$, $\gamma^*(\alpha) \succ_i \mu(\alpha)$.

(M*) If $\gamma^*(\alpha) \neq \mu(\alpha)$, then, for every $\mu' \in \mathcal{M}$, for every $S \in E(\mu(\alpha), \mu')$, it holds for some $i \in S$ that $\mu(\alpha) \succeq_i \gamma^*(\alpha, S, \mu')$.

Conditions (I) and (M*) together require that there is no history at which a coalition S can deviate an obtain a higher long-run payoff for each of its members. Proposition 1 in Ray and Vohra (2019) states that any farsighted stable set with a single payoff vector is absolutely maximal. Recalling that the proof of Theorem 3 remains valid when we replace weak improvements by strict improvements, we obtain the following result.

Theorem 5. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F = N$. A set of matchings is a pairwise myopic-farsighted stable set if and only if it is absolutely maximal.*

5. The heterogeneous society

Several results in the matching literature point towards the core as the set of reasonable outcomes, but are not able to discriminate between different core elements. Does the introduction of heterogeneity in terms of farsightedness among the two sides of the market allow us to discriminate between different core elements? A closely related issue is whether the most farsighted side is able to obtain its optimal stable matching. For instance, is it always possible to reach the woman-optimal stable matching μ^{W} from any matching $\mu \neq \mu^{\mathrm{W}}$ by means of a pairwise myopic-farsighted improving path in case the woman side is more farsighted than the man side? The answer is affirmative under certain conditions.

Example 3. Consider the marriage problem $(M, W, \succ)$, which corresponds to Example 2.31 of Roth and Sotomayor (1990) with the roles of men and women reversed. It holds that $M = \{m_1, m_2, m_3\}$ and $W = \{w_1, w_2, w_3\}$. Assume $F = \{w_1, w_2, w_3\}$, so all women are farsighted and all men are myopic. The preferences of the players are as follows.

m_1	m_2	m_3	w_1	w_2	w_3
w_1	w_3	w_1	m_2	m_1	m_1
w_2	w_1	w_2	m_1	m_2	m_2
w_3	w_2	w_3	m_3	m_3	m_3.

By applying the deferred acceptance algorithm of Gale and Shapley (1962), it can be easily verified that the woman-optimal stable matching is equal to

$$\begin{aligned} \mu^{\mathrm{W}}(m_1) &= w_1, \\ \mu^{\mathrm{W}}(m_2) &= w_3, \\ \mu^{\mathrm{W}}(m_3) &= w_2. \end{aligned}$$

The matching $\overline{\mu}$ defined by

$$\begin{aligned} \overline{\mu}(m_1) &= w_3, \\ \overline{\mu}(m_2) &= w_1, \\ \overline{\mu}(m_3) &= w_2, \end{aligned}$$

is strictly preferred by w_1 and w_3 to μ^{W} and does not make a difference for w_2. However, the pair (m_1, w_2) can block $\overline{\mu}$, so $\overline{\mu}$ does not belong to the core.

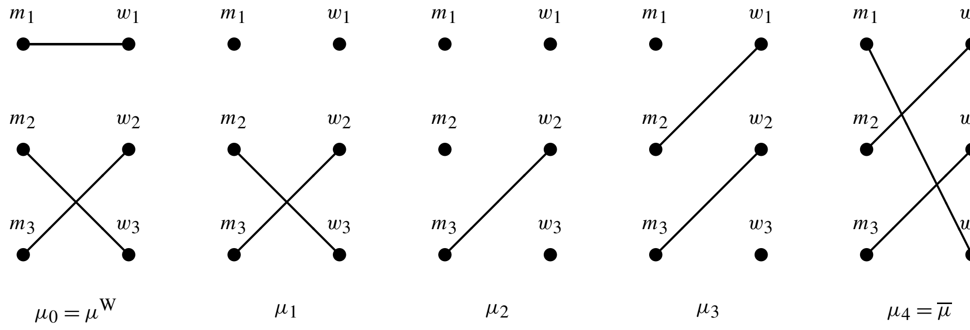

Fig. 2. Myopic-farsighted improving path in Example 3 to move from μ^{W} to $\overline{\mu}$.

We show that $\overline{\mu} \in h(\mu^{\mathrm{W}})$, so it is possible that farsighted women leave the woman-optimal stable matching by means of a pairwise myopic-farsighted improving path. To see this, consider the pairwise myopic-farsighted improving path $\mu_0, \ldots, \mu_4$ with $\mu_0 = \mu^{\mathrm{W}}$ and $\mu_4 = \overline{\mu}$, where $\mu_1 = \mu_0 - (m_1, w_1)$, $\mu_2 = \mu_1 - (m_2, w_3)$, $\mu_3 = \mu_2 + (m_2, w_1)$, and $\mu_4 = \mu_3 + (m_1, w_3)$. This pairwise myopic-farsighted improving path is illustrated in Fig. 2.

The move to μ_1 is initiated by w_1 who is farsighted and therefore wants to destroy her match with m_1 in the anticipation of ending up in a match with m_2. Similarly, the move from μ_1 to μ_2 is initiated by w_3 who is farsighted and is willing to divorce m_2 in the expectation of being matched with m_1. The transition to $\mu_4 = \overline{\mu}$ is completed by the subsequent marriages of the single players m_2 and w_1 and the single players m_1 and w_3. $\triangle$

The non-core element $\overline{\mu}$ in Example 3 is strictly preferred by two out of three women to μ^{W}, whereas the third woman is indifferent. The next example shows that $\{\overline{\mu}\}$ is a pairwise myopic-farsighted stable set. This example is important since it shows that in heterogeneous societies, it is possible for the farsighted side to do better than in any core element. However, as shown in Sections 3 and 4, this is not possible in the homogeneous case where all players are either myopic or farsighted. In homogeneous societies a pairwise myopic-farsighted stable set only contains core elements.

Example 4. We take the same primitives as in Example 3. In Example 3 it has been shown that $\overline{\mu} \in h(\mu^{\mathrm{W}})$. We show now that for every $\mu \in \mathcal{M} \setminus \{\overline{\mu}\}$ it holds that $\overline{\mu} \in h(\mu)$. It then follows that $\{\overline{\mu}\}$ is a pairwise myopic-farsighted stable set.

Let some $\mu \in \mathcal{M} \setminus \{\overline{\mu}\}$ be given such that $\mu(w_1) \neq m_1$. We define $\mu_0 = \mu$ and $\mu_1 = \mu_0 + (m_1, w_1)$. Notice that $\overline{\mu}(w_1) = m_2 \succeq_{w_1} \mu_0(w_1)$ and $w_1 \succ_{m_1} \mu_0(m_1)$, so m_1 and w_1 are willing to form the match and reach μ_1. Next, we take $\mu_2 = \mu_1 - (m_1, w_1)$. The purpose of this step is to make m_1 single. Woman w_1 is willing to do so since $\overline{\mu}(w_1) = m_2 \succ_{w_1} m_1 = \mu_1(w_1)$. Next, we make m_3 single by setting $\mu_3 = \mu_2 + (m_3, w_1)$ and $\mu_4 = \mu_3 - (m_3, w_1)$. It can be verified that m_3 and w_1 are willing to form the match at μ_2 and w_1 is willing to destroy this match next. Notice that $\mu_4(m_2) \neq w_1$, whereas m_1 and m_3 are single at μ_4. We now define $\mu_5 = \mu_4 + (m_1, w_3)$, $\mu_6 = \mu_5 + (m_2, w_1)$, and $\mu_7 = \mu_6 + (m_3, w_2)$. It is easily verified that the latter three moves involve strict improvements for the marrying couple. Since $\mu_7 = \overline{\mu}$, we have shown that $\mu_0, \ldots, \mu_7$ is a pairwise myopic-farsighted improving path from μ to $\overline{\mu}$.

