

The identification of preferences from equilibrium prices ^{1 2}

P. - A. Chiappori ³ I. Ekeland ⁴ F. Kübler ⁵
H. M. Polemarchakis ⁶

Discussion Paper No. 99xx (October, 1999)

CORE, Université Catholique de Louvain

¹This text presents results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by its authors. The National Science Foundation provided financial support through the grant SBR 9729559.

²Donald Brown, Gérard Debreu, Stefano Demichelis, John Geanakoplos, Fabrizio Germano, Itzhak Gilboa, Roger Guesnerie, Werner Hildenbrand, Rosa Matzkin, Chris Shannon and participants in seminars at the University of Bonn, CentER, the University of Chicago, CORE, the Instituto Veneto during the 1999 Summer School in Economic Theory, at Purdue University during the 1999 NBER conference on General Equilibrium, the University of Paris - I, the University of Tel Aviv and the University of Toulouse made helpful comments.

³Department of Economics, University of Chicago

⁴CEREMADE and Institut de Finance, Université de Paris - IX, Dauphine

⁵Department of Economics, Stanford University

⁶CORE, Université Catholique de Louvain

Abstract

The fundamentals of an exchange economy, the preferences of individuals, can be identified from the competitive equilibrium correspondence, which associates equilibrium prices of commodities to allocations of endowments; the argument extends to production economies. The essential step is the identification of fundamentals from aggregate demand as a function of the prices of commodities and the distribution of income. The graph of the equilibrium correspondence or of the aggregate demand function satisfy non - trivial restrictions. The identification of fundamentals allows for the prediction of the response of individuals and the economy to changes in the organization of production and exchange, while restrictions on the equilibrium correspondence or the aggregate demand function imply that general equilibrium theory has testable implications.

Key words: aggregation, equilibrium, identification, testability.

JEL classification numbers: D 10, D 50.

1 Introduction

Economic theory derives relationships between the fundamentals of the economy, some of which may not be observable, and observed individual or aggregate behavior. Such relationships raise two general problems: testability and identification.

Testability obtains if the theory generates restrictions that have to be satisfied by observed behavior. Identification requires that the fundamentals that are compatible with the observed behavior be unique. If the argument for identification is constructive, which is more demanding, one speaks of recoverability.

Here, either aggregate demand can be observed, locally, as a function of prices and the incomes of individuals or equilibrium prices can be observed, locally, as a function of the endowments of individuals. Individual optimization generates testable restrictions on observed behavior, the aggregate demand function or the equilibrium correspondence. Also identification is possible: the preferences of individuals can be uniquely recovered from the sole observation of either aggregate demand or the equilibrium correspondence.

Identification

The lack of available data and problems with econometric estimation procedures notwithstanding, it is an important theoretical question whether the necessary information concerning the unobservable characteristics of individuals can be identified from their observable, market behavior. The identification of preferences from observed behavior has strong positive as well as normative implications. The transfer paradox, introduced by Leontief (1936) and stated with generality in Donsimoni and Polemarchakis (1994), makes it clear that knowledge of the utility functions is necessary in order to identify welfare effects of transfers. More generally, explanation and prediction, as well as normative analysis, require individual or aggregate behavior, which is observable, to identify the fundamentals of the economy, which are not.

The identification of the preference relation of an individual from his demand behavior as the prices of commodities and his income vary is evident, under standard regularity assumptions examined in detail in Mas - Colell (1977). Integrability, introduced by Samuelson (1956) and further considered in Debreu (1972, 1976), concerns conditions for the existence of a complete and transitive preference relation that generates demand behavior; identification presupposes the existence of an underlying preference relation, often one that satisfies additional regularity conditions.

However, if only aggregate demand behavior is observable, the identification of the preference relations of individuals is more surprising: it might be possible to disaggregate the observed behavior into different families of individual demand functions generated by different profiles of utilities; indeed, an example

with quasi - linear preferences shows that this may be the case.

Here, the following, strong result obtains: under a condition of non - vanishing income effects, the Slutsky decomposition of the demand functions of individuals can be exploited to identify their preferences from aggregate behavior. Specifically, the observation of either aggregate excess demand (as a function of prices and endowments) or the equilibrium correspondence, a priori less informative than aggregate demand, suffices for the identification of the preferences of individuals; which is surprising, since, on the equilibrium correspondence, prices and endowments do not vary independently, and there are, typically, finitely many prices associated with each profile of endowments. Identification from the equilibrium correspondence requires variation in the allocation of endowments. Nevertheless, observation of the distribution of income associated with alternative allocations of endowments and associated competitive equilibrium prices suffices for identification. Interestingly enough, both results hold irrespective of the number of individuals relative to the number of commodities in the economy.

This results provides an affirmative answer to the question whether the theory of general competitive equilibrium allows for the prediction of the response of the economy to changes in the organization of production and exchange, changes in economic policy in particular.