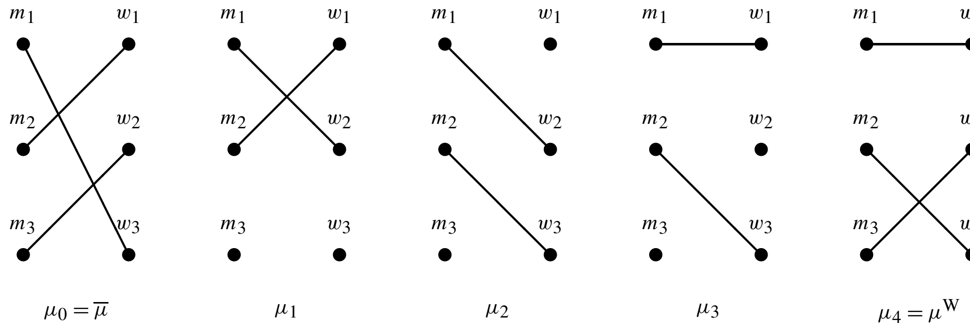

Fig. 3. Myopic-farsighted improving path in Example 3 to move from $\overline{\mu}$ to μ^{W}.

Now let some $\mu \in \mathcal{M}$ be given such that $\mu(w_1) = m_1$. We can use exactly the same argument as in the previous paragraph, only skipping the very first step since now m_1 and w_1 are already linked. △

In Example 5, we show that $\mu^{\mathrm{W}} \in h(\overline{\mu})$ despite the fact that all women are farsighted, $\overline{\mu}$ is strictly preferred by two out of three women to μ^{W}, whereas the third woman is indifferent.

Example 5. We take the same primitives as in Example 4. We show that $\mu^{\mathrm{W}} \in h(\overline{\mu})$. So even though the three women are farsighted, it is possible that they move from $\overline{\mu}$ to a matching that none of them strictly prefers and that is strictly worse for women w_1 and w_3. To verify this statement, consider the pairwise myopic-farsighted improving path $\mu_0, \ldots, \mu_4$ with $\mu_0 = \overline{\mu}$ and $\mu_4 = \mu^{\mathrm{W}}$, where $\mu_1 = \mu_0 + (m_1, w_2)$, $\mu_2 = \mu_1 + (m_2, w_3)$, $\mu_3 = \mu_2 + (m_1, w_1)$, and $\mu_4 = \mu_3 + (m_3, w_2)$. This pairwise myopic-farsighted improving path is illustrated in Fig. 3.

Since m_1 is myopic and w_2 is farsighted, it is possible to establish a match between them as man m_1 strictly prefers $w_2 = \mu_1(m_1)$ to $w_3 = \mu_0(m_1)$ and woman w_2 is indifferent since $\mu_0(w_2) = \mu_4(w_2)$. Since at μ_1 woman w_3 has become single, she is willing to form a match with m_2, her partner in the end matching of the sequence, moving from μ_1 to μ_2. Since at μ_2 woman w_1 is single, she is willing to team up with m_1, her partner in the end matching of the sequence, leading to the matching μ_3. Woman w_2 has become single at μ_3 and marries m_3 in order to move to the end matching μ_4. The men are myopically improving in each step of the sequence. △

Moreover, under certain conditions guaranteeing that the woman side is more farsighted than the man side, it can be shown in general that $\{\mu^{\mathrm{W}}\}$ is a pairwise myopic-farsighted stable set. In order to guarantee that one side is more farsighted than the other side, two cases are formally analyzed in the Appendix. First, we assume all men to be myopic, whereas women can be either myopic or farsighted. Next, we assume all women to be farsighted, whereas any given man can be either myopic or farsighted.

For every $m \in M$, let $w^*(m) \in W \cup \{m\}$ denote the top choice of m, so $w^*(m) \succeq_m w$ for every $w \in W$ and $w^*(m) \succeq_m m$. We say that *top-ranked women are farsighted* if for every $m \in M$ it holds that $w^*(m) \in F \cup \{m\}$. When top-ranked women are farsighted, then the top choice of every man is either a farsighted woman or to remain single. Intuitively this corresponds to the requirement that the farsighted side is desirable. It is automatically satisfied when all women are farsighted.

Theorem 6 in the Appendix states that in a marriage problem with set of farsighted players $F \subset W$ being such that top-ranked women are farsighted, $\{\mu^{\mathrm{W}}\}$ is a pairwise myopic-farsighted stable set. Thus, if the farsighted side is desirable, then that side is able to induce its optimal stable matching. Second, Theorem 7 in the Appendix states that in a marriage problem with set of farsighted players $F \supset W$, $\{\mu^{\mathrm{W}}\}$ is a pairwise myopic-farsighted stable set. Thus, in case all women are farsighted, they can achieve the woman-optimal stable matching, irrespective of whether men are myopic or farsighted. The fact that we can reach a designated core element is striking, certainly when taking into account that the celebrated result of Roth and Vande Vate (1990) only shows that some core element can always be reached by means of a pairwise myopic improving path from any initial matching.

6. Concluding discussion

We propose the pairwise myopic-farsighted stable set to study stable matchings under the assumption that players can be both myopic and farsighted. For the special case where all players are myopic, our concept predicts the set of matchings in the core. When all players are farsighted, we provide the characterization of pairwise myopic-farsighted stable sets: a set of matchings is a pairwise myopic-farsighted stable set if and only if it is a singleton consisting of a core element. This result confirms the characterization obtained by Mauleon et al. (2011) with a completely different effectivity function and provides a new special case where the farsighted stable set is absolutely maximal (Ray and Vohra, 2019) and coincides with the Strong Rational Expectations Farsighted Stable Set (Dutta and Vohra, 2017). In heterogeneous societies where myopic and farsighted players interact, matchings outside the core can be stable and the most farsighted side can always obtain its optimal stable matching.

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Appendix

Proof of Theorem 1. Suppose μ is not individually rational. Then there is $m \in M$ and $w \in W$ such that $\mu(m) = w$, $m \succ_m w$ or $w \succ_w m$. Without loss of generality, assume $m \succ_m w$. We show first that $\mu \notin h(\mu - (m, w))$ by distinguishing two cases.

Case 1. $m \in M \setminus F$.

Suppose $\mu_0, \ldots, \mu_L$ is a pairwise myopic-farsighted improving path from $\mu_0 = \mu - (m, w)$ to $\mu_L = \mu$. We prove by induction that, for every $\ell = 0, \ldots, L$, $\mu_\ell(m) \succeq_m m$. Since $\mu_0(m) = m$, it clearly holds that $\mu_0(m) \succeq_m m$. Assume, for some $\ell = 0, \ldots, L-1$, it holds that $\mu_\ell(m) \succeq_m m$. We show that $\mu_{\ell+1}(m) \succeq_m m$. We distinguish between three cases when moving from μ_ℓ to

$\mu_{\ell+1}$: a. m creates a match with a woman different from $\mu_\ell(m)$; b. m becomes single; c. m is not affected. In Case a it holds that $\mu_{\ell+1}(m) \succ_m \mu_\ell(m) \succeq_m m$, in Case b that $\mu_{\ell+1}(m) = m \succeq_m m$, and in Case c that $\mu_{\ell+1}(m) = \mu_\ell(m) \succeq_m m$. We have shown that $\mu_{\ell+1}(m) \succeq_m m$. It follows that $\mu_L(m) \succeq_m m$, yielding a contradiction to $m \succ_m \mu_L(m) = w$. Consequently, there is no pairwise myopic-farsighted improving path from $\mu - (m, w)$ to μ if $m \in M \setminus F$.

Case 2. $m \in F$.
Suppose $\mu_0, \ldots, \mu_L$ is a pairwise myopic-farsighted improving path from $\mu_0 = \mu - (m, w)$ to $\mu_L = \mu$. Since $\mu - (m, w) \succ_m \mu$, it is not possible that m is involved in the formation of a match with a woman when moving from μ_0 to μ_1, so $\mu_1(m) = m$. By repeating this argument, we find that $\mu_1(m) = \cdots = \mu_L(m) = m$, a contradiction to $\mu_L(m) = w$. Consequently, there is no pairwise myopic-farsighted improving path from $\mu - (m, w)$ to μ if $m \in F$.

Combining Cases 1 and 2, we have shown that $\mu \notin h(\mu - (m, w))$.

On the other hand, $\mu - (m, w) \in h(\mu)$, so by $\mu \in V$ and internal stability of V it follows that $\mu - (m, w) \notin V$. By external stability of V it holds that there is $\mu' \in h(\mu - (m, w))$ such that $\mu' \in V$.

Let $\mu_1, \ldots, \mu_L$ with $\mu_1 = \mu - (m, w)$ and $\mu_L = \mu'$ be a pairwise myopic-farsighted improving path from $\mu - (m, w)$ to μ'. We distinguish between two cases.

Case 1. $m \in M \setminus F$.
It holds that $\mu - (m, w) \succ_m \mu$. Therefore, it follows that $\mu_0, \mu_1, \ldots, \mu_L$ with $\mu_0 = \mu$ is a pairwise myopic-farsighted improving path from μ to μ' and therefore $\mu' \in h(\mu)$. This contradicts the fact that V is internally stable.

Case 2. $m \in F$.
Since $\mu' \in h(\mu - (m, w))$ and $m \in F$, it follows as before that $\mu'(m) \succeq_m m$. Therefore it holds that $\mu_0, \mu_1, \ldots, \mu_L$ with $\mu_0 = \mu$ is a pairwise myopic-farsighted improving path from μ to μ' and therefore $\mu' \in h(\mu)$. This contradicts the fact that V is internally stable. □

Proof of Theorem 3.
Step 1. Let $\mu, \mu' \in M$ be such that μ and μ' are individually rational. If $\mu' \in g(\mu)$, then $\mu' \in h(\mu)$.
Let $\mu_0, \ldots, \mu_L$ be such that $\mu_0 = \mu$, $\mu_L = \mu'$, and, for every $\ell \in \{0, \ldots, L-1\}$, there is a coalition $S_\ell \in N$ that can enforce $\mu_{\ell+1}$ from μ_ℓ and, for every $i \in S_\ell$, $\mu_L(i) \succ_i \mu_\ell(i)$.

We start by showing that if, for some $i \in N$, $\mu_L(i) = i$, then, for every $\ell \in \{0, \ldots, L-1\}$, $i \notin S_\ell$. Let $i \in N$ be such that $\mu_L(i) = i$. Since $\mu_0(i) \succeq_i i$, it follows that $i \notin S_0$ and $\mu_1(i) = \mu_0(i)$ or $\mu_1(i) = i$, so $\mu_1(i) \succeq_i i$. By repeating this argument, we obtain the desired conclusion that for every $\ell \in \{0, \ldots, L-1\}$, $i \notin S_\ell$. It follows that, for every $i \in S_\ell$, $\mu_L(i) \succ_i i$.

Let some $\ell \in \{0, \ldots, L-1\}$ be given. We define

$$\begin{array}{rcl} D_\ell^0 & = & \{m \in S_\ell \mid \mu_\ell(m) \in W \setminus S_\ell\}, \\ D_\ell^1 & = & \{w \in S_\ell \mid \mu_\ell(w) \in M \setminus S_\ell\}, \\ D_\ell^2 & = & \{w \in S_\ell \mid \mu_\ell(w) \in S_\ell\}, \end{array}$$

as the men in S_ℓ who at μ_ℓ destroy their match with a woman outside S_ℓ, the women in S_ℓ who at μ_ℓ destroy their match with a man outside S_ℓ, and the women in S_ℓ who at μ_ℓ destroy their match with a man inside S_ℓ. Let $D_\ell = D_\ell^0 \cup D_\ell^1 \cup D_\ell^2$. The cardinality of D_ℓ is denoted by d_ℓ. We define the set $A_\ell = \{m \in S_\ell \mid \mu_{\ell+1}(m) \in W\}$ as the set of men that form a match when going from μ_ℓ to $\mu_{\ell+1}$. Note that enforceability of $\mu_{\ell+1}$ implies that $\mu_{\ell+1}(m) \in S_\ell$. The cardinality of A_ℓ is denoted by a_ℓ. Take an arbitrary order of the individuals in D_ℓ, say $i_0, \ldots, i_{d_\ell - 1}$, and an arbitrary order of the men in A_ℓ, say $m_0, \ldots, m_{a_\ell - 1}$. We define $\nu_{\ell,0} = \mu_\ell$. For $k \in \{0, \ldots, d_\ell - 1\}$, we define the matching $\nu_{\ell,k+1} = \nu_{\ell,k} - (i_k, \mu_\ell(i_k))$. For $k \in \{0, \ldots, a_\ell - 1\}$, we define the matching $\nu_{\ell,d_\ell+k+1} = \nu_{\ell,d_\ell+k} + (m_k, \mu_{\ell+1}(m_k))$.

Consider the sequence of matchings

$$\nu_{0,0}, \ldots, \nu_{0,d_0+a_0-1}, \nu_{1,0}, \ldots, \nu_{L-1,0}, \ldots, \nu_{L-1,d_{L-1}+a_{L-1}-1}, \nu_{L-1,d_{L-1}+a_{L-1}}.$$

Notice that $\nu_{L-1,d_{L-1}+a_{L-1}} = \mu_L$. We argue that this sequence is a pairwise myopic-farsighted improving path from $\nu_{0,0} = \mu$ to $\nu_{L-1,d_{L-1}+a_{L-1}} = \mu'$.

Let some $\ell \in \{0, \ldots, L-1\}$ and some $k \in \{0, \ldots, d_\ell - 1\}$ be given. It holds that $\nu_{\ell,k+1} = \nu_{\ell,k} - (i_k, \mu_\ell(i_k))$ and $\mu_L(i_k) \succ_{i_k} \mu_\ell(i_k) = \nu_{\ell,k}(i_k)$.