The argument for identification builds on earlier arguments of Brown and Matzkin (1990), who argued that identification of income effects is possible from the equilibrium correspondence. Identification obtains for an economy under certainty or, equivalently, for an economy with a complete market either in contingent commodities or in elementary securities; in a companion paper, Kübler, Chiappori, Ekeland and Polemarchakis (1999), it extends to an economy with an incomplete asset market.

Testability

The two issues, testability and identification, are distinct. Observable restrictions may fail to arise even if identification is possible. Conversely, in many cases, while testable restrictions obtain, they are not sufficient to guarantee identification. For instance, in an economy where the number of individuals is greater than one but strictly smaller than the number of commodities, restrictions exist on both excess and market demands, considered as functions of prices only. However, the knowledge of excess or market demand as a function of prices never allows the identification of the preferences of individuals.

The preferences of individuals provide a typical examples of fundamentals that are not observable. Testability, then, amounts the following question: what restrictions can be derived on observed behavior from the hypothesis that individuals maximize some unobservable utility function? Observations may involve different degrees of aggregation. At the most disaggregated level, one can observe the demands of individuals as prices and incomes vary. With less or no

disaggregation, one can observe the aggregate demand. More appropriately, observations are restricted to equilibrium prices as endowments vary.

It is well known that testability obtains at the disaggregated level. For smooth demand functions, and under mild regularity conditions, homogeneity, adding up and Slutsky symmetry and negative semi - definiteness are necessary and sufficient for compatibility with utility maximization. With finite data sets, revealed preference theory also provides necessary and sufficient conditions. The case of aggregate observations is more difficult.

In seminal papers, Sonnenschein (1973, 1974) posed the question whether the individualistic foundations of general equilibrium theory, the fact that aggregate demand results from the aggregation of utility - maximizing behavior of individuals, could generate non - trivial testable restrictions on the form of the excess demand or market demand functions ¹.

For excess demand as a function of the prices of commodities, Debreu (1974) and Mantel (1974) provided a negative answer, which confirmed initial results in Sonnenschein (1973, 1974): as long as the number of individuals aggregated is large relative to the number of commodities, aggregate excess demand need not satisfy any restrictions beyond homogeneity and Walras' law as prices vary in a compact set of strictly positive prices; Balasko (1986) argued that aggregate excess demand as prices tend to the boundary of their domain provides restrictions; Geanakoplos (1984) gave an explicit construction of the utility functions that can be attributed to individuals to yield a given aggregate excess demand function.

For market demand as a function of the prices of commodities, McFadden, Mas - Colell, Mantel and Richter (1974) showed that the negative result obtained for excess demand may not hold: the positivity restrictions on the consumption, and hence demands of individuals may interfere with the decomposition of aggregate demand into individual demand functions derived by utility maximization; equivalently, the revealed preference restrictions satisfied by individuals, together with the positivity of individual consumption may restrict the aggregate response to variations in prices. For finite sets of observations, Andreu (1983) showed that the negative result carries over to the case of market demand.

The restrictions implied by individual optimization on aggregate demand as a function of prices diminish as the number of individuals aggregated increases; recently, Chiappori and Ekeland (1999, a) gave a complete characterization of the local properties of market demand for arbitrary numbers of individuals and commodities; Geanakoplos and Polemarchakis (1986) had characterized the first order restrictions satisfied by aggregate excess demand at a point. In particular, Chiappori and Ekeland (1999, a) proved that any sufficiently smooth, in fact

¹ "Excess demand" refers to demand net of the endowment, while "market demand" refers to demand inclusive of any endowment; use of the simple term "demand" either indicates that the distinction is not important; importantly, either excess demand or market demand can vary with prices only or with prices and endowment or income.

analytic, function that satisfies adding - up can be locally decomposed as the aggregate market demand of an economy where the number of individuals is at least equal to the number of commodities; it follows that constraints stemming from non - negativity are the only testable properties of aggregate market demand.

The overall impression from the work following the conundrum posed by Sonnenschein (1973, 1974) was that individual rationality fails to generate observable implications for a general specification of endowments and preferences as is standard in the theory of general competitive equilibrium following Arrow and Debreu (1954) and McKenzie (1954); which was interpreted to confirm that the theory does not allow explanation and prediction.

In response, Hildenbrand (1983) and, later, Grandmont (1992), in a framework directly appropriate for general equilibrium analysis, succeeded in obtaining restrictions for aggregate demand through a diversity requirement on the distribution of the characteristics of individuals; the approach is novel and ingenious, and it provides a way out of the impasse for general equilibrium theory that may prove fruitful indeed.