Let some $\ell \in \{0, \ldots, L-1\}$ and some $k \in \{0, \ldots, a_\ell - 1\}$ be given. For $w_k = \mu_{\ell+1}(m_k)$, it holds that $\nu_{\ell,k+1} = \nu_{\ell,k} + (m_k, w_k)$. Moreover, we have $\mu_L(m_k) \succ_{m_k} \mu_\ell(m_k)$, $\mu_L(m_k) \succ_{m_k} m_k$, $\mu_L(w_k) \succ_{w_k} \mu_\ell(w_k)$, and $\mu_L(w_k) \succ_{w_k} w_k$. It also holds that $\nu_{\ell,k}(m_k) = \mu_\ell(m_k)$ or $\nu_{\ell,k}(m_k) = m_k$, and $\nu_{\ell,k}(w_k) = \mu_\ell(w_k)$ or $\nu_{\ell,k}(w_k) = w_k$. In all cases, we find that $\mu_L(m_k) \succ_{m_k} \nu_{\ell,k}(m_k)$ and $\mu_L(w_k) \succ_{w_k} \nu_{\ell,k}(w_k)$.

We have shown that $\mu' \in h(\mu)$.

Step 2. Let $\mu, \mu' \in M$ be such that μ' is individually rational. If $\mu' \in h(\mu)$, then $\mu' \in g(\mu)$.

Let $m, w \in N$ be such that $\mu(m) = w$. We show that $\mu'(m) \succ_m \mu(m)$ or $\mu'(w) \succ_w \mu(w)$ or $\mu'(m) = w$. Suppose $\mu(m) \succeq_m \mu'(m)$, $\mu(w) \succeq_w \mu'(w)$, and $\mu'(m) \neq w$. Since $\mu(m) \neq \mu'(m)$ and $\mu(w) \neq \mu'(w)$, it follows that $\mu(m) \succ_m \mu'(m)$ and $\mu(w) \succ_w \mu'(w)$. Let $\mu_0, \ldots, \mu_L$ with $\mu_0 = \mu$ and $\mu_L = \mu'$ be a pairwise myopic-farsighted improving path from μ to μ'. It follows immediately that m and w cannot be involved in the move from μ_0 to μ_1, so $\mu_1(m) = w$. Repeating this argument, it follows that $\mu_L(m) = w$, leading to a contradiction since $\mu_L(m) = \mu'(m) \neq w$.

We now construct a coalitional farsighted strictly improving path ν_0, ν_1, ν_2 from $\nu_0 = \mu$ to $\nu_2 = \mu'$. For each pair $m, w \in N$ such that $\mu(m) = w \neq \mu'(m)$, select m if $\mu'(m) \succ_m \mu(m)$ and select w otherwise. By the result of the previous paragraph, we have $\mu'(w) \succ_w \mu(w)$ in the latter case. Let N^1 be the, possibly empty, set of selected players. We define $\nu_0 = \mu$ and we let ν_1 be the matching obtained when all players in N^1 destroy their match. Notice that the coalition N^1 can enforce ν_1 and, for every $i \in N^1$,

$$\nu_2(i) = \mu'(i) \succ_i \mu(i).$$

At ν_1, every $m \in M$ is single or matched to $\mu'(m)$.

Let

$$M^2 = \{m \in M \mid \mu'(m) \neq \nu_1(m)\}$$

be the set of men that are married at μ' to a woman different from $\nu_1(m)$. By the construction of ν_1, it holds that all these men are single. Let $W^2 = \mu'(M^2)$ be the matches of the men in M^2 at μ'. By the construction of ν_1, all women in W^2 are single at ν_1. Since μ' is individually rational, it holds that

$$\mu'(m) \succ_m m = \nu_1(m), \quad m \in M^2,$$
$$\mu'(w) \succ_w w = \nu_1(w), \quad w \in W^2.$$

The coalition $M^2 \cup W^2$ can enforce $\nu_2 = \mu'$ by forming the remaining marriages and all coalition members are strictly better off by doing so. We have shown that $\mu' \in g(\mu)$.

Step 3. V is a pairwise myopic-farsighted stable set if and only if it is a singleton consisting of a core element.

$\Rightarrow$
By Theorem 1, it holds that every matching in V is individually rational. It now follows from Steps 1 and 2 that V is a vNM farsightedly stable set and Theorem 2 of Mauleon et al. (2011) now implies that V is a singleton consisting of a core element.
$\Leftarrow$
Let μ' be a core element. It suffices to show that, for every $\mu \in M \setminus \{\mu'\}$, it holds that $\mu' \in h(\mu)$. Let some $\mu \in M \setminus \{\mu'\}$ be given. We first remove any matches from μ that violate individual rationality. We define the sets M^1 and W^1 by

$$\begin{aligned} M^1 &= \{m \in M \mid m \succ_m \mu(m)\}, \\ W^1 &= \{w \in W \mid \mu(w) \notin M^1 \text{ and } w \succ_w \mu(w)\}. \end{aligned}$$

The cardinality of M^1 is denoted by k^1 and the cardinality of W^1 by k^2. Take an arbitrary order of the men in M^1, say $m_0, \dots, m_{k^1-1}$, and an arbitrary order of the women in W^1, say $w_0, \dots, w_{k^2-1}$. Define $\mu_0 = \mu$. For $\ell \in \{0, \dots, k^1 - 1\}$, we define the matching $\mu_{\ell+1} = \mu_\ell - (m_\ell, \mu(m_\ell))$. For $\ell \in \{0, \dots, k^2 - 1\}$, we define the matching $\mu_{k^1+\ell+1} = \mu_{k^1+\ell} - (w_\ell, \mu(w_\ell))$. It holds that the matching $\mu_{k^1+k^2}$ is individually rational. If $\mu_{k^1+k^2} = \mu'$, then, obviously, $\mu_0, \dots, \mu_{k^1+k^2}$ is a pairwise myopic-farsighted improving path from μ to μ', so $\mu' \in h(\mu)$. Otherwise, it holds that $\mu_{k^1+k^2} \in M \setminus \{\mu'\}$. By Theorem 1 of Mauleon et al. (2011) it holds that $\{\mu'\}$ is a vNM farsightedly stable set, so in particular $\mu' \in g(\mu_{k^1+k^2})$. Since the matchings $\mu_{k^1+k^2}$ and μ' are individually rational, it follows from Step 1 that $\mu' \in h(\mu_{k^1+k^2})$. Since μ' is individually rational, it holds that the composition of $\mu_0, \dots, \mu_{k^1+k^2}$ and the pairwise myopic-farsighted improving path from $\mu_{k^1+k^2}$ to μ' is a pairwise myopic-farsighted improving path from μ to μ', so $\mu' \in h(\mu)$. $\square$

Theorem 6. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F \subset W$ and such that top-ranked women are farsighted. Then $\{\mu^{\mathrm{W}}\}$ is a pairwise myopic-farsighted stable set.*

Proof. Since $\{\mu^{\mathrm{W}}\}$ is a singleton, Condition (i) of Definition 2, internal stability, is satisfied. Condition (ii) of Definition 2, external stability, follows from Lemmas 1 and 3 below. $\square$

We prove first that the woman-optimal stable matching μ^{W} can be reached from any matching μ with the property that, for every $w \in W$, $\mu^{\mathrm{W}}(w) \succeq_w \mu(w)$. Since the core has a lattice structure and the woman-optimal stable matching μ^{W} is weakly preferred by all women to any other core element, Lemma 1 covers all matchings μ that belong to the core.