Alternatively, in a critical and insightful contribution, Brown and Matzkin (1996) pointed out that, contrary to the approach in Sonnenschein (1973, 1974) and then Debreu (1974), Mantel (1974) and the work that followed: (1) demand is not easily — or, for that matter, in principal — observable out of equilibrium; (2) prices are not the only variables that determine the demand of individuals. Prices movements reflect fluctuations in the fundamentals of an economy. The relationship between these fundamentals and the resulting equilibrium prices is a natural focus for empirical observation. What theory should aim at are observable relationships between equilibrium prices and fundamentals. Following a revealed preference approach, Brown and Matzkin (1996) gave a set of testable restrictions that apply to finite data sets.

Here, following Brown and Matzkin (1996), and in contrast with the line of Debreu (1974), Mantel (1974) and Sonnenschein (1973, a, b) one observes aggregate demand as, not only the prices of commodities, but, also, the endowments or incomes of individuals vary. Alternatively, one observes the set of equilibrium prices, locally, as a function of the endowments or incomes of individuals. A set of necessary and sufficient conditions that fully characterize the local structure of either aggregate demand or the equilibrium correspondence. These results extend and confirm the findings of Brown and Matzkin (1996) that restrictions on the equilibrium correspondence or the aggregate demand do exist, and competitive equilibrium theory has testable implications, and, as a consequence, is refutable.

A natural, related question, is whether restrictions remain if only the aggregate endowment is observable. The answer is negative: when the number of individuals is at least equal to the number of commodities and the distribution of income is not observable, any smooth manifold coincides, locally, with the equilibrium manifold of an exchange economy.

The two lines contrasted above — the analysis of equilibrium comparative statics by Brown and Matzkin (1996) and the analysis of aggregate demand as a function of prices by Sonnenschein (1973 a, b), Debreu (1974) and Mantel (1974) — lead to very different conclusions. Importantly, the analysis of comparative statics relies on individual data, the endowments of individuals, while the analysis of aggregate demand does not. One may conclude that disaggregated data are necessary and sufficient for individual optimization to lead to observable restrictions even on aggregate data.

2 The economy and identification

Individuals are indexed by $i \in \mathcal{I} = \{1, \dots, I\}$, a finite, non - empty set.

Commodities are indexed by $l \in \mathcal{L} = \{1, \dots, L\}$, a finite, non - empty set, and a bundle of commodities is ² $x = (\dots, x_l, \dots)'$.

The preferences of an individual are represented by the utility function, u^i , with domain the consumption set.

Utility functions, u_1^i and u_2^i , are ordinally equivalent if u_2^i is a strictly monotonically increasing transformation of u_1^i .

The endowment of an individual is e^i , a bundle of commodities.

Assumption 1 *For every individual,*

1. *The consumption set is the set of non - negative bundles of commodities;*
2. *the utility function, u^i , is continuous and quasi - concave;*
3. *in the interior of the consumption set, the utility function is twice continuously differentiable, differentiably strictly increasing ³: $Du^i(x) \gg 0$, and strictly quasi - concave ⁴: if $y \in [Du^i(x)]^\perp \setminus \{0\}$, then $y' D^2 u^i(x) y < 0$;*
4. *for a sequence of strictly positive consumption bundles, $(x_n : n = 1, \dots)$, and for $\bar{x}$, a non - zero consumption bundle on the boundary of the consumption set, $(\lim_{n \rightarrow \infty} x_n = \bar{x}) \Rightarrow (\lim_{n \rightarrow \infty} (\|Du^i(x_n)\|)^{-1} Du^i(x_n) x_n = 0)$;*
5. *$e^i \gg 0$: the endowment is a consumption bundle in the interior of the consumption set.*

²“ $'$ ” denotes the transpose.

³“ $\gg$ ”, “ $>$ ” and “ $\geq$ ” are vector inequalities; also, “ $\ll$ ”, “ $<$ ” and “ $\leq$ ”.

⁴“ $[\]$ ” denotes the span of a set of vectors or the column span of a matrix; “ $\perp$ ” denotes the orthogonal complement.

The profile of utility functions is

$$u^{\mathcal{I}} = (\dots, u^i, \dots),$$

and the allocation of endowments is

$$e^{\mathcal{I}} = (\dots, e^i, \dots).$$

Profiles of utility functions, $u_1^{\mathcal{I}}$ and $u_2^{\mathcal{I}}$, are ordinally equivalent if, for every individual, the utility functions u_1^i and u_2^i are ordinally equivalent.

The profile of utility functions is fixed, while the allocation of endowments varies.

The aggregate endowment is $e^a = \sum_{i \in \mathcal{I}} e^i$.

Prices of commodities are $p = (\dots, p_l, \dots) \gg 0$.

The optimization problem of an individual is

$$\begin{aligned} \max \quad & u^i(x), \\ \text{s.t.} \quad & px \leq pe^i. \end{aligned}$$

The solution to the individual optimization problem, $x^i(p, e^i)$, which exists, is unique and lies in the interior of the consumption set, defines x^i , the demand function of the individual.