Lemma 1. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F \subset W$ and such that top-ranked women are farsighted. For every $\mu \in M \setminus \{\mu^{\mathrm{W}}\}$ such that, for every $w \in W$, $\mu^{\mathrm{W}}(w) \succeq_w \mu(w)$ it holds that $\mu^{\mathrm{W}} \in h(\mu)$.*

Proof. Let $\mu \in M \setminus \{\mu^{\mathrm{W}}\}$ be a matching such that, for every $w \in W$, $\mu^{\mathrm{W}}(w) \succeq_w \mu(w)$. We construct a pairwise myopic-farsighted improving path $\mu_0, \dots, \mu_L$ from $\mu_0 = \mu$ to $\mu_L = \mu^{\mathrm{W}}$. Let

$$W^1 = \{w \in W \setminus F \mid \mu^{\mathrm{W}}(w) \succ_w \mu(w) \text{ and } \mu(w) \in M\}$$

be the, possibly empty, set of myopic women w who strictly prefer $\mu^{\mathrm{W}}(w)$ to $\mu(w)$ and who are not single at μ. Let k^1 be the cardinality of W^1. The set of men married at μ to a woman

$w \in W^1$ is denoted by $M^1 = \mu(W^1)$. Let M^0 be the, possibly empty, subset of M^1 such that, for every $m \in M^0$, $w^*(m) = m$. Let k^0 be the cardinality of M^0. Take any order of the men in M^1, say $m_0, \ldots, m_{k^1-1}$, such that the first k^0 men belong to M^0.

For $\ell \in \{0, \ldots, k^0 - 1\}$, we define the matching $\mu_{\ell+1} = \mu_\ell - (m_\ell, \mu(m_\ell))$, so the k^0 men in M^0 sequentially destroy their matches. For $\ell \in \{k^0, \ldots, k^1 - 1\}$, we define the matching $\mu_{\ell+1} = \mu_\ell + (m_\ell, w^*(m_\ell))$, so the $k^1 - k^0$ men in the set M^1 sequentially marry their top choices. We argue that the sequence of matchings $\mu_0, \ldots, \mu_{k^1}$ is the first part of a pairwise myopic-farsighted improving path from μ to μ^{W} by showing that

(i) for every $\ell \in \{0, \ldots, k^1 - 1\}$, $\mu_{\ell+1}(m_\ell) = w^*(m_\ell) \succ_{m_\ell} \mu_\ell(m_\ell)$,

(ii) for every $\ell \in \{k^0, \ldots, k^1 - 1\}$, $\mu_L(w^*(m_\ell)) = \mu^{\mathrm{W}}(w^*(m_\ell)) \succeq_{w^*(m_\ell)} \mu_\ell(w^*(m_\ell))$.

Let some $\ell \in 0, \ldots, k^1 - 1$ be given. The strict preference in (i) holds because m_ℓ is married to the myopic woman $\mu_\ell(m_\ell) \in W \setminus F$, whereas his top choice is either to be single or to marry a farsighted woman.

We now show that (ii) holds. Let some $\ell \in k^0, \ldots, k^1 - 1$ be given. If $\mu_\ell(w^*(m_\ell)) = \mu(w^*(m_\ell))$, then it holds that $\mu^{\mathrm{W}}(w^*(m_\ell)) \succeq_{w^*(m_\ell)} \mu_\ell(w^*(m_\ell))$ by assumption. Otherwise, there is $\ell' < \ell$ such that

$$w^*(m_{\ell'}) = w^*(m_\ell) \text{ and } \mu_\ell(w^*(m_\ell)) = \mu_{\ell'}(w^*(m_{\ell'})) = m_{\ell'}.$$

Suppose $m_{\ell'} \succ_{w^*(m_{\ell'})} \mu^{\mathrm{W}}(w^*(m_{\ell'}))$. Then $\mu^{\mathrm{W}}(m_{\ell'}) \neq w^*(m_{\ell'})$, so it follows that $w^*(m_{\ell'}) \succ_{m_{\ell'}} \mu^{\mathrm{W}}(m_{\ell'})$. Now the pair $(m_{\ell'}, w^*(m_{\ell'}))$ can block μ^{W}, a contradiction. Consequently, it holds that $\mu^{\mathrm{W}}(w^*(m_{\ell'})) \succeq_{w^*(m_{\ell'})} m_{\ell'}$, which is equivalent to $\mu^{\mathrm{W}}(w^*(m_\ell)) \succeq_{w^*(m_\ell)} \mu_\ell(w^*(m_\ell))$, so (ii) holds.

At μ_{k^1}, every man in M^1 is either single or married to his top choice.

Let

$$M^2 = \left\{m \in M \mid \mu_{k^1}(m) \in F \setminus \{\mu^{\mathrm{W}}(m)\}\right\}$$

be the, possibly empty, set of men that are married at μ_{k^1} to a farsighted woman different from $\mu^{\mathrm{W}}(m)$. For every $m \in M^2$ it holds that either $m \in M^1$ and $\mu_{k^1}(m) = w^*(m) \neq \mu^{\mathrm{W}}(m)$, or $m \in M \setminus M^1$ and $\mu_{k^1}(m) \in F$ is such that $\mu^{\mathrm{W}}(\mu_{k^1}(m)) \succ_{\mu_{k^1}(m)} m$, where in the latter case we use that $\mu_{k^1}(m) = \mu(m)$. Let k^2 be the cardinality of the set M^2. Take an arbitrary order of the men in M^2, say $m_0, \ldots, m_{k^2-1}$.

For $\ell \in \{0, \ldots, k^2 - 1\}$, we define the matching $\mu_{k^1+\ell+1} = \mu_{k^1+\ell} - (m_\ell, \mu_{k^1+\ell}(m_\ell))$, so the k^2 farsighted women married to the men in M^2 sequentially destroy their matches under μ_{k^1} and all men in M^2 become single.

We argue that the sequence of matchings $\mu_{k^1}, \ldots, \mu_{k^1+k^2}$ is the second part of a pairwise myopic-farsighted improving path from μ to μ^{W} by showing that, for every $\ell \in \{0, \ldots, k^2 - 1\}$, $\mu^{\mathrm{W}}(\mu_{k^1+\ell}(m_\ell)) \succ_{\mu_{k^1+\ell}(m_\ell)} m_\ell$.

Let some $\ell \in \{0, \ldots, k^2 - 1\}$ be given. If $m_\ell \in M \setminus M^1$, then the assertion above is obviously true, so consider the case $m \in M^1$. Suppose $m_\ell \succeq_{\mu_{k^1+\ell}(m_\ell)} \mu^{\mathrm{W}}(\mu_{k^1+\ell}(m_\ell))$. Since $\mu_{k^1+\ell}(m_\ell) = w^*(m_\ell) \neq \mu^{\mathrm{W}}(m_\ell)$ by construction of M^2, we have $m_\ell \succ_{w^*(m_\ell)} \mu^{\mathrm{W}}(w^*(m_\ell))$ and $w^*(m_\ell) \succ_{m_\ell} \mu^{\mathrm{W}}(m_\ell)$, so the pair $(m_\ell, w^*(m_\ell))$ blocks μ^{W}, a contradiction. Consequently, it holds that $\mu^{\mathrm{W}}(\mu_{k^1+\ell}(m_\ell)) \succ_{\mu_{k^1+\ell}(m_\ell)} m_\ell$.

At μ_{k^1}, every man $m \in M^1$ is single or married to his top choice $w^*(m) \in W$. If this top choice is different from $\mu^W(m)$, then his match is destroyed when moving from μ_{k^1} to $\mu^{k^1+k^2}$. Therefore, at $\mu_{k^1+k^2}$, every man $m \in M^1$ is single or matched to $\mu^{\mathrm{W}}(m)$. Also, every man $m \in M \setminus M^1$ is single or matched to $\mu^{\mathrm{W}}(m)$.

Let

$$M^3 = \{m \in M \mid \mu_{k^1+k^2}(m) = m \text{ and } \mu^{\mathrm{W}}(m) \in W\}$$

be the, possibly empty, set of men that are single at $\mu_{k^1+k^2}$ and married at μ^{W}. Let

$$W^3 = \{w \in W \mid \mu_{k^1+k^2}(w) = w \text{ and } \mu^{\mathrm{W}}(w) \in M\}$$

be the, possibly empty, set of women that are single at $\mu_{k^1+k^2}$ and married at μ^{W}. Let $k^3 = |M^3| = |W^3|$ be the cardinality of these sets. Take an arbitrary order of the men in M^3, say $m_0, \ldots, m_{k^3-1}$.

For $\ell \in \{0, \ldots, k^3 - 1\}$, we define $\mu_{k^1+k^2+\ell+1} = \mu_{k^1+k^2+\ell} + (m_\ell, \mu^{\mathrm{W}}(m_\ell))$, so the men in M^3 sequentially marry the women in W^3 to whom they are matched under μ^{W} until we arrive at $\mu_{k^1+k^2+k^3} = \mu^{\mathrm{W}}$. It holds that

$$\begin{array}{rcll} \mu_{k^1+k^2+\ell+1}(m_\ell) & = & \mu^{\mathrm{W}}(m_\ell) \succ_{m_\ell} m_\ell = \mu_{k^1+k^2+\ell}(m_\ell), & \\ \mu_{k^1+k^2+\ell+1}(w_\ell) & = & \mu^{\mathrm{W}}(w_\ell) \succ_{w_\ell} w_\ell = \mu_{k^1+k^2+\ell}(w_\ell), & \text{if } w_\ell \in W \setminus F, \\ \mu_{k^1+k^2+k^3}(w_\ell) & = & \mu^{\mathrm{W}}(w_\ell) \succ_{w_\ell} w_\ell = \mu_{k^1+k^2+\ell}(w_\ell), & \text{if } w_\ell \in F, \end{array}$$

so the conditions in Definition 1 are satisfied. □

The next lemma is known as the blocking lemma and is due to J.S. Hwang. It is presented as Lemma 3.5 in Roth and Sotomayor (1990). For an arbitrary matching $\mu \in \mathcal{M}$, we define the set of women that strictly prefer μ to μ^{W} by $W(\mu)$, so

$$W(\mu) = \{w \in W \mid \mu(w) \succ_w \mu^{\mathrm{W}}(w)\}.$$

Since μ^W is the woman-optimal stable matching, we have for every $\mu \in C$ that $W(\mu) = \emptyset$.

Lemma 2. *Let $(M, W, \succ)$ be a marriage problem and let $\mu \in \mathcal{M}$ be an individually rational matching. If $W(\mu)$ is non-empty, then there is a pair $(m, w) \in \mu(W(\mu)) \times (W \setminus W(\mu))$ that blocks μ.*

Lemma 2 states that if the set of women that strictly prefer the individually rational matching μ to μ^{W} is non-empty, then there is a blocking pair (m, w) such that m is married to a woman strictly preferring μ and w is weakly preferring μ^{W}. Lemma 2 is crucial for the proof of Lemma 3, which complements the case studied in Lemma 1.

Lemma 3. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F \subset W$ and such that top-ranked women are farsighted. For every $\mu \in \mathcal{M}$ such that $W(\mu) \neq \emptyset$ it holds that $\mu^{\mathrm{W}} \in h(\mu)$.*

Proof. Let $\mu \in \mathcal{M}$ be a matching such that $W(\mu) \neq \emptyset$. We construct a pairwise myopic-farsighted improving path $\mu_0, \ldots, \mu_L$ from $\mu_0 = \mu$ to $\mu_L = \mu^{\mathrm{W}}$. Let

$$W^1 = \{w \in \mu(M) \mid \mu(w) \succ_{\mu(w)} w \text{ or } w \succ_w \mu(w)\}$$

be the, possibly empty, set of women that are involved in a match that is not individually rational for at least one of the partners involved and denote the cardinality of W^1 by k^1. Take an arbitrary order of the women in W^1, say $w_0, \ldots, w_{k^1-1}$. For $\ell \in \{0, \ldots, k^1 - 1\}$, we define the matching $\mu_{\ell+1} = \mu_\ell - (\mu(w_\ell), w_\ell)$, so the player who is involved in a match under μ that is not individually rational destroys his or her match.

Consider the set $W(\mu_{k^1})$ of women $w \in W$ that strictly prefer $\mu_{k^1}(w)$ to $\mu^{\mathrm{W}}(w)$. The set $W(\mu_{k^1})$ is empty if and only if all women in $W(\mu)$ were matched at μ to a man that preferred to be single. For $\ell \in \{k^1, k^1+1, \ldots\}$, whenever $W(\mu_\ell) \neq \emptyset$, select some $(m_\ell, w_\ell) \in \mu_\ell(W(\mu_\ell)) \times (W \setminus W(\mu_\ell))$ that blocks μ_ℓ. Such a pair (m_ℓ, w_ℓ) is guaranteed to exist by Lemma 2. We define the matching $\mu_{\ell+1} = \mu_\ell + (m_\ell, w_\ell)$.

We argue next that after a finite number of steps, say k^2, $W(\mu_{k^1+k^2}) = \emptyset$. Since for every $\ell \geq k^1$, the man involved in the block is married, it follows that the cardinality of the set $\mu_\ell(W(\mu_\ell))$ of men married to women in $W(\mu_\ell)$ is weakly decreasing in ℓ and that these sets are nested in one another. It holds that $\mu_{\ell+1}(m_\ell) = w_\ell \succ_{m_\ell} \mu_\ell(m_\ell)$ and, for every $m \in \mu_\ell(W(\mu_\ell)) \setminus \{m_\ell\}$, $\mu_{\ell+1}(m) = \mu_\ell(m)$. Man m_ℓ strictly improves and all other men in $\mu_\ell(W(\mu_\ell))$ remain married to the same partner. It follows that cycling is impossible, so after a finite number of steps k^2 we have $W(\mu_{k^1+k^2}) = \emptyset$.