The demand function of the individual depends on the endowment, e^i , only through its value, pe^i , the income of the individual. With income, $t^i > 0$, as the exogenous variable, the optimization problem of an individual is

$$\begin{aligned} \max \quad & u^i(x), \\ \text{s.t.} \quad & px \leq t^i. \end{aligned}$$

A solution, $x^i(p, t^i)$, which exists, is unique and lies in the interior of the consumption set, defines demand as a function of the prices of commodities and the income of the individual.

For ordinally equivalent utility functions, the demand functions coincide.

The demand function identifies the utility function of an individual up to ordinal equivalence: by the separating hyperplane theorem, the demand function is surjective; since, at a solution to the individual optimization problem, the gradient of utility function and the vector of prices are collinear, the demand function identifies the utility function up to a strictly monotonically increasing transformation.

Remark In the presence of individual production possibilities described by the production function f , the optimization problem of an individual takes the

form

$$\begin{aligned} \max \quad & u^i(x), \\ \text{s.t} \quad & px \leq p(e^i + y), \\ & f(y) = 0. \end{aligned}$$

This is equivalent to the problem

$$\begin{aligned} \max \quad & v^i(x), \\ \text{s.t} \quad & px \leq pe^i, \end{aligned}$$

where the function v^i is defined by

$$v^i(x) = \max\{u^i(x + y) : f(y) = 0\}.$$

From the previous argument, v^i is identified by the demand of the individual. If the production function, f is known or, alternatively, if profit maximizing production decisions are observable, the utility function, u^i , can be identified; otherwise it cannot be.

The demand function is continuously differentiable — Debreu (1972); price effects are

$$D_p x^i = \begin{pmatrix} \vdots, \\ \dots \quad \frac{\partial x_l^i}{\partial p_k} \quad \dots \\ \vdots \end{pmatrix},$$

and income effects are

$$D_{t^i} x^i = \begin{pmatrix} \vdots \\ \frac{\partial x_l^i}{\partial t^i} \\ \vdots \end{pmatrix};$$

if $t^i = pe^i$, then

$$D_{e^i} x^i = (D_{t^i} x^i)p = \begin{pmatrix} \vdots, \\ \dots \quad \frac{\partial x_l^i}{\partial t^i} p_k \quad \dots \\ \vdots \end{pmatrix}.$$

Aggregate demand

Across individuals,

$$x^a(p, e^{\mathcal{I}}) = \sum_{i \in \mathcal{I}} x^i(p, e^i),$$

which defines x^a , the aggregate demand function.

With the incomes of individuals as exogenous variables, the aggregate demand function is defined by

$$x^a(p, t^{\mathcal{I}}) = \sum_{i \in \mathcal{I}} x^i(p, t^i),$$

where $t^{\mathcal{I}} = (\dots, t^i, \dots)$ is the distribution of income.

For ordinarily equivalent profiles of utility functions, the aggregate demand functions coincide.

The aggregate demand function may not identify the profile of utility functions.

Example The utility functions of individuals are defined by

$$u^i(x) = x_1 + \left(\frac{1}{\alpha}\right) \sum_{l=2}^L \delta_l^i x_l^\alpha, \quad \alpha < 1, \quad \delta_l^i > 0, l = 2, \dots, L.$$

The aggregate demand functions for commodities 2, ..., L is defined by

$$x_l^a(p) = \sum_{i=1}^I \min\{0, (\delta_l^i)^{\frac{1}{1-\alpha}} p_l^{\frac{1}{\alpha-1}}\}, \quad l = 2, \dots, L.$$

The aggregate demand function fails to identify the parameters δ_l^i , for $i = 1, \dots, I$, and, as a consequence, the utility functions of individuals.

Example There are two individuals and two commodities.

The utility functions of individuals are defined by

$$u^i(x) = \ln(x_1 - \epsilon) + \ln(x_2 + \epsilon),$$

and

$$u^2(x) = \ln(x_1 + \epsilon) + \ln(x_2 - \epsilon),$$

where ϵ is small enough to guarantee positive consumption in the relevant range.

The aggregate demand functions for commodity 1 is defined by

$$x_1^a(p) = \frac{1}{2p_1} \sum_{i=1}^2 p e^i.$$

The aggregate demand function fails to identify the parameter ϵ , and, as a consequence, the utility functions of individuals.

In the examples utility functions fail the boundary condition in assumption 1, but this is not important — locally, at prices and incomes that guarantee interior consumption the utility functions cannot be distinguished from utility functions that satisfy assumption 1.

The demand functions of individuals in both examples display vanishing income effects, which accounts for the failure of the aggregate demand function to identify the profile of utility functions.