Since $W(\mu_{k^1+k^2}) = \emptyset$, the matching $\mu_{k^1+k^2}$ is either equal to μ^{W} or satisfies the assumptions of Lemma 1. In the former case we are done, in the latter case we proceed as in the proof of Lemma 1 to complete the construction of the pairwise myopic-farsighted improving path leading to μ^{W}. It remains to be verified that for every $\ell \in \{0, \ldots, k^1+k^2-1\}$ the conditions of Definition 1 are satisfied.

Consider some $\ell \in \{0, \ldots, k^1-1\}$ and let (m_ℓ, w_ℓ) be such that $\mu_{\ell+1} = \mu_\ell - (m_\ell, w_\ell)$. It holds that $m_\ell \succ_{m_\ell} \mu_\ell(m_\ell)$ or $w_\ell \succ_{w_\ell} \mu_\ell(w_\ell)$. In the former case, we have that

$$\mu_{\ell+1}(m_\ell) = m_\ell \succ_{m_\ell} \mu_\ell(m_\ell),$$

and in the latter case that

$$\begin{array}{rcll} \mu_{\ell+1}(w_\ell) & = & w_\ell \succ_{w_\ell} \mu_\ell(w_\ell), & \text{if } w_\ell \in W \setminus F, \\ \mu_L(w_L) & = & \mu^{\mathrm{W}}(w_\ell) \succeq_{w_\ell} w_\ell \succ_{w_\ell} \mu_\ell(w_\ell), & \text{if } w_\ell \in F, \end{array}$$

so the conditions of Definition 1 are satisfied.

Consider some $\ell \in \{k^1, \ldots, k^1+k^2-1\}$. Since (m_ℓ, w_ℓ) blocks μ_ℓ, it follows that

$$\mu_{\ell+1}(m_\ell) \succ_{m_\ell} \mu_\ell(m_\ell) \text{ and } \mu_{\ell+1}(w_\ell) \succ_{w_\ell} \mu_\ell(w_\ell).$$

It holds that $w_\ell \in W \setminus W(\mu_\ell)$, so

$$\mu^{\mathrm{W}}(w_\ell) \succeq_{w_\ell} \mu_\ell(w_\ell).$$

Irrespective of whether w_ℓ is farsighted or myopic, the conditions of Definition 1 are therefore satisfied. □

Now, we assume all women to be farsighted, whereas any given man can be either myopic or farsighted.

Theorem 7. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F \supset W$. Then $\{\mu^{\mathrm{W}}\}$ is a pairwise myopic-farsighted stable set.*

Proof. Since $\{\mu^{\mathrm{W}}\}$ is a singleton set, Condition (i) of Definition 2, internal stability, is satisfied. Condition (ii) of Definition 2, external stability, follows from Lemmas 4 and 5 below. □

We prove first that the woman-optimal stable matching μ^{W} can be reached from any matching μ different from μ^{W} with the property that, for every $w \in W$, $\mu^{\mathrm{W}}(w) \succeq_w \mu(w)$. This covers the case where μ is a core element different from μ^{W}.

Lemma 4. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F \supset W$. For every $\mu \in M \setminus \{\mu^{\mathrm{W}}\}$ such that, for every $w \in W$, $\mu^{\mathrm{W}}(w) \succeq_w \mu(w)$ it holds that $\mu^{\mathrm{W}} \in h(\mu)$.*

Proof. Let $\mu \in M \setminus \{\mu^{\mathrm{W}}\}$ be a matching such that, for every $w \in W$, $\mu^{\mathrm{W}}(w) \succeq_w \mu(w)$. We construct a pairwise myopic-farsighted improving path $\mu_0, \ldots, \mu_L$ from $\mu_0 = \mu$ to $\mu_L = \mu^{\mathrm{W}}$. Let

$$W^1 = \{w \in W \mid \mu^{\mathrm{W}}(w) \succ_w \mu(w) \text{ and } \mu(w) \in M\}$$

be the, possibly empty, set of women who strictly prefer $\mu^{\mathrm{W}}(w)$ to $\mu(w)$ and who are married at μ. Let k^1 be the cardinality of W^1. Take an arbitrary order of the women in W^1, say $w_0, \ldots, w_{k^1-1}$.

For $\ell \in \{0, \ldots, k^1 - 1\}$, we define the matching $\mu_{\ell+1} = \mu_\ell - (\mu_\ell(w_\ell), w_\ell)$, so the k^1 women in W^1 sequentially destroy their matches. We show that the sequence of matchings $\mu_0, \ldots, \mu_{k^1}$ is the first part of a pairwise myopic-farsighted improving path from μ to μ^{W} by showing that for every $\ell \in \{0, \ldots, k^1 - 1\}$ we have $\mu_L(w_\ell) = \mu^{\mathrm{W}}(w_\ell) \succ_{w_\ell} \mu_\ell(w_\ell)$. This follows since $\mu_\ell(w_\ell) = \mu(w_\ell)$ and by the definition of W^1.

At μ_{k^1} every woman is either single or married to her partner at μ^{W}. It then follows that at μ_{k^1} every man is either single or married to his partner at μ^{W}. Let

$$W^2 = \{w \in W \mid \mu^{\mathrm{W}}(w) \succ_w \mu_{k^1}(w)\}$$

be the set of women that strictly prefer the match at μ^{W} to being single. Let k^2 be the cardinality of the set W^2. Take an arbitrary order of the women in W^2, say $w_0, \ldots, w_{k^2-1}$. For $\ell = 0, \ldots, k^2 - 1$, we define $m_\ell = \mu^{\mathrm{W}}(w_\ell)$.

For $\ell = 0, \ldots, k^2 - 1$, we define the matching $\mu_{k^1+\ell+1} = \mu_{k^1+\ell} + (m_\ell, w_\ell)$, so the k^2 women in W^2 sequentially marry their partner at μ^{W}. It holds that $\mu_L = \mu_{k^1+k^2} = \mu^{\mathrm{W}}$. Observe that the sequence of matchings $\mu_{k^1}, \ldots, \mu_{k^1+k^2}$ is the final part of a pairwise myopic-farsighted improving path from μ to μ^{W} since it holds that, for every $\ell \in \{0, \ldots, k^2 - 1\}$,

$$\begin{array}{rcll}
\mu_{k^1+\ell+1}(m_\ell) & = & \mu^{\mathrm{W}}(m_\ell) \succ_{m_\ell} m_\ell = \mu_{k^1+\ell}(m_\ell), & \text{if } m_\ell \in M \setminus F, \\
\mu_L(m_\ell) & = & \mu^{\mathrm{W}}(m_\ell) \succ_{m_\ell} m_\ell = \mu_{k^1+\ell}(m_\ell), & \text{if } m_\ell \in F, \\
\mu_L(w_\ell) & = & \mu^{\mathrm{W}}(w_\ell) \succ_{w_\ell} w_\ell = \mu_{k^1+\ell}(w_\ell). & \square
\end{array}$$

We now turn to the case where the matching μ is such that some women prefer μ to μ^{W}, so the set of women $W(\mu)$ that strictly prefer their match at μ to the one at μ^{W} is not empty.