Assumption 2 *For every individual,*

1. *the income effect for every commodity, $\partial x_l^i / \partial t^i$, is a twice differentiable function of income and*

$$\frac{\partial^2 x_l^i}{\partial (t^i)^2} \neq 0;$$

2. *there exist commodities, m and n , other than the numeraire, such that*

$$\frac{\partial}{\partial t^i} \left(\ln \frac{\partial^2 x_m^i}{(\partial t^i)^2} \right) \neq \frac{\partial}{\partial t^i} \left(\ln \frac{\partial^2 x_n^i}{(\partial t^i)^2} \right).$$

Income effects do not vanish for any commodity, while there are two commodities for which the partial elasticities of the income effects with respect to revenue do not vanish.

Remark Assumption 2 implies that there are at least three commodities: $L \geq 3$; a different argument is required for economies with two commodities, $L = 2$.

Remark Homothetic utility functions do not satisfy assumption 2. Indeed, the second example shows that identification is not possible for homothetic utility functions. Intuitively, this is caused by the fact that homothetic utilities permit aggregation.

Remark Assumption 2 fails for quasi - linear preferences, for which the income effect vanishes as well as for homothetic preferences for which the income effect is linear in income. If demand is non - linear in income, and if income effect do not vanish, assumption 2 is satisfied for an open and dense set of prices and incomes; which suffices, since continuity then allows for identification.

The generalized rank of a demand vector, defined by Lewbel (1991), makes the point.

As a function of the prices of commodities and the income of an individual, (p, t^i) , the demand for a commodity writes as

$$x_l^i(p, t^i) = \sum_{j \in \mathcal{J}} \alpha_{l,j}^i(p) g_j^i(p, t^i),$$

where

$$\{g_j^i(p, t^i) : j \in \mathcal{J} = \{1, \dots, J\}\}$$

is a basis of the subspace spanned by $x^i(p, t^i)$ as t^i varies.

If

$$\gamma_j^i(p, t^i) = \frac{\partial^2 g_j^i(p, t^i)}{\partial (t^i)^2},$$

then

$$\frac{\partial^2 x_l^i(p, t^i)}{\partial (t^i)^2} = \sum_{j \in \mathcal{J}} \alpha_{l,j}^i(p) \gamma_j^i(p, t^i), \quad \frac{\partial^3 x_l^i(p, t^i)}{\partial (t^i)^3} = \sum_{j \in \mathcal{J}} \alpha_{l,j}^i(p) \frac{\partial \gamma_j^i(p, t^i)}{\partial t^i},$$

and assumption 2 is equivalent to the existence of commodities m and n , such that

$$\frac{\sum_{j \in \mathcal{J}} \alpha_{m,j}^i(p) \frac{\partial \gamma_j^i(p, t^i)}{\partial t^i}}{\sum_{j \in \mathcal{J}} \alpha_{m,j}^i(p) \gamma_j^i(p, t^i)} \neq \frac{\sum_{j \in \mathcal{J}} \alpha_{n,j}^i(p) \frac{\partial \gamma_j^i(p, t^i)}{\partial t^i}}{\sum_{j \in \mathcal{J}} \alpha_{n,j}^i(p) \gamma_j^i(p, t^i)}.$$

If the demand system is of rank $J = 1$, then the inequality can never hold; indeed, this is the case if and only if the underlying utility function is homothetic. But if the rank of the system is more than one, then the inequality holds for an open and dense set of prices and incomes. The inequality is equivalent to

$$\sum_{j \in \mathcal{J}} \sum_{k=j+1}^J (\alpha_{m,j}^i \alpha_{n,k}^i - \alpha_{n,j}^i \alpha_{m,k}^i) \left(\frac{\partial \gamma_j^i}{\partial t^i} \gamma_k^i - \frac{\partial \gamma_k^i}{\partial t^i} \gamma_j^i \right) \neq 0.$$

By the definition of g_j^i , the left hand side can only be equal to zero on an open set of incomes if $(\alpha_{m,j}^i \alpha_{n,k}^i - \alpha_{n,j}^i \alpha_{m,k}^i) = 0$, for $j, k \in \mathcal{J}$. But this implies that the α^i is collinear for the two commodities n, m ; while this can hold at given prices, p , for two given commodities, n, m , one can always find commodities and a perturbation in prices, such that this no longer holds.

Proposition 1 *The aggregate demand function identifies the profile of utility functions up to ordinal equivalence.*

Proof It suffices that the aggregate demand function identify the demand function of every individual.

The argument is developed in a sequence of steps.

The solution to the optimization problem of an individual is determined by the necessary and sufficient first order conditions

$$Du^i - \lambda^i p = 0,$$

$$px - pe^i = 0.$$

Differentiating the first order conditions and setting

$$\begin{pmatrix} K^i & -v^i \\ -v^{i'} & b^i \end{pmatrix} = \begin{pmatrix} D^2 u^i & -p' \\ -p & 0 \end{pmatrix}^{-1},$$

and

$$S^i = \lambda^i K^i,$$

yields, by the implicit function theorem, that

$$D_p x^i = S^i - v^i(x^i - e^i)',$$

$$D_{t^i} x^i = v^i,$$

and, as a consequence,

$$dx^i = (S^i - v^i(x^i - e^i)')dp + v^{i'}dt^i.$$

This is the Slutsky decomposition ⁵ of the jacobian of the demand function of an individual; the matrix, S^i , of substitution effects, $s_{l,k}^i = (\partial x_l^i / \partial p_k)_{\bar{u}^i}$, is symmetric and negative semi-definite, it has rank $(L-1)$, and satisfies $pS^i = 0$, and the vector, v^i , of income effects, $v_l^i = \partial x_l^i / \partial t^i$, satisfies $pv^i = 1$.

For the aggregate demand function,

$$D_p x^a = \sum_{i \in \mathcal{I}} (S^i - v^i(x^i - e^i)'),$$

$$D_{t^i} x^a = v^i,$$

and, as a consequence,

$$dx^a = \sum_{i \in \mathcal{I}} (S^i - v^i(x^i - e^i)')dp + \sum_{i \in \mathcal{I}} v^{i'}dt^i.$$

To simplify notation, without loss of generality, all derivatives are evaluated at prices of commodities p with $p_1 = 1$.

Step 1 Since $D_{e_1^i} x^a = v^i$, the aggregate demand function identifies the income effects for every individual.

⁵Antonelli (1886), Slutsky (1915)

Step 2 The functions

$$f_{j,k} = \frac{\partial x_j^a}{\partial p_k} - \frac{\partial x_k^a}{p_j} - \sum_{i \in \mathcal{I}} (v_j^i e_k^i - v_k^i e_j^i), \quad j, k \in \mathcal{L} \setminus \{1\}, j \neq k,$$

for pairs of distinct commodities other than the numeraire, are determined by the aggregate demand function.

By direct substitution and the symmetry of the matrices of substitution effects,

$$f_{j,k} = \sum_{i \in \mathcal{I}} (v_k^i x_j^i - v_j^i x_k^i).$$

Step 3 By direct computation,

$$v_k^{i'} x_j^i - v_j^{i'} x_k^i = \frac{\partial}{\partial t^i} f_{j,k},$$

$$v_k^{i''} x_j^i - v_j^{i''} x_k^i = \frac{\partial^2}{\partial (t^i)^2} f_{j,k} - (v_k^{i'} v_j^i - v_j^{i'} v_k^i),$$

where $v_l^{i'} = (\partial^2 v_l^i / \partial (t^i)^2)$ and $v_l^{i''} = (\partial^2 v_l^i / \partial (t^i)^2)$. The aggregate demand function identifies the functions of income effects of individuals and, hence, their derivatives, while the endowments of individuals are observable; at prices of commodities and allocation of endowments $(p, t^{\mathcal{I}})$ this is a system of $(L-1)(L-2)$ linear equations in the $(L-1)$ variables $x_2^i, \dots, x_L^i$.

Step 4 In order to identify the demand of an individual, it suffices to select a subset of the equations and show that the associated matrix of coefficients has full column rank.

Without loss of generality, $m = 2$, and $n = 3$,

The subset of equations is

$$v_k^{i'} x_2^i - v_2^{i'} x_k^i = \frac{\partial f_{2,k}}{\partial t^i},$$

$$v_3^{i''} x_2^i - v_2^{i''} x_3^i = \frac{\partial^2 f_{2,3}}{\partial (t^i)^2} - (v_3^{i'} v_2^i - v_2^{i'} v_3^i),$$

and the associated matrix of coefficients is

$$\begin{pmatrix} v_3^{i'} & -v_2^{i'} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots \\ v_k^{i'} & 0 & \dots & -v_2^{i'} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots \\ v_L^{i'} & 0 & \dots & 0 & \dots & -v_2^{i'} \\ v_3^{i''} & -v_2^{i''} & \dots & 0 & \dots & 0 \end{pmatrix}.$$

Since $v_2^{i'} \neq 0$, the matrix of coefficients is invertible if and only if the matrix

$$\begin{pmatrix} v_3^{i'} & -v_2^{i'} \\ v_3^{i''} & -v_2^{i''} \end{pmatrix}$$

has full rank. Indeed, the determinant is

$$\begin{aligned} & (-v_2^{i'} v_3^{i''})^{-1} \left(\frac{v_2^{i''}}{v_2^{i'}} - \frac{v_3^{i''}}{v_3^{i'}} \right) = \\ & - \left(\left(\frac{\partial^2 x_2^i}{\partial (t^i)^2} \right) \left(\frac{\partial^2 x_3^i}{\partial (t^i)^2} \right) \right)^{-1} \left(\frac{\partial}{\partial t^i} \left(\ln \frac{\partial^2 x_2^i}{\partial (t^i)^2} \right) - \frac{\partial}{\partial t^i} \left(\ln \frac{\partial^2 x_3^i}{\partial (t^i)^2} \right) \right) \neq 0. \end{aligned}$$

□

Remark The observation of the endowments of individuals is not important for the argument. What matters is that aggregate demand be expressed as a function of the incomes of individuals, in addition to the prices of commodities. This is an important observation if one wants to use this identification result as the basis of econometric estimation procedures: while data on individual endowments is not available there are a wide variety of data sources for individual incomes.