Lemma 5. *Let $(M, W, \succ)$ be a marriage problem with set of farsighted players $F \supset W$. For every $\mu \in M$ such that $W(\mu) \neq \emptyset$ it holds that $\mu^{\mathrm{W}} \in h(\mu)$.*

Proof. Let $\mu \in M$ be a matching such that $W(\mu) \neq \emptyset$. We construct a pairwise myopic-farsighted improving path $\mu_0, \ldots, \mu_L$ from $\mu_0 = \mu$ to $\mu_L = \mu^{\mathrm{W}}$. Let

$$W^1 = \{w \in \mu(M) \mid \mu(w) \succ_{\mu(w)} w \text{ or } w \succ_w \mu(w)\}$$

be the, possibly empty, set of women that are involved in a match that is not individually rational for at least one of the partners and denote the cardinality of W^1 by k^1. Take an arbitrary order of the women in W^1, say $w_0, \ldots, w_{k^1-1}$. For $\ell \in \{0, \ldots, k^1-1\}$, we define $m_\ell = \mu(w_\ell)$ to be the man married to w_ℓ at μ and we define the matching $\mu_{\ell+1} = \mu_\ell - (m_\ell, w_\ell)$, so the player who is involved in a match under μ that is not individually rational destroys his or her match. We argue that the sequence of matchings $\mu_0, \ldots, \mu_{k^1}$ is the first part of a pairwise myopic-farsighted improving path from μ to μ^{W}.

Let some $\ell \in \{0, \ldots, k^1-1\}$ be given. It holds that $m_\ell \succ_{m_\ell} w_\ell$ or $w_\ell \succ_{w_\ell} m_\ell$. In the former case, we have that

$$\begin{array}{rcll} \mu_{\ell+1}(m_\ell) & = & m_\ell \succ_{m_\ell} \mu_\ell(m_\ell), & \text{if } m_\ell \in M \setminus F, \\ \mu_L(w_\ell) & = & \mu^{\mathrm{W}}(m_\ell) \succeq_{m_\ell} m_\ell \succ_{m_\ell} \mu_\ell(m_\ell), & \text{if } m_\ell \in F, \end{array}$$

whereas in the latter case it holds that

$$\mu_L(w_\ell) = \mu^{\mathrm{W}}(w_\ell) \succeq_{w_\ell} w_\ell \succ_{w_\ell} \mu_\ell(w_\ell),$$

so the conditions of Definition 1 are satisfied.

Let

$$W^2 = \{w \in W(\mu_{k^1}) \mid \mu_{k^1}(w) \in F\}$$

be the, possibly empty, set of women that prefer their match at μ_{k^1} to the one at μ^{W} and that are matched to a farsighted man. We denote the cardinality of W^2 by k^2 and take an arbitrary order of the women in W^2, say $w_0, \ldots, w_{k^2-1}$. For $\ell \in \{0, \ldots, k^2-1\}$, we define $m_\ell = \mu_{k^1}(w_\ell)$ to be the man married to w_ℓ at μ_{k^1} and we define the matching $\mu_{k^1+\ell+1} = \mu_{k^1+\ell} - (m_\ell, w_\ell)$, so man m_ℓ destroys his match with w_ℓ. We argue that the sequence of matchings $\mu_{k^1}, \ldots, \mu_{k^1+k^2}$ is the second part of a pairwise myopic-farsighted improving path from μ to μ^{W}.

Let some $\ell \in \{0, \ldots, k^2-1\}$ be given. Since $m_\ell \succ_{w_\ell} \mu^{\mathrm{W}}(w_\ell)$ and μ^{W} is a core element, it holds that

$$\mu_L(m_\ell) = \mu^{\mathrm{W}}(m_\ell) \succ_{m_\ell} w_\ell = \mu_{k^1+\ell}(m_\ell),$$

so the conditions of Definition 1 are satisfied.

Consider the set $W(\mu_{k^1+k^2})$ of women that strictly prefer $\mu_{k^1+k^2}(w)$ to $\mu^{\mathrm{W}}(w)$. By construction of $\mu_{k^1+k^2}$ it holds that the men in the set $\mu_{k^1+k^2}(W(\mu_{k^1+k^2}))$, who are married to a woman in $W(\mu_{k^1+k^2})$, are all myopic. For $\ell \in \{0, 1, \ldots\}$, whenever $W(\mu_{k^1+k^2+\ell}) \neq \emptyset$, select some

$$(m_\ell, w_\ell) \in (\mu_{k^1+k^2+\ell}(W(\mu_{k^1+k^2+\ell}))) \times (W \setminus W(\mu_{k^1+k^2+\ell}))$$

that blocks $\mu_{k^1+k^2+\ell}$. Such a pair (m_ℓ, w_ℓ) is guaranteed to exist by Lemma 2. We define the matching $\mu_{k^1+k^2+\ell+1} = \mu_{k^1+k^2+\ell} + (m_\ell, w_\ell)$.

We argue next that after a finite number of steps, say k^3, $W(\mu_{k^1+k^2+k^3}) = \emptyset$. Since for every $\ell \geq 0$, the man involved in the block is married to a woman in $W(\mu_{k^1+k^2+\ell})$, it follows that the cardinality of the set $\mu_{k^1+k^2+\ell}(W(\mu_{k^1+k^2+\ell}))$ is weakly decreasing in ℓ and that these sets are nested in one another. It holds that $\mu_{k^1+k^2+\ell+1}(m_\ell) = w_\ell \succ_{m_\ell} \mu_{k^1+k^2+\ell}(m_\ell)$ and, for every $m \in \mu_{k^1+k^2+\ell}(W(\mu_{k^1+k^2+\ell})) \setminus \{m_\ell\}$, $\mu_{k^1+k^2+\ell+1}(m) = \mu_{k^1+k^2+\ell}(m)$. Man m_ℓ is strictly improving and all other men in $\mu_{k^1+k^2+\ell}(W(\mu_{k^1+k^2+\ell}))$ remain married to the same partner. It follows that cycling is impossible, so after a finite number of steps k^3 we have $W(\mu_{k^1+k^2+k^3}) = \emptyset$.

We argue that the sequence of matchings $\mu_{k^1+k^2}, \ldots, \mu_{k^1+k^2+k^3}$ is the third part of a pairwise myopic-farsighted improving path from μ to μ^{W}.

Let some $\ell \in \{0, \ldots, k^3 - 1\}$ be given. Since the set of men $\mu_{k^1+k^2+\ell}(W(\mu_{k^1+k^2+\ell}))$ is a subset of $\mu_{k^1+k^2}(W(\mu_{k^1+k^2}))$, it holds that m_ℓ is myopic. Since (m_ℓ, w_ℓ) blocks $\mu_{k^1+k^2+\ell}$, it follows that

$$\mu_{k^1+k^2+\ell+1}(m_\ell) \succ_{m_\ell} \mu_{k^1+k^2+\ell}(m_\ell).$$

It holds that $w_\ell \in W \setminus W(\mu_{k^1+k^2+\ell})$, so

$$\mu_L(w_\ell) = \mu^{\mathrm{W}}(w_\ell) \succeq_{w_\ell} \mu_\ell(w_\ell).$$

The conditions of Definition 1 are therefore satisfied.

Since $W(\mu_{k^1+k^2+k^3}) = \emptyset$, the matching $\mu_{k^1+k^2+k^3}$ is either equal to μ^{W} or satisfies the assumptions of Lemma 4. In the former case we are done, in the latter case we proceed as in the proof of Lemma 4 to complete the construction of the pairwise myopic-farsighted improving path leading to μ^{W}. $\square$

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