Equilibrium

Competitive equilibrium prices are such that $x^a(p, e^{\mathcal{I}}) - e^a = 0$.

The competitive equilibrium correspondence associates competitive equilibrium prices to profiles of endowments,

$$\omega(e^{\mathcal{I}}) = \{p : x^a(p, e^{\mathcal{I}}) - e^a = 0, \text{ and } p_1 = 1\}.$$

For ordinally equivalent profiles of utility functions, the competitive equilibrium correspondences coincide.

Proposition 2 *The competitive equilibrium correspondence, on an open set of endowments, identifies the profile of utility functions, up to ordinal equivalence, on the associated subsets of the consumption sets of individual.*

Proof It suffices that the competitive equilibrium correspondence identify the demand function of every individual.

The argument is developed in a sequence of steps.

The graph of the competitive equilibrium correspondence has the structure of a continuously differentiable manifold.

The tangent space to the competitive equilibrium manifold is defined by

$$dx^a = de^a,$$

and, as a consequence, by

$$\sum_{i \in \mathcal{I}} (S^i - v^i(x^i - e^i)') dp + \sum_{i \in \mathcal{I}} (v^i p - I) de^i = 0.$$

The competitive equilibrium correspondence determines the competitive equilibrium manifold and, consequently, everywhere, its tangent space.

Step 1 The tangent space of the equilibrium manifold identifies the income effects of every individual, v^i , and the matrix of aggregate price effects, $\sum_{i \in \mathcal{I}} (S^i - v^i(x^i - e^i)')$.

For every individual, the matrix $(v^i p - I)$ has rank $(L-1)$, since $p(v^i p - I) = 0$, while $y \in [p]^\perp \Rightarrow (v^i p - I)y = -y$. Hence, there exists a subspace of dimension 1 — it suffices that the dimension be greater than 0 — of vectors that satisfy $(v^i p - I)y = 0$. This identifies v^i , since $p \gg 0$ — it suffices that $p \neq 0$ — while $p v^i = 1$.

The rank of the matrix of aggregate price effects is at most equal to $(L-1)$, since $\sum_{i \in \mathcal{I}} (S^i - v^i(x^i - e^i)') p = 0$. Since, for every individual, the matrix $(v^i p - I)$ has rank $(L-1)$ — it suffices that this is the case for one individual — the space $\sum_{i \in \mathcal{I}} [(v^i p - I)]$ has dimension $(L-1)$ which identifies the matrix of aggregate price effects, since

$$\sum_{i \in \mathcal{I}} (S^i - v^i(x^i - e^i)') dp = - \sum_{i \in \mathcal{I}} (v^i p - I) de^i.$$

Step 2 For competitive equilibrium prices of commodities and allocations of endowments, the jacobian of the aggregate demand function

$$D_p x^a = \sum_{i \in \mathcal{I}} (S^i - v^i(x^i - e^i)'),$$

$$D_{e^i} x^a = v^i p,$$

is identified.

The functions $f_{j,k}$ for pairs of distinct commodities other than the numeraire are defined as in the proof of lemma 2.

For the identification of the demand functions of individuals it is required that the first and second derivatives of the functions $f_{j,k}$ with respect to the revenue of every individual, e_1^i , be identified by the competitive equilibrium manifold. It suffices, then, that, for every individual, i , the projection of the set of competitive equilibria to prices of commodities and endowments of individual i , be surjective.

The competitive equilibrium correspondence identifies the profile of utility functions on the set of consumption bundles for each individual that obtain at prices of commodities and allocations of endowments on the graph of the competitive equilibrium correspondence. $\square$

Remark If, for every individual and for every commodity, for any sequence, $((p_n, e_n^i) : n = 1, \dots)$, of prices of commodities and endowments,

$$\lim_{n \rightarrow \infty} e_{1,n}^i = \infty \quad \Rightarrow \quad \lim_{n \rightarrow \infty} x_{l,n}^i(p_n, e_n^i) = \infty, \quad l \in \mathcal{L} :$$

for every individual, every commodity is normal, in a strong sense, then the competitive equilibrium correspondence identifies the utility functions of individuals on their entire domain of definition. The argument is as follows:

Given $(\bar{p}, \bar{e}^i)$, prices of commodities and an endowment for an individual, i , there exist endowments $\bar{e}^2, \dots, \bar{e}^{i-1}, \bar{e}^{i+1}, \dots, \bar{e}^L$, such that $x^a(\bar{p}, \bar{e}^{\mathcal{I}}) = \bar{e}^a$.

It suffices to set the endowment of an individual $h \neq i$; the endowments of individuals $g \neq h, i$ are set arbitrarily, at $\bar{e}^g$. The solution, $x^h(p, \tau^h)$, to the optimization problem of individual h with the value of his endowment replaced by revenue, $\tau^h > 0$, exists, it is unique and strictly positive, and it satisfies $p x^h(p, \tau^h) = \tau^h$. For every commodity, $(\lim_{n \rightarrow \infty} \tau_n^h = \infty) \Rightarrow (\lim_{n \rightarrow \infty} x_{l,n}^h(p_n, \tau_n^h) = \infty)$. If $e^h(\tau^h) = \sum_{j \in \mathcal{I} \setminus \{h\}} x^j(\bar{p}, \bar{e}^j) + x^h(\bar{p}, \tau^h) - \sum_{j \in \mathcal{I} \setminus \{h\}} x^j$, then there exists $\bar{\tau}^h$, sufficiently large, such that $\bar{e}^h = e^h(\bar{\tau}^h) \gg 0$. But, then, $\bar{p} \in \omega(\bar{e}^{\mathcal{I}})$.

Remark Variation in individual endowments are important for identification from equilibrium prices. Nevertheless, from an econometric perspective, the result is of interest even if one can only observe the incomes of individuals. An estimation procedure may work, as long as it can be assumed that the observations of income arise from a sufficiently rich variation in endowments; in particular, in the limit, one needs to observe situations where the income of only one individual varies.

2.1 Restrictions

The identification of the profile of utility functions from the aggregate demand function or from the equilibrium correspondence makes no reference to the quasi - concavity of the utility functions of individuals. A fortiori, then, the aggregate demand function or the equilibrium correspondence of a profile of utility functions that satisfy assumption 2 is restricted. If a profile that fails to satisfy assumption 2 is used to generate an aggregate demand function or an equilibrium correspondence on an open set of prices of commodities and endowment allocations, it will generate an aggregate demand function or an equilibrium correspondence that cannot arise from a profile of utility functions that satisfy assumption 2. In addition, restrictions arise from the fact that individual Slutsky matrices are negative semi - definite. In our identification argument above, this fact is not used. Therefore it is possible to uniquely recover allocations without negative semidefiniteness of the Slutsky matrix. It follows that utility maximization imposes restrictions on these allocations and that there are strong local restrictions on the equilibrium manifold.

The previous restrictions obtain under a specific assumption, namely, that individual endowments are observable. This fact is quite interesting; it suggests, indeed, that testable restrictions require that some data are available at the individual level. One substantiate this claim by supposing that only aggregate endowments can be observed: do restrictions still exist? Quite obviously, the answer depends on the number of individuals: in the extreme case of one individual, equilibrium prices are support prices as the endowment varies, which implies Slutsky restrictions.

With more than one individual, the distribution scheme defined by $e^i = (1/I)e^a$ associates a profile of endwments, $e^{\mathcal{I}}(e^a)$, with an aggregate endowment e^a in an open set.

An equilibrium price function is defined by $p(e^a)$.

It follows from the rersults in Chiappori and Ekeland (1999, a) that, as long as the number of individuals is at least as large as the number of commodities : $I \geq L$, there exists a profile of utility functions, such that, locally,

$$x^a(p(e^a), e^{\mathcal{I}}(e^a)) = e^a;$$

equivalently, the equilibrium correspondence is unrestricted.

3 Implications and extensions

In applied general equilibrium, the preferences of individuals and the technologies of firms are chosen within a small parametric class to match empirical properties of data. The restriction to a given parametric class is viewed as

essential: failure of identification makes counterfactual policy analysis problematic: different models that match the data may produce different prediction — Hansen and Heckman (1996) discuss this in depth.

The argument here is that there is no failure of identification if aggregate consumptions, incomes and equilibrium prices are observable, along varying profiles of individual endowments. It implies that it is possible to estimate preferences econometrically, from data on prices and incomes, as in Brown and Matzkin (1990).

However, data on prices and incomes is likely to be in the form of time series. In a more applicable argument, one needs to take into account that for time series data, prices and incomes might be part of one, intertemporal equilibrium, and not points on an equilibrium correspondence. According to Kübler (1999), the assumption of time separable expected utility restores global in an intertemporal model. For identification, the assumption of separable utility is not enough, since sufficiently complete asset markets allow individuals to smooth their expenditure across dates and states of the world. Examining identification in an overlapping generations model without bequest is subject to further research.

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